CIS 32 Spring 2009, Homework 4

1. Which, if any, of the following sentences is a proposition? In each case, justify your answer.

    (a) Twice two is four.

    (b) The square root of two is not a rational number.

    (c) $5 > 6$

    (d) Pigs can fly.

    (e) Please write a specimen of your signature in the space provided.

    (f) Would you believe you're standing in a place where Madonna once stood?

    (g) That's true.

   (15 points)

2. Using the following propositions:

p	Its is raining
q	I have an umbrella
r	I get wet

   Formulate the following expressions in words:

    (a) $(p \land q)$

    (b) $(p \land \neg q) \land r$

    (c) $\neg p \land \neg r$

    (d) $p \land (q \lor r)$

    (e) $\neg p \lor r$

   (10 points)

3. For each of the propositions in the previous question. write out the truth table.

   (10 points)

4. For each of the propositions in question 2, say whether it is: consistent, inconsistent and/or a tautology.

   (10 points)

5. Using the following propositions:

p	Logic is easy
q	There is a logic question
r	I pass the exam

   symbolise the following statements:

    (a) If logic is easy then I pass the exam.

    (b) I fail the exam if logic is difficult.

    (c) If there is no logic question, then I pass the exam.

    (d) Logic is difficult. I pass the exam if there is no logic question.

    (e) Either logic is easy, and I pass the exam, or logic is hard, and I fail.

    (f) If logic is difficult then I pass the exam if there is no logic question.

   (15 points)

6. Use the truth table method to decide whether the following statements are true:

   (a) $(p \Rightarrow q) \models ((p \Rightarrow r) \Rightarrow (p \Rightarrow r))$
   (b) $(p \wedge q) \models \neg(p \vee q)$
   (c) $(p \Leftrightarrow q) \models (p \Rightarrow q)$

   (15 points)

7. Using the proof rules in the lecture notes and those given below, try to prove the following:

   (a) $(p, p \Rightarrow (q \wedge r)) \vdash (p \wedge r)$
   (b) $(p, p \Rightarrow (q \wedge r)) \vdash (s \vee r)$
   (c) $(p \wedge (p \Rightarrow (q \wedge r))) \vdash (p \Rightarrow r)$
   (d) $(p \Rightarrow (q \wedge r)) \vdash (p \Rightarrow r)$
   (e) $(\neg q, p \Leftrightarrow (q \wedge r)) \vdash \neg p$

   (25 points)

   Some proof rules that aren't in the lecture notes are:

$$\frac{\vdash \phi \Leftrightarrow \psi}{\vdash \phi \Rightarrow \psi; \vdash \psi \Rightarrow \phi} \quad \Leftrightarrow\text{-E}$$

$$\frac{\vdash \phi \Rightarrow \psi; \vdash \psi \Rightarrow \phi}{\vdash \phi \Leftrightarrow \psi} \quad \Leftrightarrow\text{-I}$$

$$\frac{\vdash \neg\neg\phi}{\vdash \phi} \quad \neg\text{-E}$$

$$\frac{\phi \vdash \bot}{\vdash \neg\phi} \quad \neg\text{-I}$$

   For the last of these rules, remember that $\bot$ stands for any formula which is inconsistent (for example $\phi \wedge \neg\phi$).