

Chapter 4.3

Real-time Game Physics



Outline

- Introduction
 - Motivation for including physics in games
 - Practical development team decisions
- Particle Physics
 - Particle Kinematics
 - Closed-form Equations of Motion
- Numerical Simulation
 - Finite Difference Methods
 - Explicit Euler Integration
 - Verlet Integration
- Brief Overview of Generalized Rigid Bodies
- Brief Overview of Collision Response
- Final Comments

Real-time Game Physics

Introduction



Why Physics?

- The Human Experience
 - Real-world motions are physically-based
 - Physics can make simulated game worlds appear more natural
 - Makes sense to strive for physically-realistic motion for some types of games
- Emergent Behavior
 - Physics simulation can enable a richer gaming experience



Why Physics?

- Developer/Publisher Cost Savings
 - Classic approaches to creating realistic motion:
 - Artist-created keyframe animations
 - Motion capture
 - Both are labor intensive and expensive
 - Physics simulation:
 - Motion generated by algorithm
 - Theoretically requires only minimal artist input
 - Potential to substantially reduce content development cost



High-level Decisions

- Physics in Digital Content Creation Software:
 - Many DCC modeling tools provide physics
 - Export physics-engine-generated animation as keyframe data
 - Enables incorporation of physics into game engines that do not support real-time physics
 - Straightforward update of existing asset creation pipelines
 - Does not provide player with the same emergent-behavior-rich game experience
 - Does not provide full cost savings to developer/publisher



High-level Decisions

- Real-time Physics in Game at Runtime:
 - Enables the emergent behavior that provides player a richer game experience
 - Potential to provide full cost savings to developer/publisher
 - May require significant upgrade of game engine
 - May require significant update of asset creation pipelines
 - May require special training for modelers, animators, and level designers
 - Licensing an existing engine may significantly increase third party middleware costs



High-level Decisions

- License vs. Build Physics Engine:
 - License middleware physics engine
 - Complete solution from day 1
 - Proven, robust code base (in theory)
 - Most offer some integration with DCC tools
 - Features are always a tradeoff



High-level Decisions

- License vs. Build Physics Engine:
 - Build physics engine in-house
 - Choose only the features you need
 - Opportunity for more game-specific optimizations
 - Greater opportunity to innovate
 - Cost can be easily be much greater
 - No asset pipeline at start of development

Real-time Game Physics

The Beginning: Particle Physics



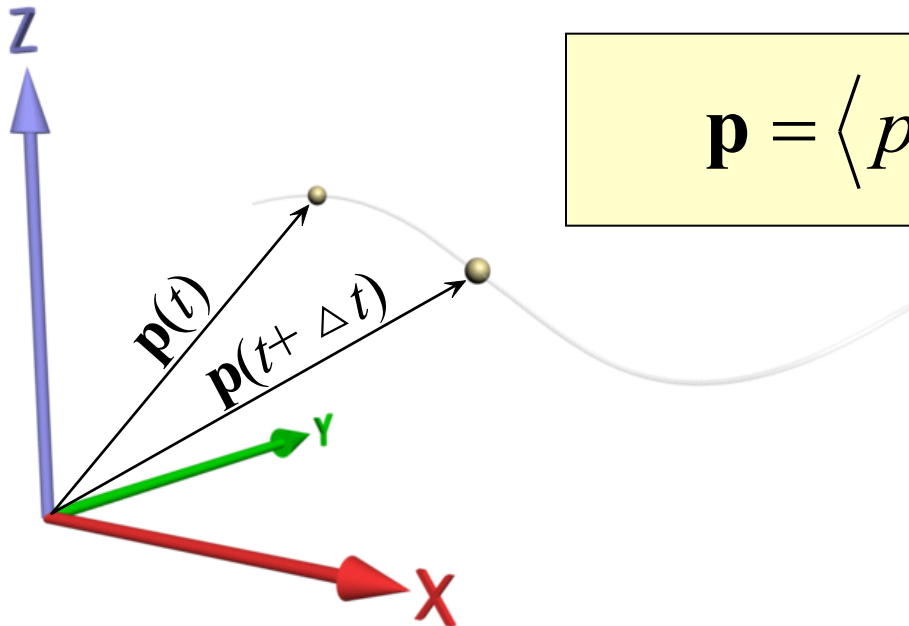
The Beginning: Particle Physics

- What is a Particle?
 - A sphere of finite radius with a perfectly smooth, frictionless surface
 - Experiences no rotational motion
- Particle Kinematics
 - Defines the basic properties of particle motion
 - Position, Velocity, Acceleration



Particle Kinematics - Position

- Location of Particle in World Space
 - SI Units: meters (m)



- Changes over time when object moves



Particle Kinematics - Velocity and Acceleration

- Velocity (SI units: m/s)
 - First time derivative of position:

$$\mathbf{V}(t) = \lim_{\Delta t \rightarrow 0} \frac{\mathbf{p}(t + \Delta t) - \mathbf{p}(t)}{\Delta t} = \frac{d}{dt} \mathbf{p}(t)$$

- Acceleration (SI units: m/s²)
 - First time derivative of velocity
 - Second time derivative of position

$$\mathbf{a}(t) = \frac{d}{dt} \mathbf{V}(t) = \frac{d^2}{dt^2} \mathbf{p}(t)$$



Newton's 2nd Law of Motion

- Paraphrased – “An object's change in velocity is proportional to an applied force”
- The Classic Equation:

$$\mathbf{F}(t) = m\mathbf{a}(t)$$

- m = mass (SI units: kilograms, kg)
- $\mathbf{F}(t)$ = force (SI units: Newtons)



What is Physics Simulation?

- The Cycle of Motion:
 - Force, $\mathbf{F}(t)$, causes acceleration
 - Acceleration, $\mathbf{a}(t)$, causes a change in velocity
 - Velocity, $\mathbf{V}(t)$ causes a change in position
- Physics Simulation:
 - Solving variations of the above equations over time to emulate the cycle of motion



Example: 3D Projectile Motion

- Constant Force
 - Weight of the projectile, $\mathbf{W} = m\mathbf{g}$
 - \mathbf{g} is constant acceleration due to gravity
- Closed-form Projectile Equations of Motion:

$$\mathbf{V}(t) = \mathbf{V}_{init} + \mathbf{g}(t - t_{init})$$

$$\mathbf{p}(t) = \mathbf{p}_{init} + \mathbf{V}_{init}(t - t_{init}) + \frac{1}{2}\mathbf{g}(t - t_{init})^2$$

- These closed-form equations are valid, *and exact**, for any time, t , in seconds, greater than or equal to t_{init}



