MAT2440 - Exam 3 Review

1. Let $a \neq 0$, b, and c be integers. Show that if $a|b$ and $a|c$, then $a|(b+c)$.
2. Convert 3452 to:
   (a) base 2
   (b) base 3
   (c) base 7
3. Convert the following number $(ABBA)_{16}$ from hexadecimal to octal.
4. Find 3^{2003} mod 99.
5. Use the Euclidean algorithm to find
   (a) $\gcd(12, 18)$
   (b) $\gcd(111, 201)$
   (c) $\gcd(345, 346)$
   (d) $\gcd(1005, 475)$
6. Which memory locations are assigned by the hashing function $h(k) = k$ mod 97 to the records of insurance company customers with these Social Security numbers?
   (a) 034-56-7981
   (b) 220-19-5744
   (c) 183-21-1232
7. What sequence of pseudorandom numbers is generated using the linear congruential generator $x_{n+1} = (3x_n + 2)$ mod 13 with seed $x_0 = 1$?
8. Let $P(n)$ be the statement that $1^2 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/6$ for the positive integer n. Prove that $P(n)$ is true for $n \geq 1$.
9. Prove that for every positive integer n,
$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = n(n+1)(n+2)/3.$$
10. Prove that 6 divides $n^3 - n$ whenever n is a positive integer.
11. Let $P(n)$ be the statement that a postage of n cents can be formed using just 3-cent and 5-cent stamps. Show that $P(n)$ is true for $n \geq 8$.

12. Determine which amounts of postage can be formed using just 4-cent and 11-cent stamps.

13. Which amounts of money can be formed using just two-dollar bills and 5-dollar bills? Prove your answer.

14. Let F be the function such that $F(n)$ is the sum of the first n positive integers. Give a recursive definition of $F(n)$.

15. Give a recursive definition of the set of positive integer powers of 3.

16. Ackermann's function is defined by the following recursive definition.

$$A(m,n) = \begin{cases} 2n & \text{if } m = 0 \\ 0 & \text{if } m \geq 1 \text{ and } n = 0 \\ 2 & \text{if } m \geq 1 \text{ and } n = 1 \\ A(m-1, A(m, n-1)) & \text{if } m \geq 1 \text{ and } n \geq 2 \end{cases}$$

Compute the following values of Ackermann's function.

(a) $A(1,0)$

(b) $A(2,2)$

17. Give a recursive algorithm for find the sum of the of the first n positive integers. (Hint: See problem 13.)

18. Give a recursive algorithm for finding the maximum of a finite list of integers, making use of the fact that the maximum of n integers is the larger of the last integer in the list and the maximum of the first $n-1$ integers in the list.

19. Give a recursive algorithm for finding $n!$ mod m where n and m are positive integers.

20. Use a merge sort to sort $b, d, a, f, g, h, z, p, o, k$ into alphabetic order. Show all steps used by the algorithm.

21. Devise a recursive algorithm for computing n^2 where n is a nonnegative integer, using the fact that $(n+1)^2 = n^2 + 2n + 1$.