Section 3.1 - Relations

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1 Relations

1.1 Definitions

Definition: A relation R from a set S to a set T is a subset of $S \times T$. I.e. it's a set of ordered pairs.

Definition: Consider $s \in S$, $t \in T$. We say that s is related to t under R if $(s, t) \in \mathbb{R}$. We can also just write $s\ R\ t$.

1.2 Notes

NB: A relation doesn't have to be a function.

If S and T are not the same, then the types of relations we're talking about associate one type of a data with another. You can think of setting up a mapping from one type to another (e.g. a map [or possibly a multimap] data structure). However, if $S = T$, were talking about a relation on a set S, between 2 elements of S.

2 Properties of Relations on a Set

1. **Reflexive**: $\forall x$ x R x
2. **Antireflexive**: x R x is never true.
3. **Symmetric**: $\forall x, y\ xRy \implies yRx$
4. **Antisymmetric**: $\forall x, y\ xRy \ \land\ yRx$ only if $x = y$.
5. **Transitive**: $\forall x, y, z\ xRy\ \land\ yRz \implies xRz$

3 Examples

1. $\leq$
2. $<$
3. congruence modulo p with respect to $\mathbb{Z}$. In other words m is congruent to n modulo p if m-n is a multiple of p.

4 Converse Relation

Definition: Consider a relation $R \subseteq (S \times T)$. Then the converse relation, $R^{\leftarrow}$ is the relation from T to S given by $(t, s) \in R^{\leftarrow} \iff (s, t) \in R$.