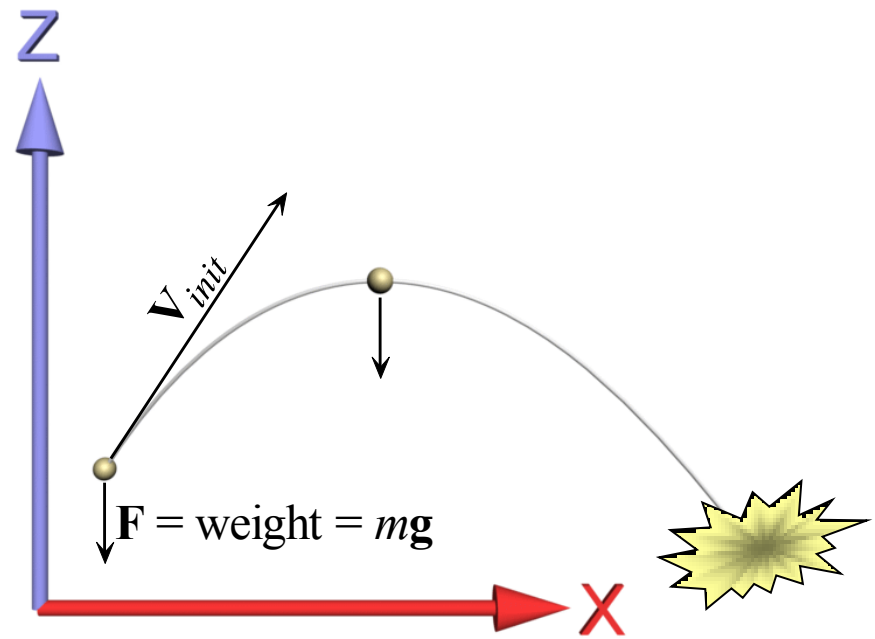
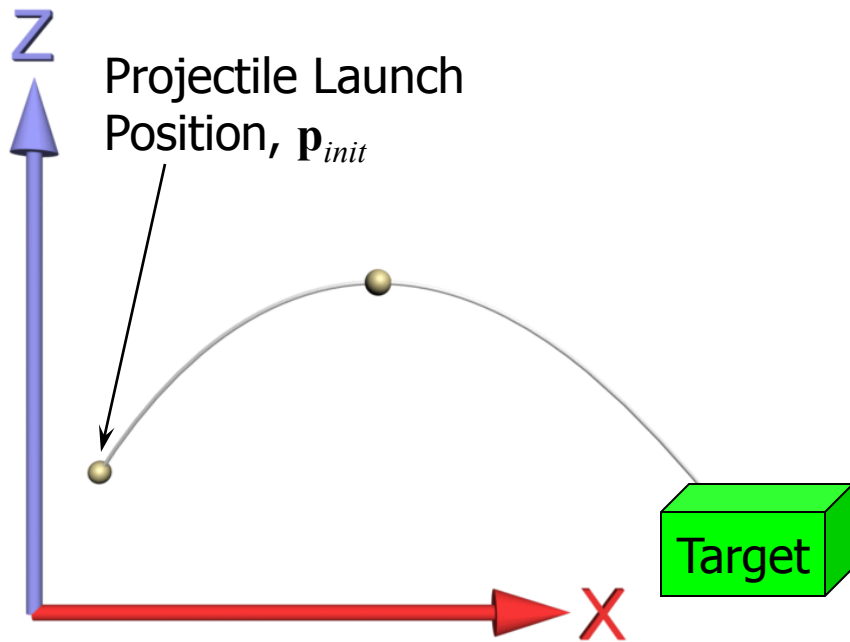
Example: 3D Projectile Motion

- Initial Value Problem
 - Simulation begins at time t_{init}
 - The initial velocity, \mathbf{V}_{init} and position, \mathbf{p}_{init} , at time t_{init} , are known
 - Solve for later values at any future time, t , based on these initial values
- On Earth:
 - If we choose positive Z to be straight up (away from center of Earth), $g_{Earth} = 9.81 \text{ m/s}^2$:

$$\mathbf{g}_{Earth} = -g_{Earth} \hat{k} = \langle 0.0, 0.0, -9.81 \rangle \text{ m/s}^2$$



Concrete Example: Target Practice





Concrete Example: Target Practice

- Choose \mathbf{V}_{init} to Hit a Stationary Target
 - \mathbf{p}_{target} is the stationary target location
 - We would like to choose the initial velocity, \mathbf{V}_{init} , required to hit the target at some future time, t_{hit} .
 - Here is our equation of motion at time t_{hit} :

$$\mathbf{p}_{target} = \mathbf{p}_{init} + \mathbf{V}_{init} (t_{hit} - t_{init}) + \frac{1}{2} \mathbf{g} (t_{hit} - t_{init})^2$$

- Solution in general is a bit tedious to derive...
- Infinite number of solutions!
- Hint: Specify the magnitude of \mathbf{V}_{init} , solve for its direction



Concrete Example: Target Practice

- Choose Scalar launch speed, V_{init} , and Let:

$$\mathbf{V}_{init} = \langle V_{init} \cos \theta \cos \phi, V_{init} \sin \theta \cos \phi, V_{init} \sin \phi \rangle$$

- Where:

$$\cos \theta = \frac{p_{target,x} - p_{init,x}}{\sqrt{(p_{target,x} - p_{init,x})^2 - (p_{target,y} - p_{init,y})^2}} ; \quad \sin \theta = \frac{p_{target,y} - p_{init,y}}{\sqrt{(p_{target,x} - p_{init,x})^2 - (p_{target,y} - p_{init,y})^2}}$$

$$\tan \phi = \frac{A \pm \sqrt{A^2 - 2g \left(\frac{A}{V_{init}} \right)^2 \left(\frac{1}{2} g \left(\frac{A}{V_{init}} \right)^2 + p_{target,z} - p_{init,z} \right)}}{g} \left(\frac{V_{init}}{A} \right)^2$$

$$A = \frac{(p_{target,y} + p_{target,x}) - (p_{init,y} + p_{init,x})}{(\cos \theta + \sin \theta)}$$



Concrete Example: Target Practice

- If Radicand in $\tan\phi$ Equation is Negative:
 - No solution. V_{init} is too small to hit the target

$$\text{if } \left(A^2 - 2g \left(\frac{A}{V_{init}} \right)^2 \left(\frac{1}{2} g \left(\frac{A}{V_{init}} \right)^2 + p_{target,z} - p_{init,z} \right) \right) < 0, \text{ then no solution!}$$

- Otherwise:
 - One solution if radicand == 0
 - If radicand > 0, TWO possible launch angles, ϕ
 - Smallest ϕ yields earlier time of arrival, t_{hit}
 - Largest ϕ yields later time of arrival, t_{hit}

