Section 1.6 - Sequences

Ari Mermelstein

February 5, 2018

1 Overview of Sequences

Consider functions who domain is $\mathbb{N}$ or $\mathbb{P}$. Notice that we can view these functions as a list of things (because they are not continuous, i.e. their values can be listed in a numerical order). For example, if we have a function $f : \mathbb{N} \to \mathbb{N} : f(n) = n^2$, we can list the elements of this function in order. $(0, 1, 4, 9, 16, \cdots)$. We therefore call these functions **sequences**.

2 Examples of Sequences

2.1 Sums

We can have sequences based around sums. We use the Greek capital letter Sigma (Σ) represent a summation. For example, consider:

$$\sum_{i=1}^{4} i = 1 + 2 + 3 + 4 = 10$$

or in general:

$$\sum_{i=1}^{n} i$$

Notice that wherever we stop, we get different answers.

We can also look at more general sums such as:

$$\sum_{k=1}^{n} a_k$$

2.2 Products

We can have sequences based around products also. The analogue for products uses a large capital pi, (Π). For example, consider:

$$\prod_{i=1}^{n} i = n! = 1 \times 2 \times 3 \cdots \times n$$

.

3 Formal Definition of a Sequence

Definition: A sequence is an infinite string of objects that can be listed using subscripts from a subset of $\mathbb{N}$. A sequence on $\mathbb{N}$ is a list $(s_0, s_1, s_2, \cdots, s_n, \cdots)$ where s_n is called the n$^{\text{th}}$ term of the sequence (s_n). Occasionally, people write $S(n)$ or $S[n]$ instead. (Remind you guys of anything?)

3.1 Examples

1. $FACT(n) = n! = (1, 1, 2, 6, 24, 120, \cdots)$.
2. $TWO(n) = 2^n = (1, 2, 4, 8, 16, 32, 64, 128, \cdots)$.
3. $(b_n)_{n\in\mathbb{P}}$ given by $b_n = \dfrac{1}{n^2}$.
4. $(s_n)_{n\in\mathbb{P}}$ given by $s_n = \log_2 n$.