MAT2440 - Exam 1 Review

1. Determine whether $(\neg p \wedge (p \vee q)) \rightarrow q$ is a tautology.
2. Show that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent.
3. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q$ are logically equivalent.
4. Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logically equivalent.
5. Let $P(x)$, $Q(x)$ and $R(x)$ be the statements "x is a professor", "x is ignorant", and "x is vain," respectively, where the domain consists of all people. Translate each of these statements into English.
   (a) $\forall x \neg (P(x) \wedge Q(x))$
   (b) $\exists x (R(x) \wedge \neg Q(x)) \rightarrow \exists y P(y)$
   (c) $(\forall x Q(x) \rightarrow \exists y \neg P(y)) \vee \forall y R(y)$
6. (Follow-up to previous problem.) Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, and $R(x)$.
   (a) All ignorant people are vain.
   (b) No professors are ignorant.
   (c) There is a person that is both vain and a professor.
7. Negate the following statements so that the negation appears only within the predicates.
   (a) $\forall x \exists y P(x, y)$
   (b) $\exists y (Q(y) \wedge \forall x \neg R(x, y))$
8. Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is " x is funny" and the domain consists of all people.
   (a) $\forall x (C(x) \rightarrow F(x))$
   (b) $\forall x (C(x) \wedge F(x))$
   (c) $\exists x (C(x) \wedge \neg F(x))$
9. Determine whether the following arguments are valid. If the argument is correct, what rule of inference is being used? If it is not, what logical error occurs?
   (a) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.

(b) Quincy likes all action movies. Quincy likes the movie *Shakespeare in Love*. Therefore, *Shakespeare in Love* is an action movie.

(c) Consider the following argument form.

$$\begin{array}{l} p \wedge q \\ p \rightarrow r \\ \underline{q \rightarrow s} \\ \therefore r \wedge s \end{array}$$

10. For each of these arguments, determine whether they are correct or incorrect and explain why.

    (a) All men are mortal. Socrates is a man. Therefore, Socrates is mortal.

    (b) A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.

11. Show that for every integer n, n^2 is even if and only if n is even.

12. Show that if n is and integer and $n^3 + 5$ is odd, then n is even using

    (a) a proof by contraposition.

    (b) a proof by contradiction.