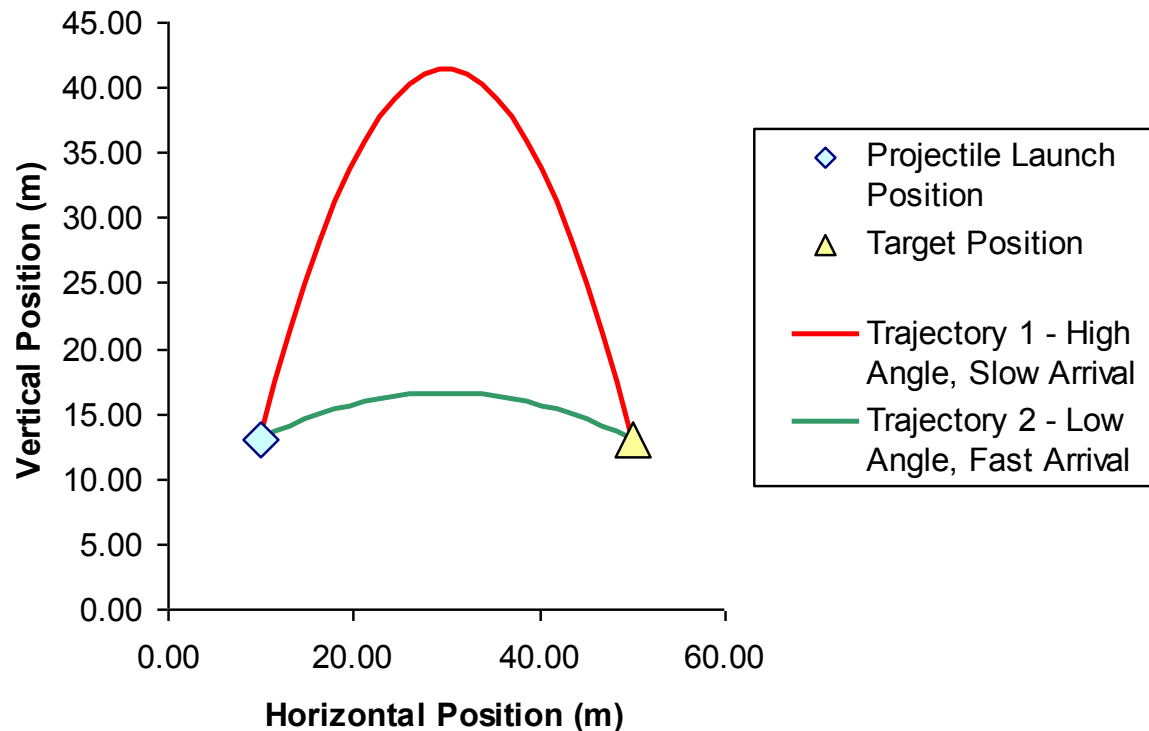


Target Practice – A Few Examples

$$V_{init} = 25 \text{ m/s}$$

Value of Radicand of $\tan\phi$ equation: **969.31**

Launch angle ϕ : 19.4 deg or 70.6 deg



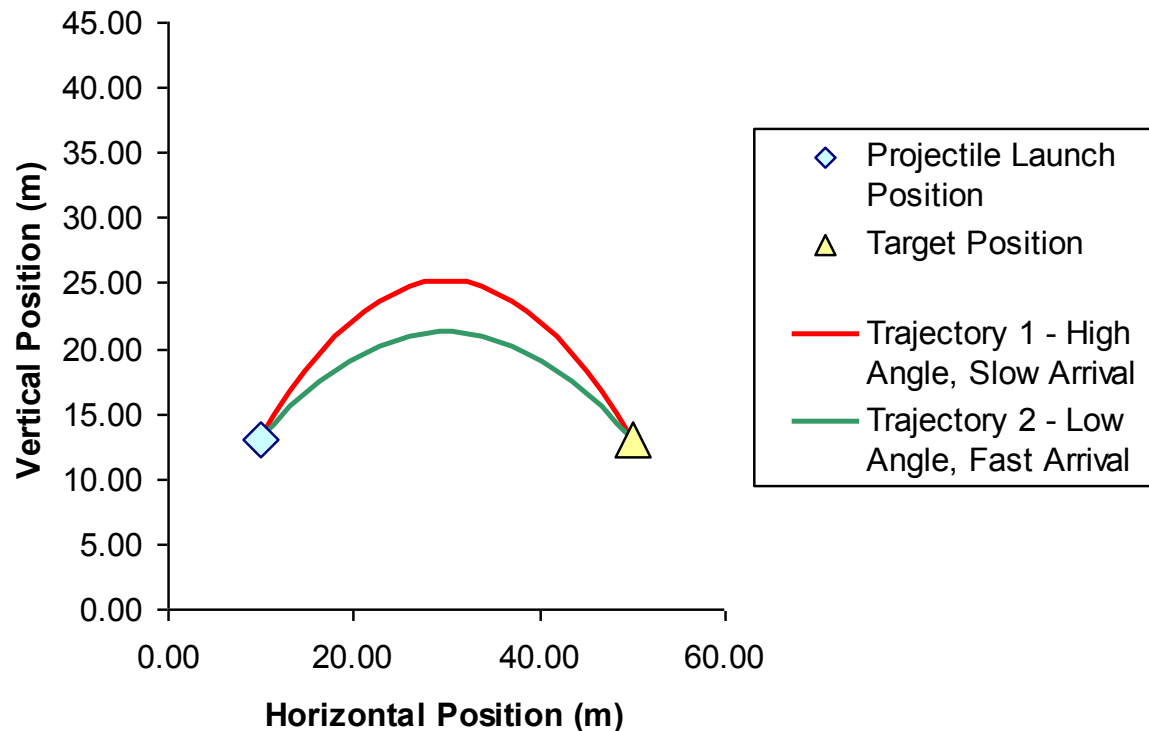


Target Practice – A Few Examples

$$V_{init} = 20 \text{ m/s}$$

Value of Radicand of $\tan\phi$ equation: **60.2**

Launch angle ϕ : 39.4 deg or 50.6 deg

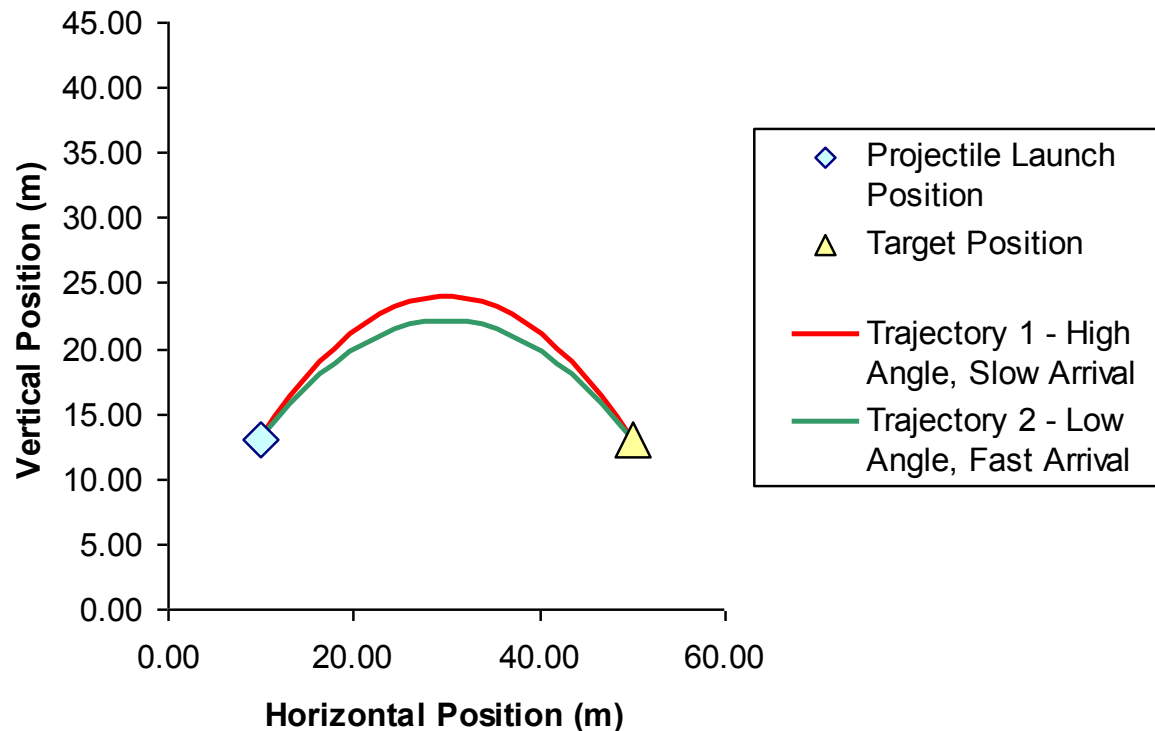


Target Practice – A Few Examples

$$V_{init} = 19.85 \text{ m/s}$$

Value of Radicand of $\tan \phi$ equation: **13.2**

Launch angle ϕ : 42.4 deg or 47.6 deg (note convergence)

