Blackboard exam 4, Mathematics Mathematics 4701, Section TY3
Starts: 4:30 pm, Thurs, Apr 22; ends: 5:20 pm (late submission loses points).
Instructor: Attila Máté

Follow these instructions carefully:

Show all your work. Your explanations count for much more that simple correct answers. Your wording must be your own; using my words will not earn any credit; your explanations must indicate that you understand the material, not simply copy the explanations from somewhere else.

Do not change the notation in the question. Changing the notation can result in a serious loss of points. In important cases, you may get zero points for changing the notation. This is especially true for proofs that you may find online or in a textbook with a different notation. Changing the notation to a notation you may find in a publicly available source may be taken as evidence of illegitimate copying, and you may be penalized appropriately.

You must work on your own; collaboration will inevitably show up with similar wordings of the explanations and invalidate your answer. Clear signs of cheating will be taken seriously.

Blackboard allows, but will indicate, late submissions. In case of multiple submissions, only the last one will count.

1. Assume that

$$c_1 n^5 \cdot 8^n + c_2 n \cdot 4^n + c_3 n^3 \cdot 9^n = 0$$

for all $n > 0$. Show that then $c_1 = 0$.

2. Given

$$A = LU = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 8 & 4 \\ 0 & 3 & 6 \\ 0 & 0 & 7 \end{pmatrix},$$

write A as $L'U'$ such that L' is a lower triangular matrix and U' is an upper triangular matrix such that the elements in the main diagonal of U' are all 1's.

3. Consider the differential equation $y' = f(x, y)$ with initial condition $y(x_0) = y_0$. Show that with $x_1 = x_0 + h$ ($h > 0$), the solution at x_1 can be obtained within an error of $O(h^3)$ by the formula

$$y_1 = y_0 + \frac{h}{2} f(x_0, y_0) + \frac{h}{2} f\big(x_0 + h, y_0 + hf(x_0, y_0)\big)$$