# Exam 1, Mathematics 4701, Section TY2 <br> 1:25 pm-3:25 pm, Feb 28, 2019, IH-137 <br> Instructor: Attila Máté 

21 copies

Follow these instructions carefully:
Work on the paper provided; do not use your own paper. Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet). On the top of each page, print your name (encircle your last name) and indicate the number of the problem you are working on by writing e.g. "Problem \#4". Always encircle your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing "Answer to Part b)" (the number of the problem should be on the top of the page). Do not use a red pen or a red pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the same amount of credit. Show all your work.
1.a) Calculate $x-\sqrt{x^{2}-2}$ for $x=1,000,000$ with 6 significant digit accuracy. Avoid the loss of significant digits.
b) Find $1-\cos 0.009$ with 10 decimal digit accuracy.
2.a) Evaluate

$$
\sqrt{x+y^{2}}
$$

for $x=5 \pm 0.04$ and $y=2 \pm 0.08$.
$b)$ The leading term of the Newton interpolation polynomial $P$ to a function $f$ with the nodes $x_{0}, x_{1}$, $\ldots x_{n}$ is

$$
f\left[x_{0}, x_{1}, \cdots, x_{n}\right] x^{n} .
$$

Using this, show that

$$
f\left[x_{0}, x_{1}, \cdots, x_{n}\right]=\frac{f^{(n)}(\xi)}{n!}
$$

for some $\xi$ in the interval spanned by $x_{0}, x_{1}, \ldots x_{n}$. (All the nodes $x_{0}, x_{1}, \ldots x_{n}$ are assumed to be distinct.)
3.a) Find the Lagrange interpolation polynomial $P(x)$ such that $P(1)=-3, P(3)=-1, P(4)=3$.
b) Estimate the error of Lagrange interpolation when interpolating $f(x)=1 / x$ at $x=2$ when using the interpolation points $x_{1}=1, x_{2}=4$, and $x_{3}=5$.
4. Find the Newton-Hermite interpolation polynomial for $f(x)$ with $f(2)=4, f^{\prime}(2)=15, f(4)=10$, $f^{\prime}(4)=39, f^{\prime \prime}(4)=28$.
a) First, write the divided difference table, using the points 2,4 in natural order.
b) Using the divided difference table, write the Newton-Hermite interpolation polynomial using the order of points $2,2,4,4,4$.
c) Using the divided difference table, write the Newton-Hermite interpolation polynomial using the order of points $4,2,4,2,4$.
5. a) Consider the equation $f(x)=0$ with $f(x)=2-x+\ln x$. Using Newton's method with $x_{0}=3$ as a starting point, find the next approximation to the solution of the equation.
b) Evaluate the derivative of $P(x)=x^{3}-4 x^{2}+6 x+4$ at $x=2$ using Horner's method. Show the details of your calculation.
c) Let $P$ and $Q$ be polynomials, let $x_{0}$ and $r$ be a numbers, and assume that

$$
P(x)=\left(x-x_{0}\right) Q(x)+r .
$$

Show that $P^{\prime}\left(x_{0}\right)=Q\left(x_{0}\right)$.

