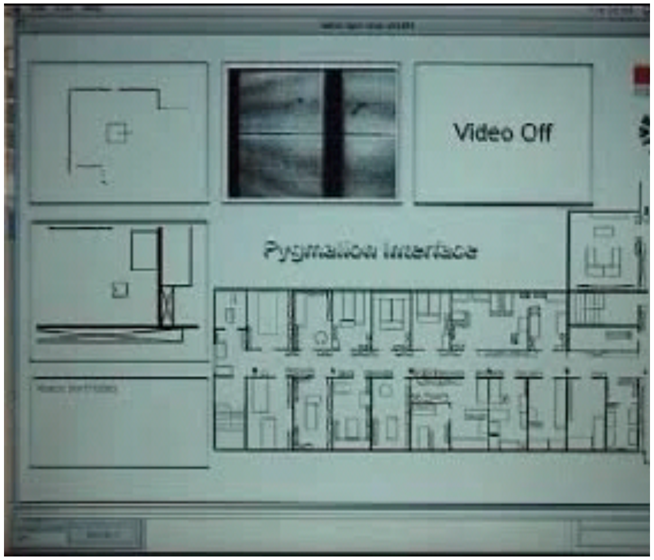
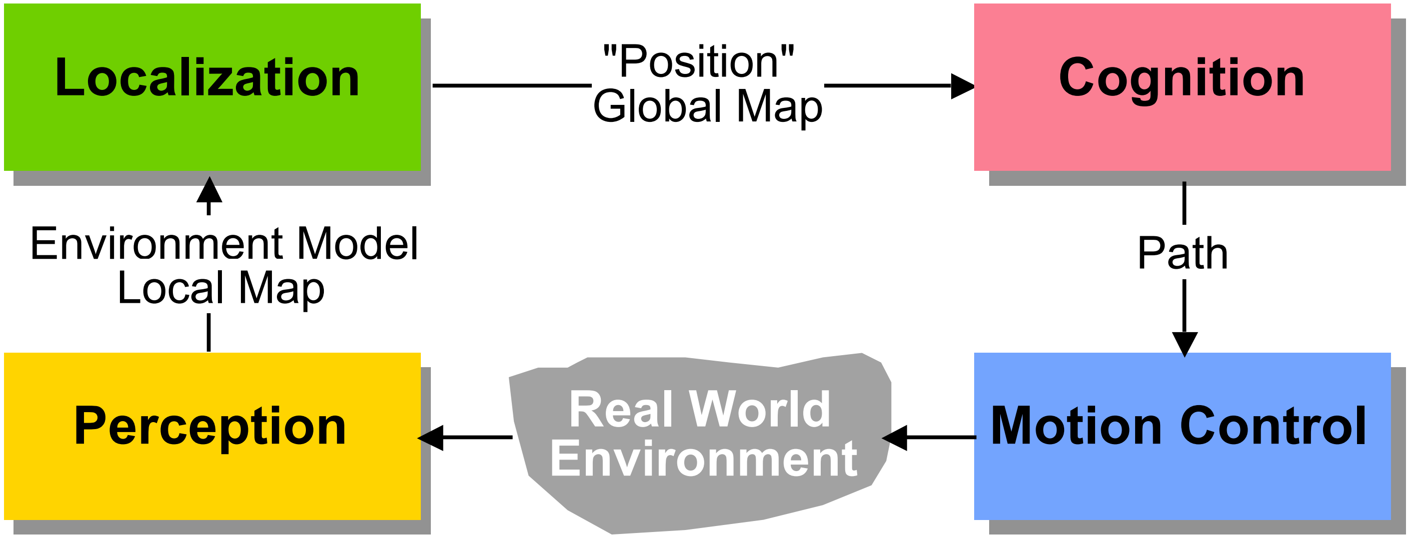
Motion Control (wheeled robots)

- Requirements for Motion Control
  - *Kinematic / dynamic model of the robot*
  - *Model of the interaction between the wheel and the ground*
  - *Definition of required motion -> speed control, position control*
  - *Control law that satisfies the requirements*





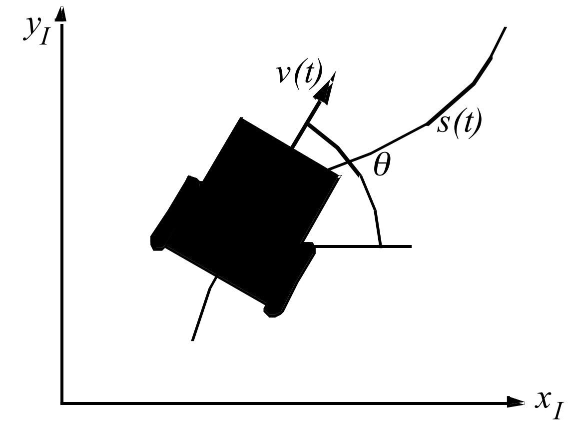
Introduction: Mobile Robot Kinematics

- Aim
  - *Description of mechanical behavior of the robot for* design *and* control
  - *Similar to robot manipulator kinematics*
  - *However, mobile robots can move unbound with respect to its environment*
    - *there is no direct way to measure the robot's position*
    - *Position must be integrated over time*
    - *Leads to inaccuracies of the position (motion) estimate*
      -> the number 1 challenge in mobile robotics
  - *Understanding mobile robot motion starts with understanding wheel* constraints *placed on the robots mobility*

