Junior Prize Exam
Spring 2011

1) Prove that there is no two-digit positive integer that is equal to the product of its digits.

2) Let n be an integer. Show that $n^2 + 1$ is not divisible by 7.

3) Let x, y, z be positive numbers such that $x + y + z = 1$. Prove that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} > 3.$$

4) Given that the product of two of its roots of the equation

$$x^3 - 14x^2 + 59x - 70 = 0$$

is 10, solve the equation.

5) One places eighth rooks on a chessboard in such a way that none of the rooks can take any other. Prove that the number of rooks on black squares is even.

6) Prove that if $n > 2$ is an integer then

$$(2n - 1)^n + (2n)^n < (2n + 1)^n.$$

7) Let r and k be integers such that $r \geq 0$ and $0 \leq k \leq 2^r$. Show that $\binom{2^r-1}{k}$ is odd.

Soon after the exam, solutions will appear on the Web Site

http://www.sci.brooklyn.cuny.edu/~mate/prize11/index.html

All computer processing for this manuscript was done under Fedora Linux. The Perl programming language was instrumental in collating the problems. $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX was used for typesetting.