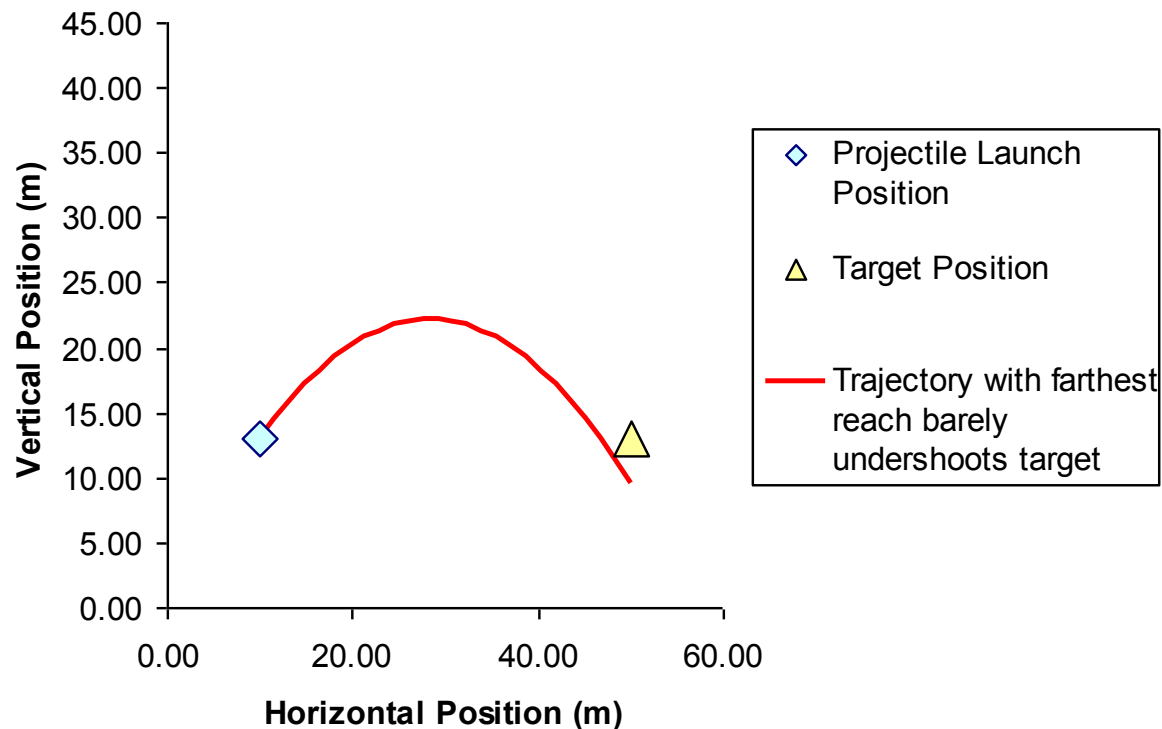


Target Practice – A Few Examples

$$V_{init} = 19 \text{ m/s}$$

Value of Radicand of $\tan\phi$ equation: **-290.4**

Launch angle ϕ : No solution! V_{init} too small to reach target!



