Section 1.5 - Functions

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1 Definition of a function

1.1 Definition

Definition: A function f assigns to each element $x \in S$ a unique value $y \in T$. We say that f is defined on S with values in T. We sometimes write this $f : S \to T$. We say that $S = Dom(f)$ (read, f is the domain of f). We also write $f(x) = y$. $\{y = f(x) : x \in Dom(f)\} = Im(f) =$The image of f.
It is often convenient to specify a set of allowable images for a function f, such as set T such that $Im(f) \subseteq T$. This set T is called the codomain of f.

1.2 Examples

1. $f_1 : \mathbb{R} \to \mathbb{R}. \quad f_1(x) = x^2.$
2. $f_2 : [0, \infty) \to \mathbb{R}. \quad f_2(x) = \sqrt{x}.$
3. $f_3 : (0, \infty) \to \mathbb{R}. \quad f_3(x) = \log_2(x).$
4. $f_4 : \mathbb{R} \to \mathbb{R}. \quad f_4(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & \text{otherwise} \end{cases}$
5. $f_5 : \mathbb{R} \to \mathbb{Z}. \quad f_5(x) = \lfloor x \rfloor =$ The largest integer $\leq x$.
6. $f_6 : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}. \quad f_6(m, n) = \left\lfloor \frac{n}{2} \right\rfloor - \left\lfloor \frac{m-1}{2} \right\rfloor.$

2 More Definitions

Definition: The graph of a function f is a subset of $S \times T$ such that $Graph(f) = \{(x, y) \in S \times T : y = f(x)\}$.

Example: The graph of $g : \{0, 1, 2, 3\} \to \mathbb{N}, g(n) = n^2$ is $Graph(g) = \{(0, 0), (1, 1), (2, 4), (3, 9)\}$.

Definition: Consider a set S and another set $A \subseteq S$ then the characteristic function of A, denoted $\chi_A = \begin{cases} 1 & x \in A \\ 0 & \text{otherwise} \end{cases}$

Example: Let $A = \{n \in \mathbb{N} : n \text{ is odd } \}$ and let $S = \mathbb{N}$. Then $\chi_A = \begin{cases} 1 & x \text{ is odd} \\ 0 & x \text{ is even} \end{cases}$
What does this remind you of?

Definition: Consider functions $f : S \to T, g : T \to U$. We define $g \circ f : S \to U$ by the rule $(g \circ f)(x) = g(f(x)) \; \forall x \in S$.

TODO: Do examples