Exam 2, Mathematics 2501, Section TR3 30 copies
5:40–4:50 pm, October 28, 2021, IH-137
Instructor: Attila Máté

Follow these instructions carefully:

Work on the paper provided; do not use your own paper. *Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet).* On the top of each page, *print* your name (*encircle your last name*) and indicate the number of the problem you are working on by writing e.g. *"Problem #4".* Always *encircle* your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing *"Answer to Part b)"* (the number of the problem should be on the top of the page). Do not use a *red* pen or a *red* pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the *same* amount of credit. **Show all your work.**

1. Let X be the random variable with density function

$$f_X(x) = \begin{cases} 4x^3 & \text{if } 0 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

a) Find the expectation of X.
b) Find the variance of X.

2. 1. Let X be an variable having distribution $N(180, 25)$ (normal distribution with expectation 180 and standard deviation 5). Find the probability that *a*) $X \ge 177$, *b*) $X \le 186$, and *c*) $177 \le X \le 186$.

3. A pair of fair coins are tossed 1200 times. Let X denote the number of times both coins come up head.

a) Write a formula for the probability $P(X = k)$ $(0 \le k \le 1200)$.
b) Write the sum for the exact probability that $290 \le X \le 310$.
c) Use a well-known approximation to calculate the approximate probability that $290 \le X \le 310$.

4. A rare disease occurs in 0.03% of the population.

a) What is the probability that out of 10000 randomly chosen individuals, exactly 4 have the disease? Write the formula expressing the exact value of this probability, but do not evaluate.

b) Use a well-known approximation to approximately calculate the probability given in Part *a*).

5.*a*) On a certain night soon after sunset in March, the probability that an observer sees a meteor (falling star) in any given second is 6/3600 (note that an hour is 3600 seconds; one mentions seconds here, since one second is a very short time under the circumstances – so 6 is the hourly rate when one measures this rate in very short time intervals). What is the probability that this observer will see exactly k meteors in a given hour $(k \ge 0)$?

b) What is the probability that the same observer will see 4 meteors in a given hour.