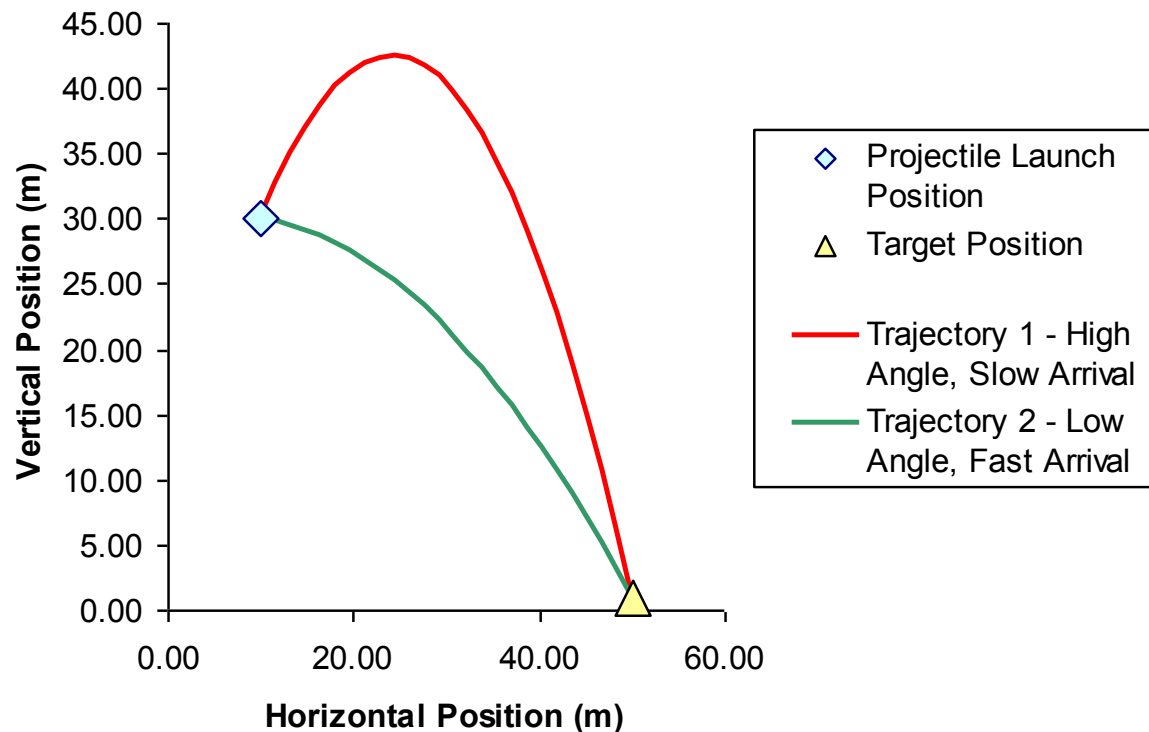


Target Practice – A Few Examples

$$V_{init} = 18 \text{ m/s}$$

Value of Radicand of $\tan\phi$ equation: **2063**

Launch angle ϕ : -6.38 deg or 60.4 deg



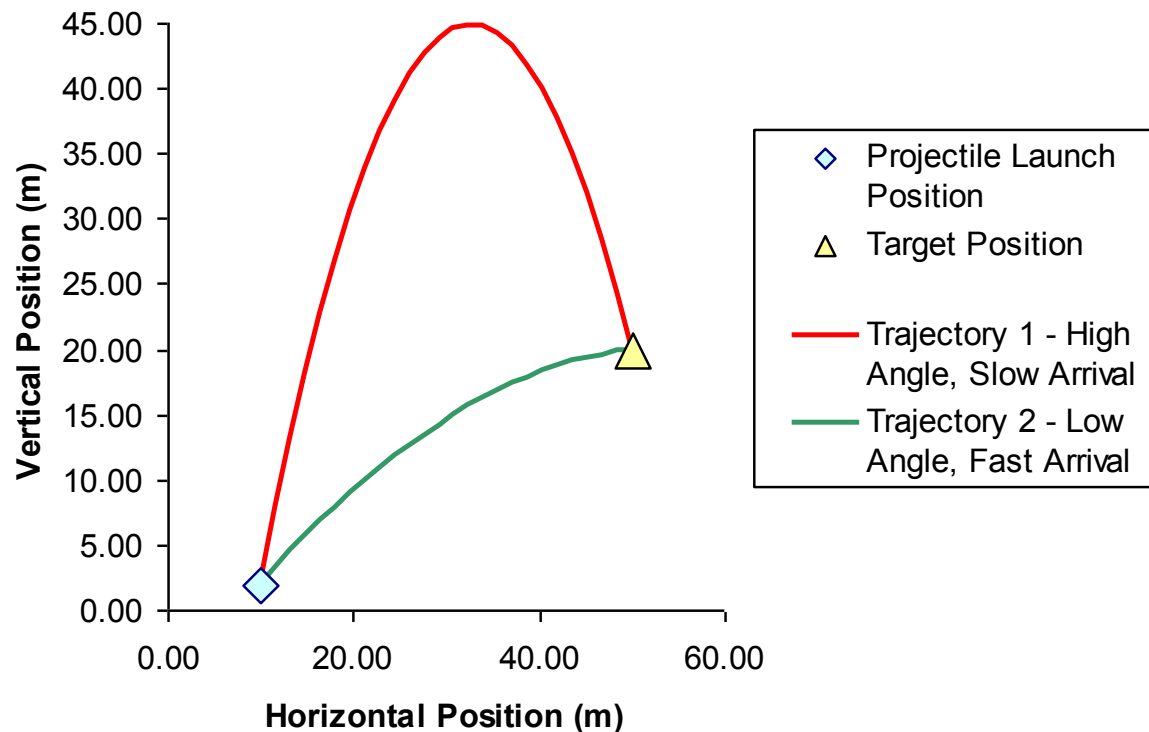


Target Practice – A Few Examples

$$V_{init} = 30 \text{ m/s}$$

Value of Radicand of $\tan\phi$ equation: **668**

Launch angle ϕ : 39.1 deg or 75.2 deg





■ Stop Here

Real-time Game Physics

**Practical Implementation:
Numerical Simulation**



What is Numerical Simulation?

- Equations Presented Above
 - They are “closed-form”
 - Valid and exact for constant applied force
 - Do not require time-stepping
 - Just determine current game time, t , using system timer
 - *e.g.*, $t = \frac{\text{QueryPerformanceCounter}}{\text{QueryPerformanceFrequency}}$ or equivalent on Microsoft® Windows® platforms
 - Plug t and t_{init} into the equations
 - Equations produce identical, repeatable, stable results, for any time, t , regardless of CPU speed and frame rate



What is Numerical Simulation?

- The above sounds perfect
- Why not use those equations always?
 - Constant forces aren't very interesting
 - Simple projectiles only
 - Closed-form solutions rarely exist for interesting (non-constant) forces
- We need a way to deal when there is no closed-form solution...

Numerical Simulation represents a series of techniques for incrementally solving the equations of motion when forces applied to an object are not constant, or when otherwise there is no closed-form solution



Finite Difference Methods

- What are They?
 - The most common family of numerical techniques for rigid-body dynamics simulation
 - Incremental “solution” to equations of motion
 - Derived using truncated Taylor Series expansions
 - See text for a more detailed introduction
- “Numerical Integrator”
 - This is what we generically call a finite difference equation that generates a “solution” over time



Finite Difference Methods

- The ***Explicit Euler*** Integrator:

$$\underbrace{\mathbf{S}(t + \Delta t)}_{\text{new state}} = \underbrace{\mathbf{S}(t)}_{\text{prior state}} + \Delta t \underbrace{\frac{d}{dt}\mathbf{S}(t)}_{\text{state derivative}}$$

- Properties of object are stored in a state vector, \mathbf{S}
- Use the above integrator equation to incrementally update \mathbf{S} over time as game progresses
- Must keep track of prior value of \mathbf{S} in order to compute the new
- For Explicit Euler, one choice of state and state derivative for particle:

$$\mathbf{S} = \langle m\mathbf{V}, \mathbf{p} \rangle$$

$$d\mathbf{S}/dt = \langle \mathbf{F}, \mathbf{V} \rangle$$



Explicit Euler Integration

$$V_{init} = 30 \text{ m/s}$$

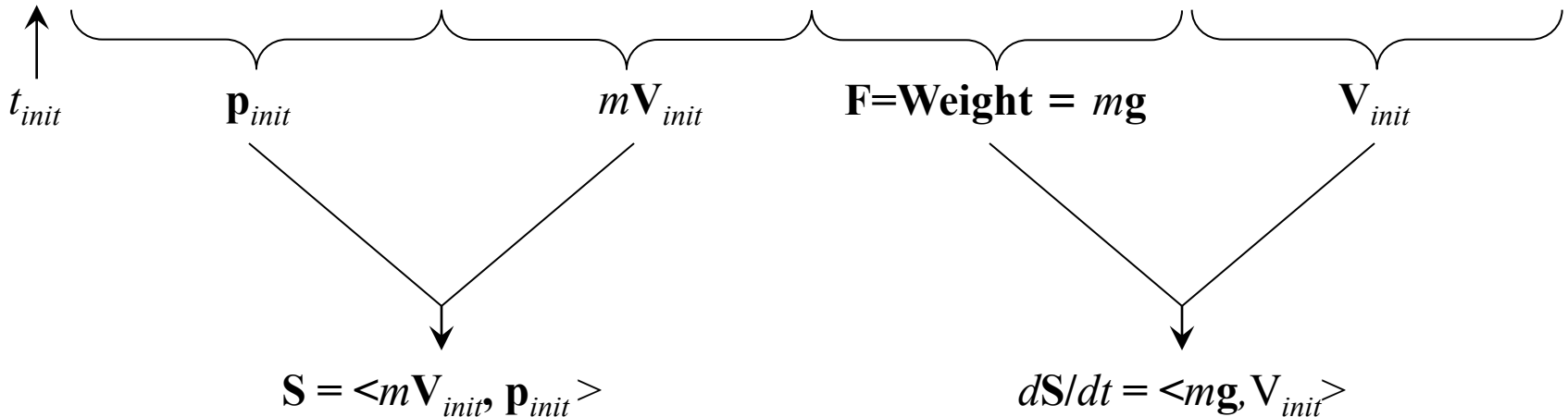
Launch angle, ϕ : 75.2 deg (slow arrival)

Launch angle, θ : 0 deg (motion in world xz plane)

Mass of projectile, m : 2.5 kg

Target at $\langle 50, 0, 20 \rangle$ meters

Time	Position (m)			Linear Momentum (kg-m/s)			Force (N)			Velocity (m/s)		
	p_x	p_y	p_z	mV_x	mV_y	mV_z	F_x	F_y	F_z	V_x	V_y	V_z
5.00	10.00	0.00	2.00	19.20	0.00	72.50	0.00	0.00	-24.53	7.68	0.00	29.00





Explicit Euler Integration

$$\mathbf{S}(t + \Delta t) = \mathbf{S}(t) + \Delta t \frac{d}{dt} \mathbf{S}(t) = \begin{bmatrix} 19.2 \\ 0.0 \\ 72.5 \\ 10.0 \\ 0.0 \\ 2.0 \end{bmatrix} + \Delta t \begin{bmatrix} 0.0 \\ 0.0 \\ -24.53 \\ 7.68 \\ 0.0 \\ 29.0 \end{bmatrix}$$