Introduction: Kinematics Model

- Goal:
  - *establish the robot speed* $\dot{\xi} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$ *as a function of the wheel speeds* $\dot{\varphi}_i$, *steering angles* β_i, *steering speeds* $\dot{\beta}_i$ *and the geometric parameters of the robot (*configuration coordinates*).*
  - *forward kinematics*

$$\dot{\xi} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(\dot{\varphi}_1, \ldots \dot{\varphi}_n, \beta_1, \ldots \beta_m, \dot{\beta}_1, \ldots \dot{\beta}_m)$$



  - *Inverse kinematics*

$$\begin{bmatrix} \dot{\varphi}_1 & \cdots & \dot{\varphi}_n & \beta_1 & \ldots & \beta_m & \dot{\beta}_1 & \ldots & \dot{\beta}_m \end{bmatrix}^T = f(\dot{x}, \dot{y}, \dot{\theta})$$

  - *why not*

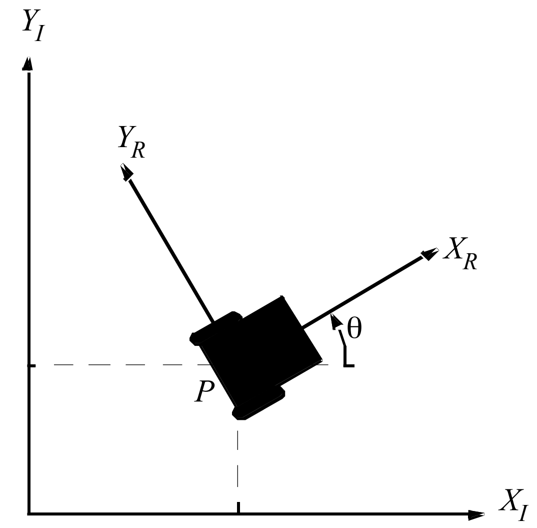
$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = f(\varphi_1, \ldots \varphi_n, \beta_1, \ldots \beta_m)$$

-> ***not straight forward***

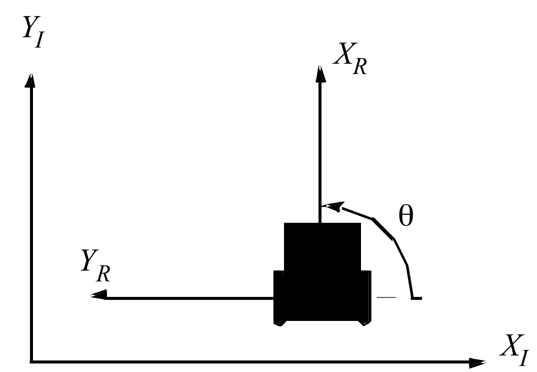
Representing Robot Position

- Representing to robot within an arbitrary initial frame
  - *Initial frame:* $\{X_I, Y_I\}$
  - *Robot frame:* $\{X_R, Y_R\}$
  - *Robot position:* $\xi_I = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$
  - *Mapping between the two frames*
  - $\dot{\xi}_R = R(\theta)\dot{\xi}_I = R(\theta) \cdot \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

