Junior Prize Exam
Spring 2024

1) Find all pairs of positive integers (x, y) such that $x^2 + y^2 = 17^2$.

2) In a manuscript of 60 sheets, the pages are numbered by the numbers $1, 2, 3, \ldots, 120$ in the usual way. Unfortunately, some sheets were lost. The sum of the page numbers on the remaining pages is 7159. How many sheets were lost?

3) Given a 5×5 matrix (a list of numbers, called entries, arranged in a rectangle with 5 rows and 5 columns), with each of the entries being 1 or -1. Form the products of the entries on each of the rows and each of the columns, obtaining 10 products altogether. Show that the sum of all these products cannot be 0.

4) Given a triangle such that all its altitudes are at least 1. Show that its area is at least $1/\sqrt{3}$.

5) Given 50 positive integers whose sum is at least 100, show that it is possible to choose 3 of them whose sum is at least 6.

6) At a dinner party, there are 6 guests, among whom everybody knows at least one other guest (knowing another is symmetric, so if A knows B, then B also knows A). Among the first five guests, each one knows a different number of other guests. How many guests does the sixth guest know?

7) One places dominoes on a 6×6 chess board in such a way that every field is covered and no dominoes overlap. Each domino completely covers two adjacent fields. Show that among the 5 horizontal and 5 vertical lines separating the fields, there is at least one that is not cut by any dominoes.

Soon after the exam, solutions will appear on the Web Site
`http://www.sci.brooklyn.cuny.edu/~mate/prize/2024/`

All computer processing for this manuscript was done under Debian Linux. The Perl programming language was instrumental in collating the problems. $\mathcal{AMS}$-TEX was used for typesetting.