Section 3.2 - Graphs and Digraphs

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1 Digraphs

1.1 Definitions

Definition: A Digraph (or directed graph) is pair G=(V, E) where V is a set of vertices (or nodes) and E is a set of (directed)edges. There is also a function $\gamma : E \to V \times V$ that tells you which pair of vertices the label in E corresponds to. It is called directed, because each edge has an initial vertex and a terminal vertex. In other words for all $e \in E$, $\gamma(e) = (x, y)$ for $x, y \in V$.

N.B. Most people define the set of Edges in terms of it's endpoints as part of the definition and don't bother with the gamma function.

1.2 Examples

2 More definitions

Definition: A path in a digraph is a sequence of edges such that the terminal vertex of one edge is the initial vertex of the next edge. The length of a path is the number of edges on it. (some people describe a path in terms of vertices where the edges are understood.) A path is said to be closed if the first vertex visited is same as the last vertex. **Definition**: A cycle is a path of length at least 1 which starts and ends at the same vertex, with no vertices repeated in between.

Definition: A digraph is acyclic if it contains no cycles. Some people call this type of graph a DAG (directed, acyclic graph).

3 Graphs

Definition: A graph is a pair G = (V,E) where V is a set of vertices and E is a set of (undirected) edges. There is also a function $\gamma : e \to \{\{x, y\} : x, y \in V\}$. In other words, for all $e \in E$, $\gamma(e) = \{x, y\}$. In other words, each edge corresponds to a pair of vertices. Conceptually, you can travel from x to y or from y to x.