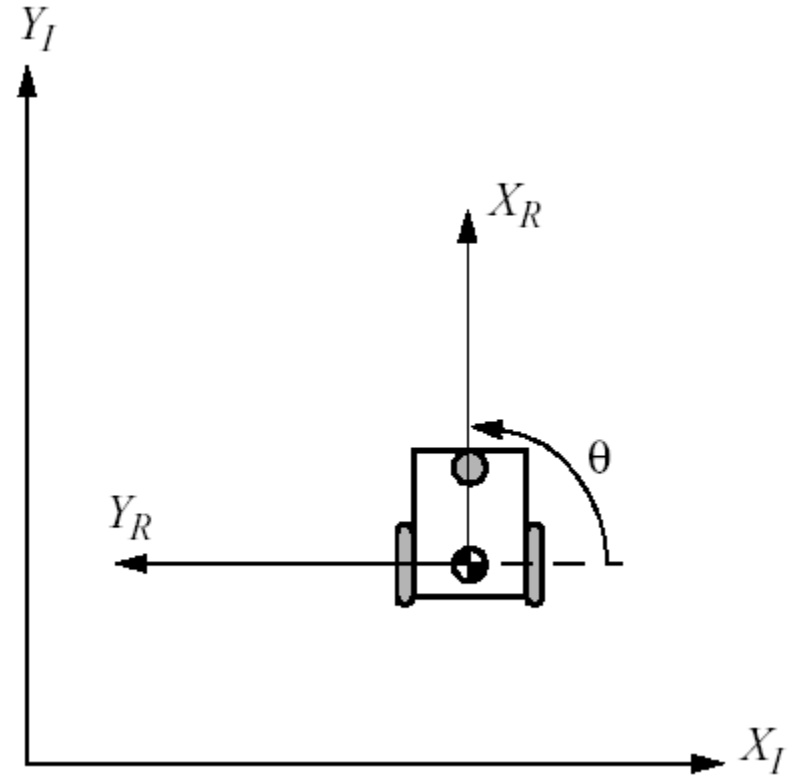


  - *Example: Robot aligned with* Y_I



Example

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



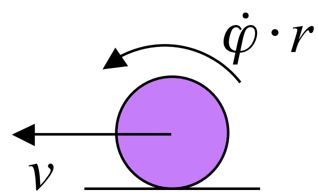
$$\dot{\xi}_R = R\left(\frac{\pi}{2}\right)\dot{\xi}_I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$