$\Delta t = .2 s$	$\Delta t = .1 s$	$\Delta t = .01 s$
$\begin{bmatrix} 19.2025 \\ 0.0 \\ 67.5951 \\ 11.5362 \\ 0.0 \end{bmatrix}$	$\begin{bmatrix} 19.2025 \\ 0.0 \\ 72.0476 \\ 10.7681 \\ 0.0 \end{bmatrix}$	$\begin{bmatrix} 19.2025 \\ 0.0 \\ 72.2549 \\ 10.0768 \\ 0.0 \end{bmatrix}$

Exact, Closed - form Solution

$\begin{bmatrix} 19.2 \\ 0.0 \\ 67.5951 \\ 11.5362 \\ 0.0 \\ 7.6038 \end{bmatrix}$	$\begin{bmatrix} 19.2 \\ 0.0 \\ 72.0476 \\ 10.1536 \\ 0.0 \\ 4.8510 \end{bmatrix}$	$\begin{bmatrix} 19.2 \\ 0.0 \\ 72.2549 \\ 10.0768 \\ 0.0 \\ 2.2895 \end{bmatrix}$
--	--	--



A Tangent: Truncation Error

- The previous slide highlights values in the numerical solution that are different from the exact, closed-form solution
- This difference between the exact solution and the numerical solution is primarily *truncation error*
- Truncation error is equal and opposite to the value of terms that were removed from the Taylor Series expansion to produce the finite difference equation
- **Truncation error, left unchecked, can accumulate to cause simulation to become unstable**
 - This ultimately produces floating point overflow
 - Unstable simulations behave unpredictably



A Tangent: Truncation Error

- Controlling Truncation Error
 - Under certain circumstances, truncation error can become zero, *e.g.*, the finite difference equation produces the exact, correct result
 - For example, when zero force is applied
 - More often in practice, truncation error is nonzero
 - Approaches to control truncation error:
 - Reduce time step, Δt
 - Select a different numerical integrator
 - See text for more background information and references



Explicit Euler Integration – Truncation Error

Lets Look at Truncation Error (position only)

$$\text{Truncation Error } (\Delta t = 0.2\text{s}) = \begin{bmatrix} 11.5362 \\ 0.0 \\ 7.800 \end{bmatrix}_{\text{numerical}} - \begin{bmatrix} 11.5362 \\ 0.0 \\ 7.6038 \end{bmatrix}_{\text{exact}} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.1962 \end{bmatrix}$$

$$\text{Truncation Error } (\Delta t = 0.1\text{s}) = \begin{bmatrix} 10.1536 \\ 0.0 \\ 4.9000 \end{bmatrix}_{\text{numerical}} - \begin{bmatrix} 10.1536 \\ 0.0 \\ 4.8510 \end{bmatrix}_{\text{exact}} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.049 \end{bmatrix}$$

$$\text{Truncation Error } (\Delta t = 0.01\text{s}) = \begin{bmatrix} 10.0768 \\ 0.0 \\ 2.2900 \end{bmatrix}_{\text{numerical}} - \begin{bmatrix} 10.0768 \\ 0.0 \\ 2.2895 \end{bmatrix}_{\text{exact}} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0005 \end{bmatrix}$$

Truncation Error

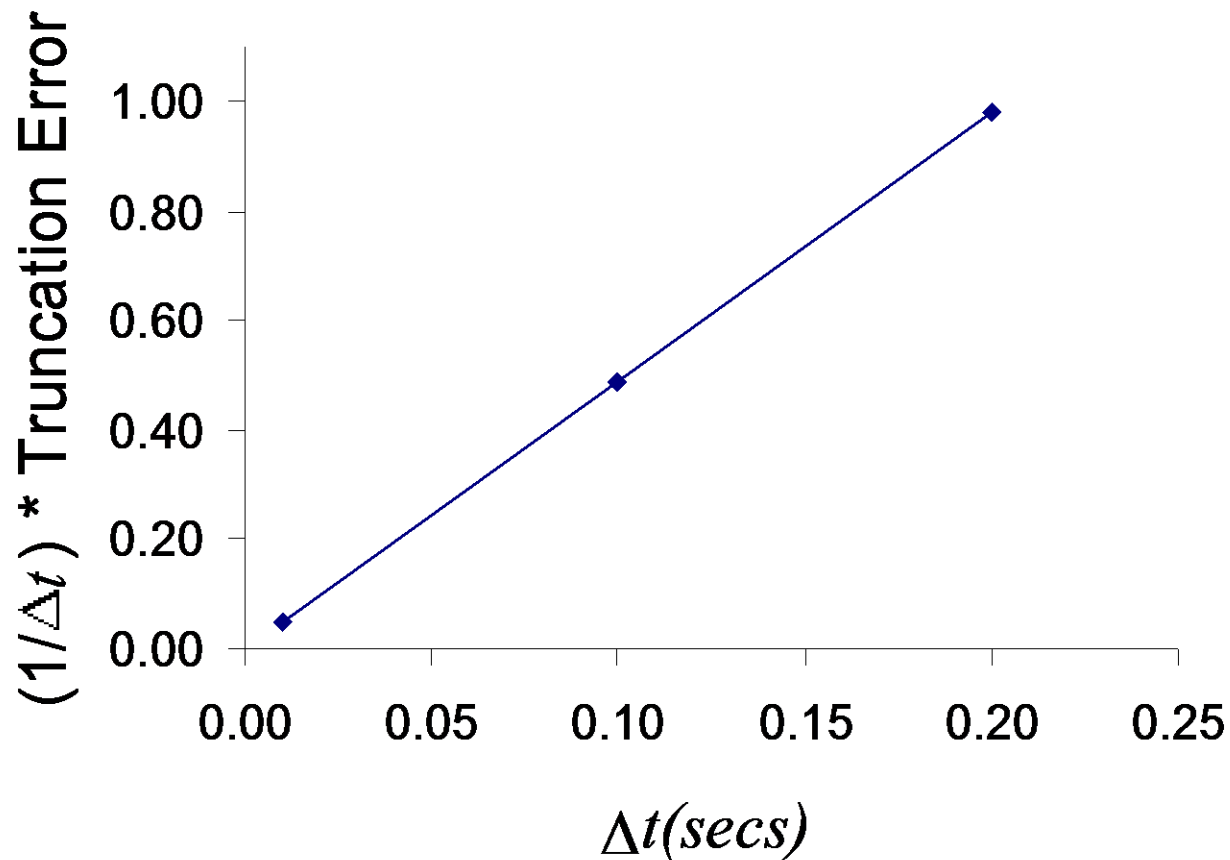
$$\begin{bmatrix} 0.0 \\ 0.0 \\ 0.1962 \end{bmatrix}$$

