JUNIOR PRIZE EXAM
SPRING 2010

1) Show that there is no integer whose square, written in the decimal system, ends in two odd digits.

2) Let $n$ be an arbitrary positive integer. Show that

$$n(n+2)(5n-1)(5n+1)$$

is divisible by 24.

3) Show that there is no triangle whose altitudes are of length 4, 7, and 10 units.

4) Show that

$$\sqrt[3]{\sqrt{5}+2}-\sqrt[3]{\sqrt{5}-2}=1.$$

5) How many nonnegative real solutions does the equation $x = 2000 \sin x$ have (of course, we are using radians here when evaluating $\sin x$). (**Note:** The approximation of $\pi$ up to five decimal places is 3.14159.)

6) If $r+s+t=3$, $r^2+s^2+t^2=1$ and $r^3+s^3+t^3=3$, compute $rst$.

7) Let $n$ be a positive integer, and assume we are given $2n-1$ irrational numbers. Prove that it is possible to select $n$ numbers $x_1, x_2, \ldots, x_n$ among them such that, given arbitrary nonnegative rational numbers $a_1, a_2, \ldots, a_n$, the number

$$\sum_{i=1}^{n} a_i x_i$$

is rational if and only if $a_1 = a_2 = \ldots = a_n = 0$.

SOON AFTER THE EXAM, SOLUTIONS WILL APPEAR ON THE WEB SITE
`http://www.sci.brooklyn.cuny.edu/~mate/prize09/index.html`

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All computer processing for this manuscript was done under Fedora Linux. $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX was used for typesetting.