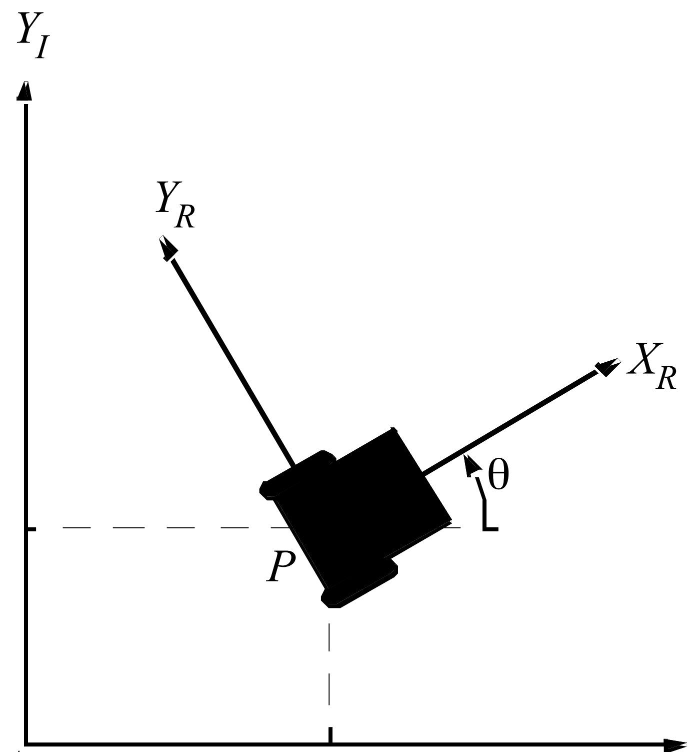
Forward Kinematic Models

- Presented on blackboard

Wheel Kinematic Constraints: Assumptions

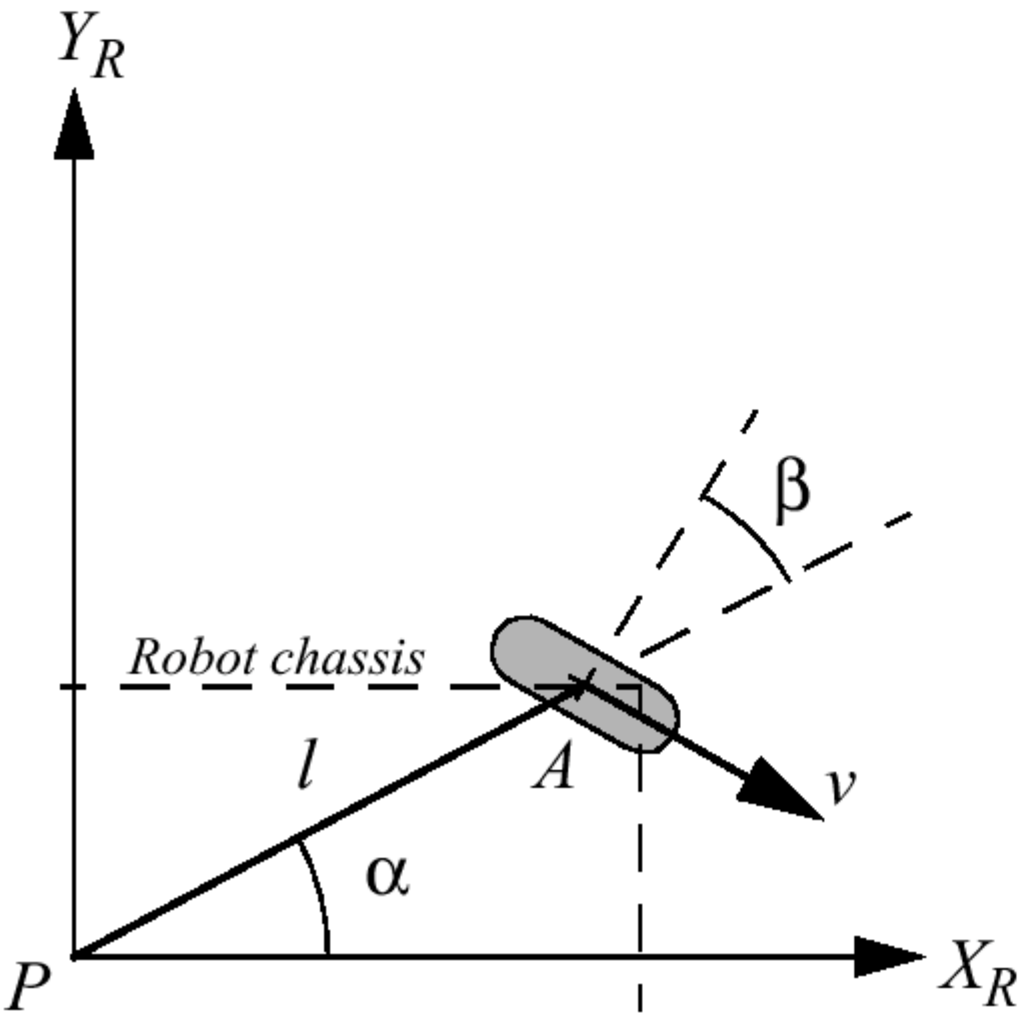
- Movement on a horizontal plane
- Point contact of the wheels
- Wheels not deformable
- Pure rolling
  - v = *0 at contact point*
- No slipping, skidding or sliding
- No friction for rotation around contact point
- Steering axes orthogonal to the surface
- Wheels connected by rigid frame (chassis)

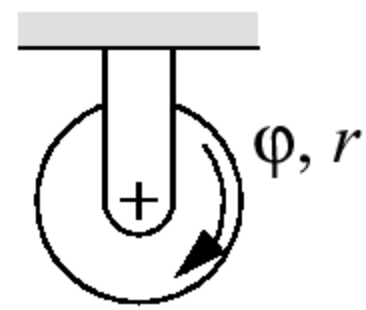




Wheel Kinematic Constraints:

Fixed Standard Wheel





Example

$$\left[\sin(\alpha+\beta) \;\; -\cos(\alpha+\beta) \;\; (-l)\cos\beta\right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

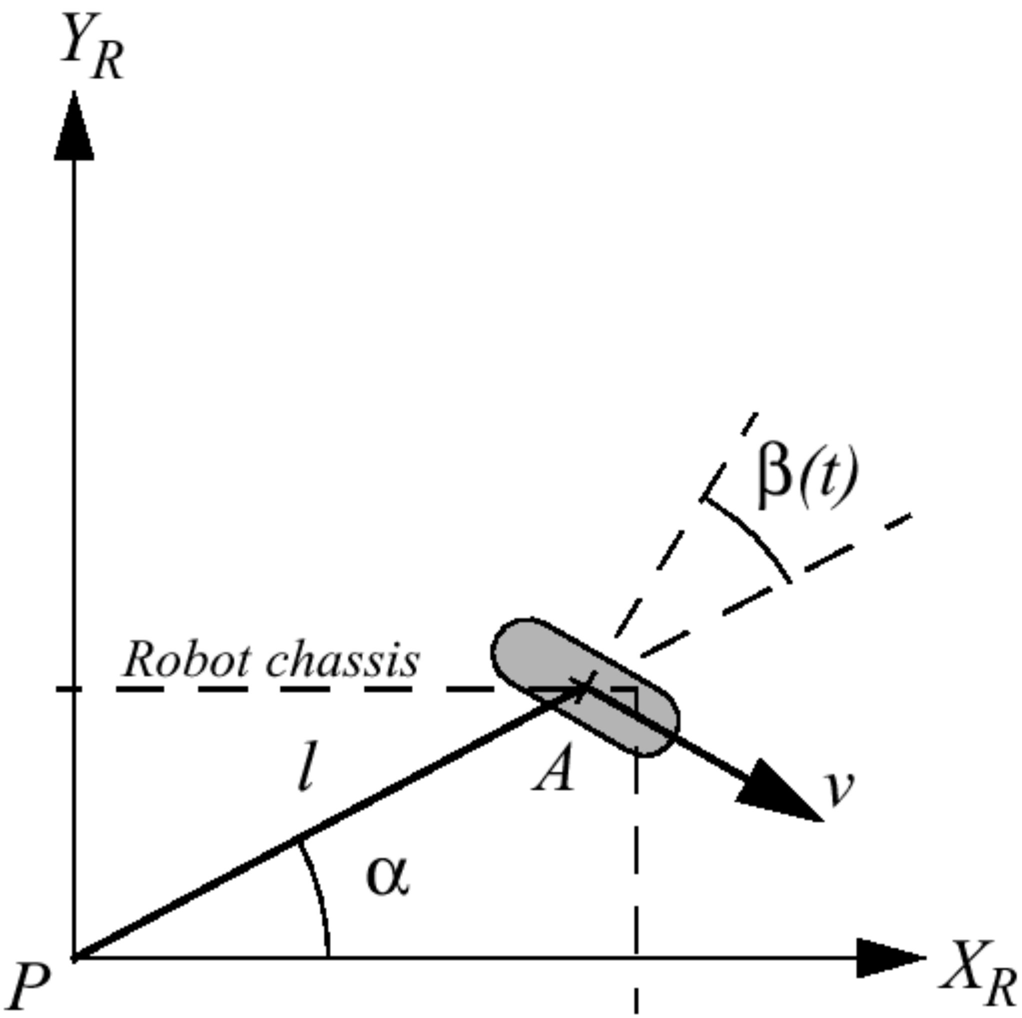
$$\left[\cos(\alpha+\beta) \;\; \sin(\alpha+\beta) \;\; l\sin\beta\right] R(\theta)\dot{\xi}_I = 0$$

