

Follow these instructions carefully:

Work on the paper provided; do not use your own paper. *Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet).* On the top of each page, *print* your name (*encircle your last name*) and indicate the number of the problem you are working on by writing e.g. “*Problem #4*”. Always *encircle* your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing “*Answer to Part b*” (the number of the problem should be on the top of the page). Do not use a *red* pen or a *red* pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the *same* amount of credit. **Show all your work.**

1. Consider a random sample of a normal distribution with standard deviation 2 and unknown expectation μ (i.e., of the distribution $\mathcal{N}(\mu, 4)$):

44.1, 48.4, 41.5, 42.0.

The hypothesis

$$H_0 : \mu = 46.0$$

is to be tested against the alternative hypothesis

$$H_1 : \mu > 46.0$$

(one-sided test).

- a) Find the p -value of this test.
- b) Test H_0 at the level 5% of significance.
- c) Test H_0 at the level 1% of significance.

2. Consider a random sample of a normal distribution with unknown standard deviation and unknown expectation μ (i.e., of the distribution $\mathcal{N}(\mu, \sigma^2)$ with unknown σ):

44.1, 48.4, 41.5, 42.0.

The hypothesis

$$H_0 : \mu = 46.0$$

is to be tested against the alternative hypothesis

$$H_1 : \mu > 46.0$$

- a) Test H_0 at the level 5% of significance.
- b) Explain why it is not feasible to find the p -value of this test without a computer.

3. In a random sample of 400 persons eating lunch at a department store cafeteria, 260 persons had dessert. Construct a 90% confidence interval for the true proportion of persons usually eating dessert in this cafeteria.

4.a) Let P and Q be events, and let I_P , and I_Q be their indicator variables. Describe the event whose probability equals the expectation

$$E(1 - (1 - I_P)(1 - I_Q))$$

- b) Given events P and Q , use indicator variables to derive the formula for $P(P \cup Q)$.

5. Let $n > 0$ be an integer, and $X \sim \text{Bin}(n, x)$ be a binomial variable. Given $\delta > 0$, show that

$$\sum_{\substack{k: 0 \leq k \leq n \\ |k - xn| \geq n\delta}} \binom{n}{k} x^k (1-x)^{n-k} \leq \frac{1}{n\delta^2}.$$