Exam 3, Mathematics 4701, Section TY2 21 copies
1:25 pm–3:25 pm, May 2, 2019, IH-137
Instructor: Attila Máté

Follow these instructions carefully:

Work on the paper provided; do not use your own paper. *Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet).* On the top of each page, *print* your name (*encircle your last name*) and indicate the number of the problem you are working on by writing e.g. *"Problem #4".* Always *encircle* your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing *"Answer to Part b)"* (the number of the problem should be on the top of the page). Do not use a *red* pen or a *red* pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the *same* amount of credit. **Show all your work.**

1. *a*) Given that $y_0 = 1$, $y_1 = 8$ and $y_{n+2} = 4y_{n+1} - 4y_n$ for every integer $n \geq 0$, write a formula expressing y_n.

b) Write a difference operator that annihilates all but the first term in the expression

$$c_1 n^7 \cdot 9^n + c_2 n^3 \cdot 6^n + c_3 n^5 \cdot 7^n,$$

while it reduces the first term to $c \cdot 9^n$, where c is a nonzero constant (it is assumed that $c_1 \neq 0$).

2. Solve the equation

$$\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} 5 & 4 & 3 \\ 0 & 4 & 1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 14 \\ -6 \end{pmatrix}.$$

3. *a*) Given

$$A = LU = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 6 & 9 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix},$$

write A as $L'U'$ such that L' is a lower triangular matrix and U' is an upper triangular matrix such that the elements in the main diagonal of U' are all 1's.

b) Explain briefly why pivoting works smoothly with the Crout algorithm (make a comparison with the Dolittle algorithm, for which pivoting does not work smoothly). Do not write the formulas used to calculate the matrix elements in these algorithms, only give a brief explanation using your own words.

4. *a*) Change the order of equations in the system

$$\begin{aligned} -3x + 2y + 7z &= \quad 3 \\ 8x - 3y - 2z &= -9 \\ 2x + 9y - 3z &= \quad 6 \end{aligned}$$

so that the resulting system can be solved by Gauss-Seidel iteration.

b) Write the equations describing the Gauss-Seidel iteration to solve the system of equations in Part *a*).

c) Write the equations describing Jacobi iteration to solve the system of equations in Part *a*).

5. *a*) Explain what an orthogonal matrix is.

b) Let $\mathbf{v}$ be an n-dimensional column vector such that $\mathbf{v}^T\mathbf{v} = 1$, and let I be the $n \times n$ identity matrix. Show that the matrix

$$H = H_{\mathbf{v}} = I - 2\mathbf{v}\mathbf{v}^T$$

is orthogonal.