

CISC 2210 Spring 2018 Quiz 3

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1 Write each of the following in Predicate Notation (1 point each)

Make sure that you use quantifiers properly and bind all variables.

1. For all $x \in \mathbb{R}$, there exists $\frac{1}{x}$ such that $(x)^{\frac{1}{x}} = 1$.
2. There exists $n \in \mathbb{N}$ such that for all $m \in \mathbb{N}$, $m < n$.

2 Determine the truth values of the following: (2 points each)

1. $\forall x \in \mathbb{R} \exists y \in \mathbb{R} x > y$.
2. $\exists x \in \mathbb{R} \exists y \in \mathbb{R} 2x = y$.

3 Write the following without using a $\neg$ in front of the notation (2 points each)

Let C = the set of Brooklyn College students.

1. It is not true that all Brooklyn College students are CS majors.
2. It's not true that some Brooklyn College students don't take any classes.

4 Infinite sets (5 points)

1. Prove that $\{n \in \mathbb{N} : n \text{ is odd}\}$ has the same cardinality as $\mathbb{N}$.
2. Consider the family of functions $\{f \mid f : \mathbb{N} \rightarrow \{0, 1\}\}$. That is the set of all Boolean functions. Prove that this set is not countable. In other words, show that it does **not** have the same cardinality as $\mathbb{N}$.
(Hint: Try to list these functions as a table. One side of the table is numbers from $\mathbb{N}$ and the other is a list of what each function does on that input. Make the same diagonal argument as we did for $\mathbb{R}$.) Please be very specific about what goes wrong when you list it as a table. Be as formal as you can.