

CIS 32 Homework 4

Fall 2003

1. Using the proof rules in the lecture notes and those given below, try to prove the following:

- (a) $(p, p \Rightarrow (q \wedge r)) \vdash (p \vee r)$
- (b) $(p \wedge (p \Rightarrow (q \wedge r))) \vdash (p \vee r)$
- (c) $(p \wedge (p \Rightarrow (q \wedge r))) \vdash (s \vee r)$

(45 points)

Some proof rules that aren't in the lecture notes are:

$$\frac{\vdash \phi \Leftrightarrow \psi}{\vdash \phi \Rightarrow \psi; \vdash \psi \Rightarrow \phi} \Leftrightarrow\text{-E}$$

$$\frac{\vdash \phi \Rightarrow \psi; \vdash \psi \Rightarrow \phi}{\vdash \phi \Leftrightarrow \psi} \Leftrightarrow\text{-I}$$

$$\frac{\vdash \neg\neg\phi}{\vdash \phi} \neg\text{-E}$$

$$\frac{\phi \vdash \perp}{\vdash \neg\phi} \neg\text{-I}$$

For the last of these rules, remember that \perp stands for any formula which is inconsistent (for example $\phi \wedge \neg\phi$).

2. Convert the following sentences to predicate logic form:
- (a) Every prime number other than 2 is odd.
 - (b) Every cloud has a silver lining.
 - (c) Nobody knows the trouble I seen.
 - (d) Everybody hates grunge music.

(e) Everybody hates all grunge music except that by Nirvana.

(20 points)

3. Let

$Days(x)$ mean that x is a day
 $R(x)$ mean that x is rainy
 $S(x)$ mean that x is sunny

Symbolise each of the following in two different ways:

- (a) Every day is sunny.
- (b) Some days are not sunny.
- (c) Every day that is not sunny is rainy.

(15 points)

4. Prove $(p, p \Rightarrow (q \wedge r)) \vdash (p \vee r)$ using resolution.

(20 points)