Section 3.3 - Matrices

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1 Definition

Definition: A matrix is a rectangular (2D) array that stores elements. It is customary to name matrices by capital letters. We denote entries inside the matrix by their indices. For example, if I write $\mathbf{A} = [a_{ij}]$, then I mean that each entry of A can be singled out by saying "a" followed by its row and column number.

1.1 Example:

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 & 3 & 2 \\ 1 & -2 & 1 & -1 & 3 \\ 3 & 0 & 1 & 2 & -3 \end{bmatrix} = [a_{ij}]$$

Find a_{11}, a_{23}, a_{31}.

1.2 Other Notation

We can also use the notation $A[i,j]$ to single out elements.

Example:

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}.$$

Find $B[1,1], B[2,3]$.

2 Describing sets of matrices

We use the notation $\mathcal{M}_{m,n}$ to denote the set of all matrices with m rows and n columns. If a matrix has m rows and n columns, we say that the matrix is a $m \times n$ matrix.

A matrix in $\mathcal{M}_{1,n}$ is called a row vector and a matrix in $\mathcal{M}_{m,1}$ is called a column vector.

3 Equality

We say that 2 matrices ($\mathbf{A} = [a_{ij}]$) is equal to $\mathbf{B} = [b_{ij}]$ if $\mathbf{A}$ and $\mathbf{B}$ has the same dimensions, and $a_{ij} = b_{ij}\ \forall\ 1 \leq i \leq m$ and $1 \leq j \leq n$.

4 Addition of Matrices

Let $\mathbf{A}, \mathbf{B} \in \mathcal{M}_{m,n}$. Then $\mathbf{A} + \mathbf{B}[i,j] = A[i,j] + B[i,j]\ \forall\ 1 \leq i \leq m$ and $1 \leq j \leq n$.

Example:

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 0 \\ -1 & 3 & 2 \\ -3 & 1 & 2 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 1 & 0 & 5 & 3 \\ 2 & 3 & -2 & 1 \\ 4 & -2 & 0 & 2 \end{bmatrix} \mathbf{C} = \begin{bmatrix} 3 & 1 & -2 \\ -5 & 0 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

Find $\mathbf{A}+\mathbf{C}, \mathbf{B}+\mathbf{B}$.

5 Scalar Product

The product of a matrix **A** and a scalar c is a matrix $\mathbf{cA}[i,j] = c\mathbf{A}[i,j]$ ex. find **2C**.

6 Transpose of a Matrix

6.1 Definition

<u>**Definition**</u>: Given a matrix $A \in \mathcal{M}_{m,n}$, the transpose of A, denoted A^T is a new matrix $A^T \in \mathcal{M}_{n,m}$ such that the ith column of A^T is not the ith row and vice versa.

6.2 Example

For B above, $B^T = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & -2 \\ 5 & 2 & 0 \\ 3 & 1 & 2 \end{bmatrix}$.

7 Matrix multiplication

Only defined for pairs of matrices where the number of columns of the first matches the number of rows of the second. For example, you can multiply $\mathbf{A} \in \mathcal{M}_{m,n}$ with $\mathbf{B} \in \mathcal{M}_{n,p}$. The result is an $m \times p$ matrix. **Beware**: Matrix multiplication is **not(!!)** commutative. Even if **AB** and **BA** are both defined, you may not get the same answer for both. Matrix multiplication is **associative** however.

Suppose **A** is an $m \times n$ matrix and **B** is an $n \times p$ matrix. then **AB** is an $m \times p$ matrix such that:

$\mathbf{AB}_{i,k} = \sum_{j=1}^{n} A[i,j] * B[j,k]$.

In other words, to find entry ik we need to multiply and add row i in A with column k in B.

7.1 example:

$\mathbf{A} = \begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

Find $\mathbf{AB}, \mathbf{BA}$.

Do examples.

8 Adjacency Matrices

<u>**Definition**</u>: An adjacency matrix of a graph $G = (V, E, \gamma)$ is a matrix $A = [a_{ij}] \in \mathcal{M}_{|V|,|V|}$ such that A[i,j] = the number of edges from vertex i to vertex j.

Do examples.