$$\begin{bmatrix} 0.0 \\ 0.0 \\ 0.049 \end{bmatrix}$$

$$\begin{bmatrix} 0.0 \\ 0.0 \\ 0.0005 \end{bmatrix}$$



Explicit Euler Integration – Truncation Error



is a linear (first-
explicit Euler
order-Accurate in

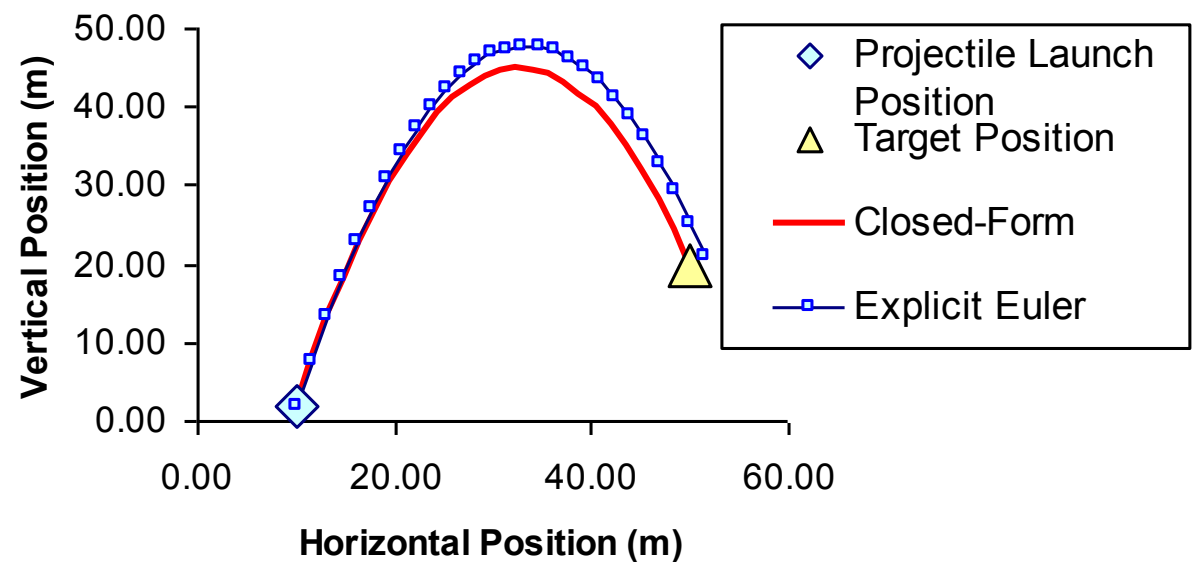
by " $O(\Delta t)$ "



Explicit Euler Integration - Computing Solution Over Time

- The solution proceeds step-by-step, each time integrating from the prior state

Time	Position (m)			Linear Momentum (kg-m/s)			Force (N)			Velocity (m/s)		
	p_x	p_y	p_z	mV_x	mV_y	mV_z	F_x	F_y	F_z	V_x	V_y	V_z
5.00	10.00	0.00	2.00	19.20	0.00	72.50	0.00	0.00	-24.53	7.68	0.00	29.00
5.20	11.54	0.00	7.80	19.20	0.00	67.60	0.00	0.00	-24.53	7.68	0.00	27.04
5.40	13.07	0.00	13.21	19.20	0.00	62.69	0.00	0.00	-24.53	7.68	0.00	25.08
5.60	14.61	0.00	18.22	19.20	0.00	57.79	0.00	0.00	-24.53	7.68	0.00	23.11
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
10.40	51.48	0.00	20.87	19.20	0.00	-59.93	0.00	0.00	-24.53	7.68	0.00	-23.97





Finite Difference Methods

- The **Verlet** Integrator:

$$\underbrace{\mathbf{S}(t + \Delta t)}_{\text{new state}} = 2 \underbrace{\mathbf{S}(t)}_{\text{prior state 1}} - \underbrace{\mathbf{S}(t - \Delta t)}_{\text{prior state 2}} + (\Delta t)^2 \underbrace{\left(\frac{d^2}{dt^2} \mathbf{S}(t) \right)}_{\text{state derivative}}$$

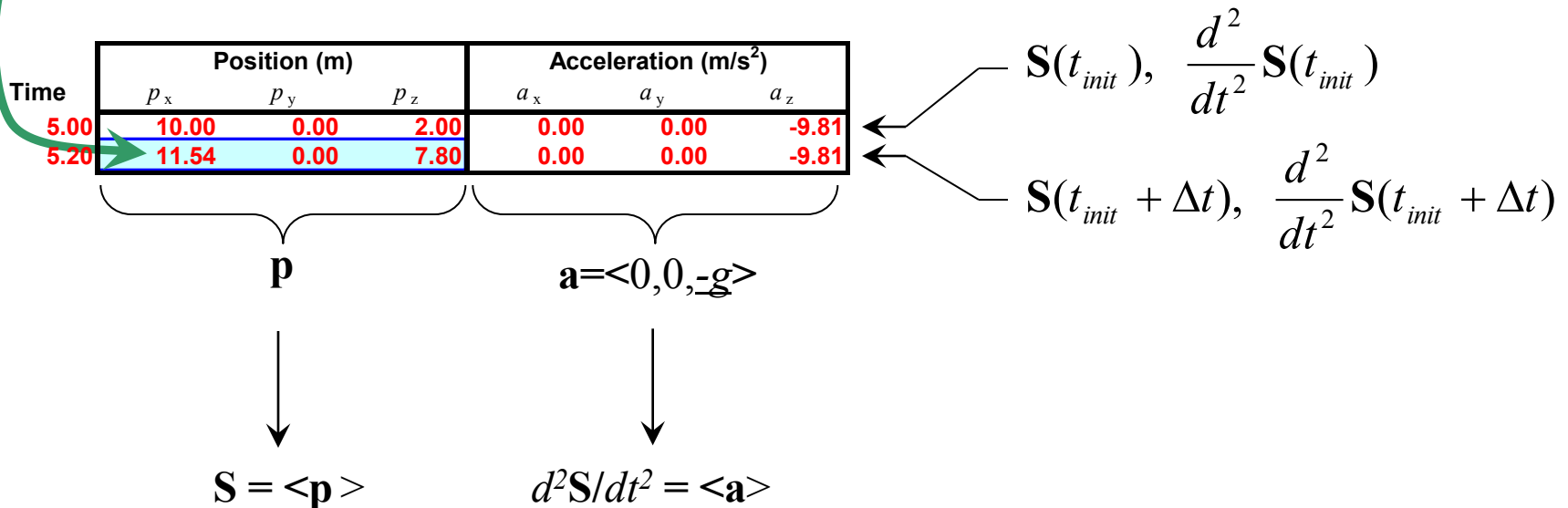
- Must store state at two prior time steps, $\mathbf{S}(t)$ and $\mathbf{S}(t-\Delta t)$
- Uses second derivative of state instead of the first
- Valid for constant time step only (as shown above)
- For Verlet, choice of state and state derivative for a particle:

$$\mathbf{S} = \langle \mathbf{p} \rangle \quad d^2\mathbf{S}/dt^2 = \langle \mathbf{F} / m \rangle = \langle \mathbf{a} \rangle$$



Verlet Integration

- Since Verlet requires two prior values of state, $\mathbf{S}(t)$ and $\mathbf{S}(t-\Delta t)$, you must use some method other than Verlet to produce the first numerical state after start of simulation, $\mathbf{S}(t_{init} + \Delta t)$
- Solution: Use explicit Euler integration to produce $\mathbf{S}(t_{init} + \Delta t)$, then Verlet for all subsequent time steps





Verlet Integration

- The solution proceeds step-by-step, each time integrating from the prior two states

Time	Position (m)			Acceleration (m/s ²)			
	p_x	p_y	p_z	a_x	a_y	a_z	
$S(t-\Delta t)$	5.00	10.00	0.00	2.00	0.00	0.00	-9.81
	5.20	11.54	0.00	7.80	0.00	0.00	-9.81
	5.40	13.07	0.00	13.21	0.00	0.00	-9.81
$S(t)$	5.60	14.61	0.00	18.22	0.00	0.00	-9.81
	5.80	16.14	0.00	22.85	0.00	0.00	-9.81
	6.00	17.68	0.00	27.08	0.00	0.00	-9.81
	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$S(t+\Delta t)$	10.40	51.48	0.00	20.87	0.00	0.00	-9.81

