CIS 32 Homework 4

Spring 2003

1. Determine, for each of the following propositional logic formulae, whether it is:

   - a tautology;
   - consistent;
   - inconsistent.

   (a) $p \vee \neg p$

   (b) $(p \Rightarrow q) \Rightarrow ((p \Rightarrow r) \Rightarrow (p \Rightarrow r))$

   (c) $p \vee q \vee \neg r$

   (d) $(p \wedge q) \Rightarrow r$

   (e) $p \Rightarrow (q \Rightarrow p)$

   (15 points)

2. Identify the propositions in each of the following examples, and rewrite them as propositional logic formulae. Draw up truth tables for each of the examples.

   (a) If the reactor is on, then the warning system should be on.

   (b) If the temperature is high, and the reactor is on, then the red light should be on, and the green light should be off.

   (c) If the temperature is low, and the reactor is on, then the green light should be on and the red light should be off.

   (15 points)

3. Use the truth table method to decide whether the following statements are true:

   (a) $(p \Rightarrow q) \models ((p \Rightarrow r) \Rightarrow (p \Rightarrow r))$

   (b) $(p \wedge q) \models \neg(p \vee q)$

   (c) $(p \Leftrightarrow q) \models (p \Rightarrow q)$

(15 points)

4. Using the proof rules in the lecture notes and those given below, try to prove the following:

   (a) $(p, p \Rightarrow (q \wedge r)) \vdash (p \vee r)$

   (b) $(p \wedge (p \Rightarrow (q \wedge r))) \vdash (p \vee r)$

   (c) $(p \wedge (p \Rightarrow (q \wedge r))) \vdash (s \vee r)$

   (25 points)

   Some proof rules that aren't in the lecture notes are:

$$\frac{\vdash \phi \Leftrightarrow \psi}{\vdash \phi \Rightarrow \psi; \vdash \psi \Rightarrow \phi} \Leftrightarrow\text{-E}$$

$$\frac{\vdash \phi \Rightarrow \psi; \vdash \psi \Rightarrow \phi}{\vdash \phi \Leftrightarrow \psi} \Leftrightarrow\text{-I}$$

$$\frac{\vdash \neg\neg\phi}{\vdash \phi} \neg\text{-E}$$

$$\frac{\phi \vdash \bot}{\vdash \neg\phi} \neg\text{-I}$$

   For the last of these rules, remember that $\bot$ stands for any formula which is inconsistent (for example $\phi \wedge \neg\phi$).

5. Convert the following sentences to predicate logic form:

   (a) Every prime number other than 2 is odd.

   (b) Every cloud has a silver lining.

   (c) Nobody knows the trouble I seen.

   (d) Everybody hates grunge music.

   (e) Everybody hates all grunge music except that by Nirvana.

(15 points)

6. Let

$Days(x)$	mean that x is a day
$R(x)$	mean that x is rainy
$S(x)$	mean that x is sunny

Symbolise each of the following in two different ways:

(a) Every day is sunny.

(b) Some days are not sunny.

(c) Every day that is not sunny is rainy.

(15 points)