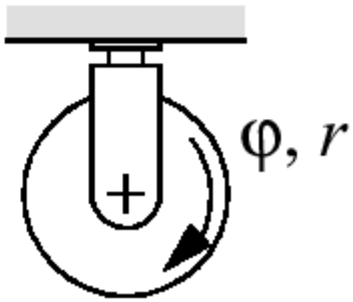
- Suppose that the wheel A is in position such that
- a = 0 and b = 0
- This would place the contact point of the wheel on X_I with the plane of the wheel oriented parallel to Y_I. If q = 0, then ths sliding constraint reduces to:

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = 0$$

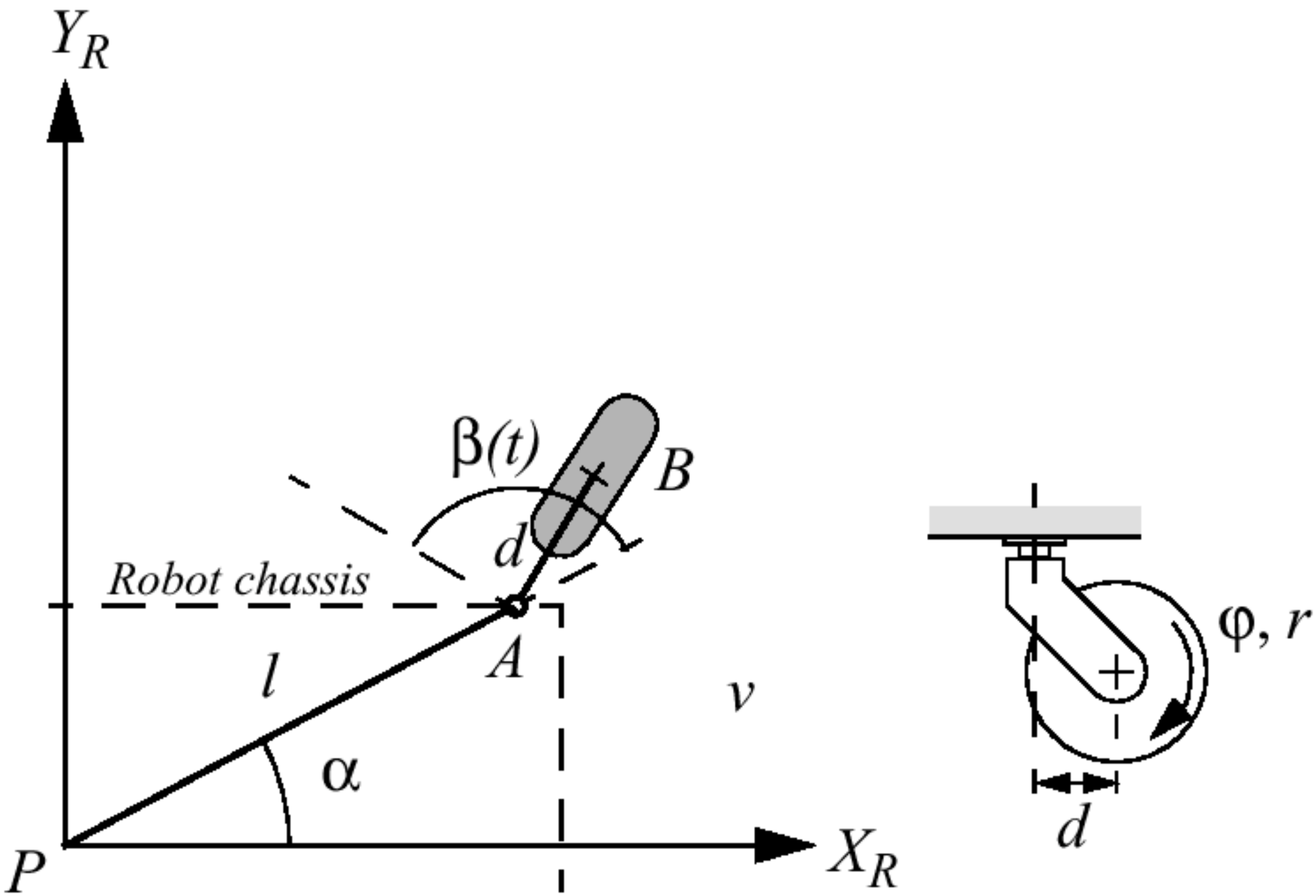
Wheel Kinematic Constraints:

Steered Standard Wheel



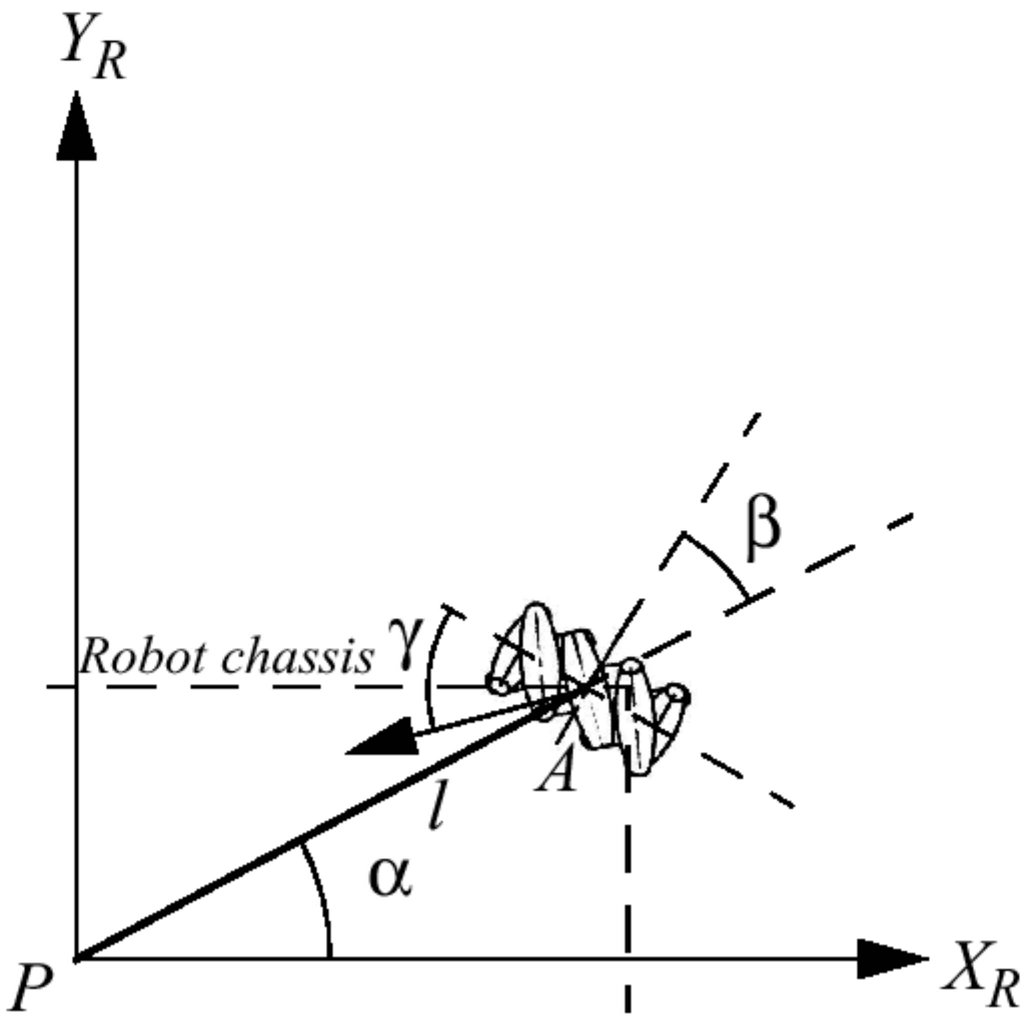


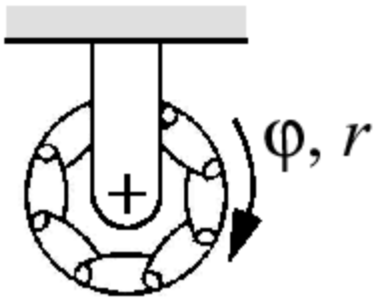
Wheel Kinematic Constraints: Castor Wheel



Wheel Kinematic Constraints:

