

*Follow these instructions carefully:*

Work on the paper provided; do not use your own paper. *Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet).* On the top of each page, *print* your name (*encircle your last name*) and indicate the number of the problem you are working on by writing e.g. “*Problem #4*”. Always *encircle* your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing “*Answer to Part b*” (the number of the problem should be on the top of the page). Do not use a *red* pen or a *red* pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the *same* amount of credit. **Show all your work.**

1. Let  $A$  and  $B$  be two events such that  $P(A) = .6$ ,  $P(B) = .7$ , and  $P(A \cap B) = .4$ .

- a) Find  $P(A \cup B)$ .
- b) Find  $P(A \setminus B)$ .

2.a) From an urn containing 8 red balls and 6 green balls, six balls are taken without replacement. Determine the probability that 4 of the balls are red and 2 of them are green.

b) Give the probability if the same experiment is performed with replacement, and the same outcome is obtained.

3.a) In a factory, parts are manufactured by three machines,  $M_1$ ,  $M_2$ , and  $M_3$  in proportions 20 : 10 : 70. The percentages 6%, 3%, and 5% of these parts are defective, respectively. Find the probability that a randomly chosen part is defective.

b) Find the probability that a defective part was manufactured on the third machine.

4. A fair die is rolled on four times in a row. What is the probability that the number 5 or 6 is rolled at least once?

5.a) Let  $X$  be a random variable that can assume only positive integer values, and assume its probability function is  $P(X = n) = A/3^n$  for some constant  $A$  ( $n \geq 1$ ). Find  $A$ .

b) Let  $X$  be a continuous random variable that can assume values between 0 and 3, and assume its density function is  $f_X(x) = B(x^2 + 1)$  with some constant  $B$  ( $0 \leq x \leq 3$ ). Find  $B$ .