Senior Prize Exam
Spring 2025

1) Let α, β, and γ be such that $\alpha+\beta+\gamma=\pi$. Prove that

$$\frac{\sin\alpha+\sin\beta-\sin\gamma}{\sin\alpha+\sin\beta+\sin\gamma}=\tan\frac{\alpha}{2}\tan\frac{\beta}{2}.$$

2) What is the remainder if 18^{82} is divided by 11?

3) Determine a and b in the equation

$$x^3+ax^2+bx+6=0$$

such that one of its roots be 2, and another be 3.

4) Assume all but one edges of a tetrahedron have length ≤ 1 (no restriction is placed on the length of the remaining one edge). Prove that the volume of the tetrahedron is $\le 1/8$.

5) Let ABC be a triangle such that the coordinates of its vertices are integers, and its area is a positive integer (taking the area of the unit square of the coordinate system to be 1), Show that the midpoint of one of the sides of the triangle has integer coordinates.

6) Show that the integral

$$\int_0^{\sqrt{\pi}}\cos x^2\,dx$$

is positive.

7) Let f be a real-valued function of two real variables such that all second order partial derivatives of f exist and are continuous everywhere. Assume that $f(x,y)=0$ whenever $xy=0$. Show that there is a constant $C>0$ such that

$$|f(x,y)|\le C|x||y| \quad \text{whenever} \quad |x|,|y|\le 1.$$

Soon after the exam, solutions will appear on the Web Site
http://www.sci.brooklyn.cuny.edu/~mate/prize/2025/

All computer processing for this manuscript was done under Debian Linux. The Perl programming language was instrumental in collating the problems. $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX was used for typesetting.