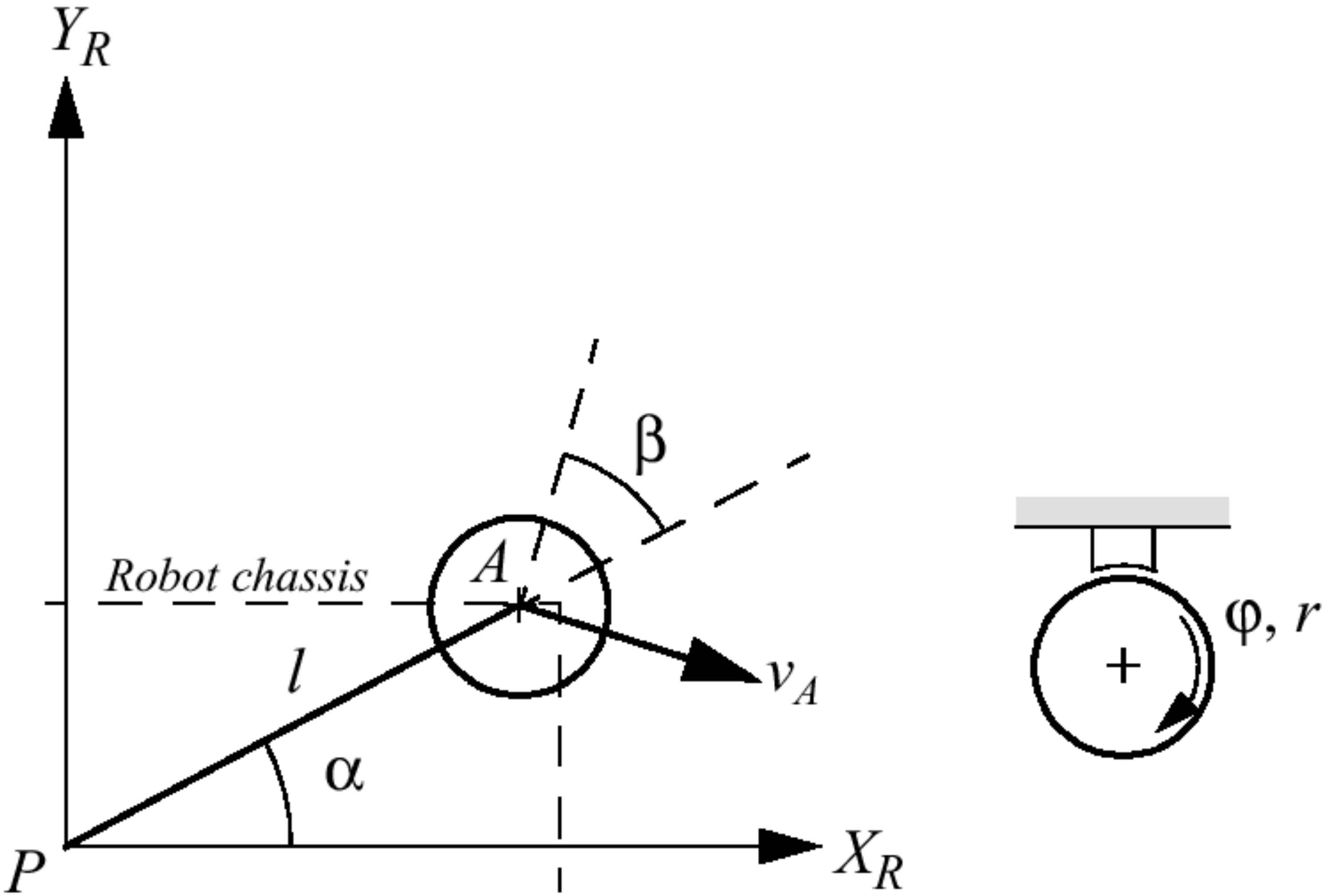
Swedish Wheel





Wheel Kinematic Constraints:

Spherical Wheel



Robot Kinematic Constraints

- Given a robot with M wheels
  - *each wheel imposes zero or more constraints on the robot motion*
  - *only fixed and steerable standard wheels impose constraints*
- What is the maneuverability of a robot considering a combination of different wheels?
- Suppose we have a total of $N=N_f+N_s$ standard wheels
  - *We can develop the equations for the constraints in matrix forms:*
  - *Rolling*

$$J_1(\beta_s)R(\theta)\dot{\xi}_I + J_2\dot{\varphi} = 0 \qquad \varphi(t) = \begin{bmatrix} \varphi_f(t) \\ \varphi_s(t) \end{bmatrix}_{(N_f+N_s)\times 1} \qquad J_1(\beta_s) = \begin{bmatrix} J_{1f} \\ J_{1s}(\beta_s) \end{bmatrix}_{(N_f+N_s)\times 3} \qquad J_2 = diag(r_1 \cdots r_N)$$

