

JUNIOR PRIZE EXAM
SPRING 2025

- 1) Let α , β , and γ be such that $\alpha + \beta + \gamma = \pi$. Prove that

$$\frac{\sin \alpha + \sin \beta - \sin \gamma}{\sin \alpha + \sin \beta + \sin \gamma} = \tan \frac{\alpha}{2} \tan \frac{\beta}{2}.$$

- 2) What is the remainder if 18^{82} is divided by 11?

- 3) Determine a and b in the equation

$$x^3 + ax^2 + bx + 6 = 0$$

such that one of its roots be 2, and another be 3.

- 4) Determine the value of p in the equation $x^2 - px + 36 = 0$ such that we have

$$\frac{1}{x_1} + \frac{1}{x_2} = \frac{5}{12}$$

holds for the roots x_1 and x_2 of the equation.

- 5) Let a , b , and c be real numbers such that $a^2 + b^2 + c^2 = 1$. Show that

$$-\frac{1}{2} \leq ab + bc + ca \leq 1.$$

- 6) Show that every positive integer that is not divisible by 10 has a multiple that begins and ends with the same digit when written in the decimal system (without starting zeros).

- 7) How many ways can you arrange the numbers 0, 1, 2, 3, 4, 5, and 6, in a sequence such that the sum of any four consecutive ones among them is divisible by 3.

SOON AFTER THE EXAM, SOLUTIONS WILL APPEAR ON THE WEB SITE

<http://www.sci.brooklyn.cuny.edu/~mate/prize/2025/>