$\frac{d^2}{dt^2} \mathbf{S}(t)$

- For constant acceleration, Verlet integration produces results *identical* to those of explicit Euler
- But, results are different when non-constant forces are applied
- Verlet Integration tends to be more stable than explicit Euler for generalized forces

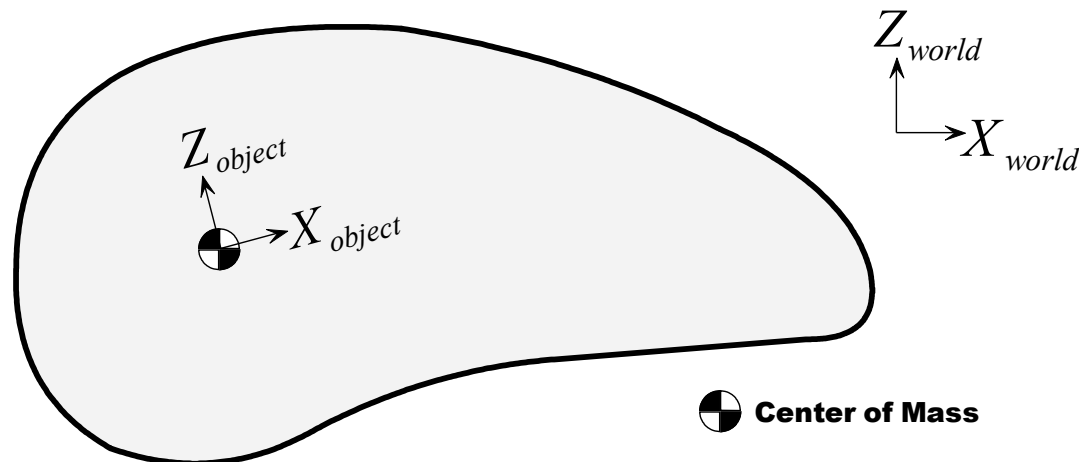
Real-time Game Physics

Generalized Rigid Bodies



Generalized Rigid Bodies

- Key Differences from Particles
 - Not necessarily spherical in shape
 - Position, \mathbf{p} , represents object's center-of-mass location
 - Surface may not be perfectly smooth
 - Friction forces may be present
 - Experience rotational motion in addition to translational (position only) motion





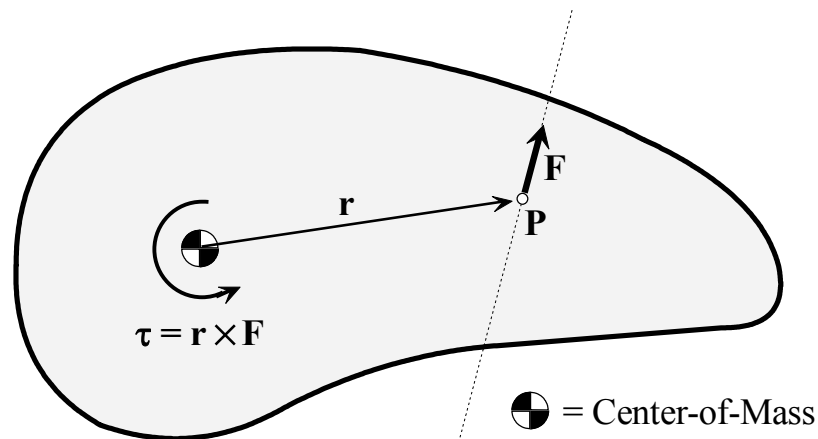
Generalized Rigid Bodies – Simulation

- Angular Kinematics
 - Orientation, 3x3 matrix \mathbf{R} or quaternion, q
 - Angular velocity, ω
 - As with translational/particle kinematics, all properties are measured in world coordinates
- Additional Object Properties
 - Inertia tensor, \mathbf{J}
 - Center-of-mass
- Additional State Properties for Simulation
 - Orientation
 - Angular momentum, $\mathbf{L}=\mathbf{J}\omega$
 - Corresponding state derivatives

Generalized Rigid Bodies - Simulation

■ Torque

- Analogous to a force
- Causes rotational acceleration
 - Cause a change in angular momentum
- Torque is the result of a force (friction, collision response, spring, damper, etc.)





Generalized Rigid Bodies – Numerical Simulation

- Using Finite Difference Integrators
 - Translational components of state $\langle m\mathbf{V}, \mathbf{p} \rangle$ are the same
 - \mathbf{S} and $d\mathbf{S}/dt$ are expanded to include angular momentum and orientation, and their derivatives
 - Be careful about coordinate system representation for \mathbf{J} , \mathbf{R} , etc.
 - Otherwise, integration step is identical to the translation only case
- Additional Post-integration Steps
 - Adjust orientation for consistency
 - Adjust updated \mathbf{R} to ensure it is orthogonal
 - Normalize q
 - Update angular velocity, ω
 - See text for more details



Collision Response

- Why?
 - Performed to keep objects from interpenetrating
 - To ensure behavior similar to real-world objects
- Two Basic Approaches
 - Approach 1: Instantaneous change of velocity at time of collision
 - Benefits:
 - Visually the objects never interpenetrate
 - Result is generated via closed-form equations, and is perfectly stable
 - Difficulties:
 - Precise detection of time and location of collision can be prohibitively expensive (frame rate killer)
 - Logic to manage state is complex



Collision Response

- Two Basic Approaches (continued)
 - Approach 2: Gradual change of velocity and position over time, following collision
 - Benefits
 - Does not require precise detection of time and location of collision
 - State management is easy
 - Potential to be more realistic, if meshes are adjusted to deform according to predicted interpenetration
 - Difficulties
 - Object interpenetration is likely, and parameters must be tweaked to manage this
 - Simulation can be subject to numerical instabilities, often requiring the use of implicit finite difference methods



Final Comments

- Instantaneous Collision Response
 - Classical approach: Impulse-momentum equations
 - See text for full details
- Gradual Collision Response
 - Classical approach: Penalty force methods
 - Resolve interpenetration over the course of a few integration steps
 - Penalty forces can wreak havoc on numerical integration
 - Instabilities galore
 - Implicit finite difference equations can handle it
 - But more difficult to code
 - Geometric approach: Ignore physical response equations
 - Enforce purely geometric constraints once interpenetration has occurred



Fixed Time Step Simulation

- Numerical simulation works best if the simulator uses a fixed time step
 - *e.g.*, choose $\Delta t = 0.02$ seconds for physics updates of 1/50 second
 - Do not change Δt to correspond to frame rate
 - Instead, write an inner loop that allows physics simulation to catch up with frame rate, or wait for frames to catch up with physics before continuing
 - This is easy to do
- Read the text for more details and references!



Final Comments

- Simple Games
 - Closed-form particle equations may be all you need
 - Numerical particle simulation adds flexibility without much coding effort
 - Collision detection is probably the most difficult part of this
- Generalized Rigid Body Simulation
 - Includes rotational effects and interesting (non-constant) forces
 - See text for details on how to get started



Final Comments

- Full-Up Simulation
 - The text and this presentation just barely touch the surface
 - Additional considerations
 - Multiple simultaneous collision points
 - Articulating rigid body chains, with joints
 - Friction, rolling friction, friction during collision
 - Mechanically applied forces (motors, etc.)
 - Resting contact/stacking
 - Breakable objects
 - Soft bodies
 - Smoke, clouds, and other gases
 - Water, oil, and other fluids