  - *Lateral movement*

$$C_1(\beta_s)R(\theta)\dot{\xi}_I = 0 \qquad C_1(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}_{(N_f+N_s)\times 3}$$

Example: Differential Drive Robot

- Presented on blackboard

Mobile Robot Maneuverability

- The maneuverability of a mobile robot is the combination
  - *of the mobility available based on the sliding constraints*
  - *plus additional freedom contributed by the steering*

- Three wheels is sufficient for static stability
  - *additional wheels need to be synchronized*
  - *this is also the case for some arrangements with three wheels*

- It can be derived using the equation seen before
  - *Degree of mobility* δ_m
  - *Degree of steerability* δ_s
  - *Robots maneuverability* $\delta_M = \delta_m + \delta_s$

Mobile Robot Maneuverability: Degree of Mobility

- To avoid any lateral slip the motion vector $R(\theta)\dot{\xi}_I$ has to satisfy the following constraints:

$$C_{1f} R(\theta)\dot{\xi}_I = 0$$

$$C_{1s}(\beta_s) R(\theta)\dot{\xi}_I = 0$$

$$C_1(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}$$

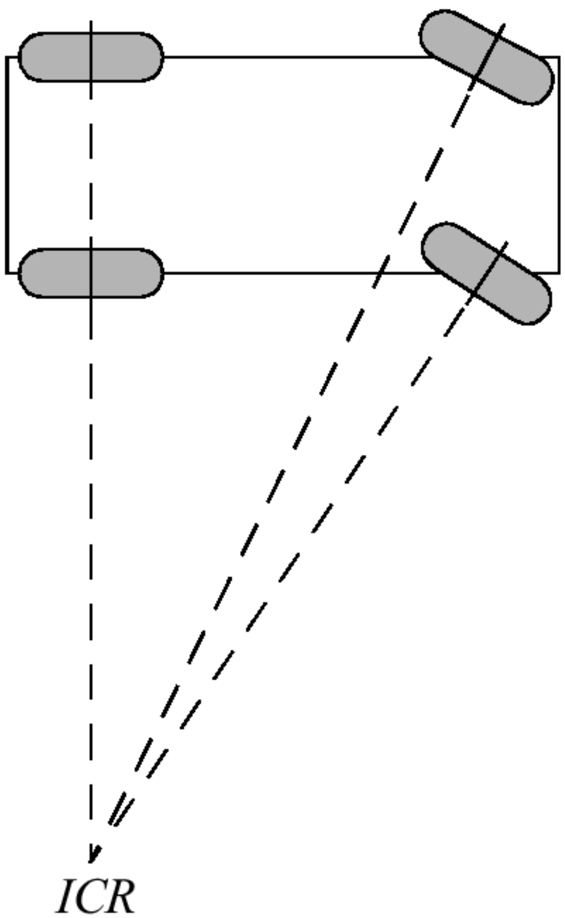
- Mathematically:
  - $R(\theta)\dot{\xi}_I$ *must belong to the* null space *of the projection matrix* $C_1(\beta_s)$
  - Null space *of* $C_1(\beta_s)$ *is the space* N *such that for any vector* n *in* N

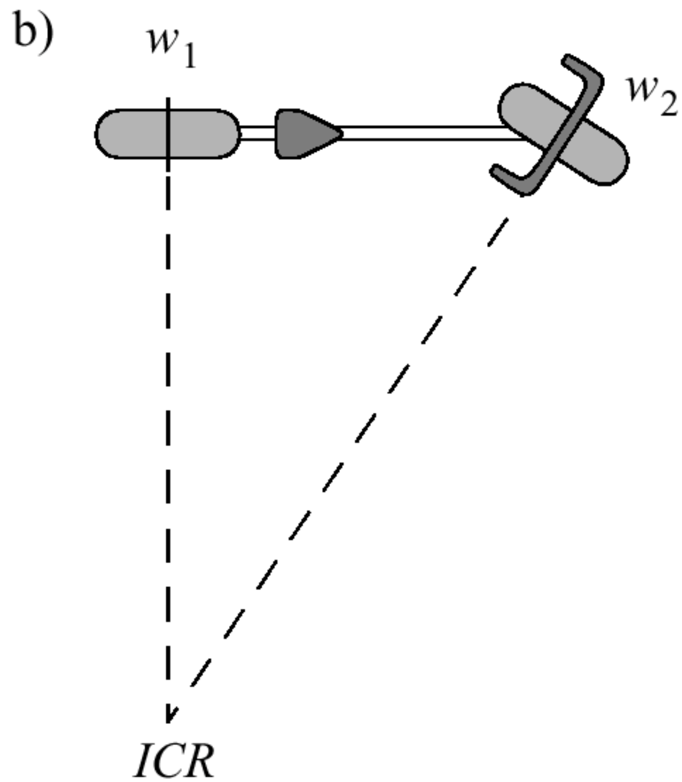
$$C_1(\beta_s) \cdot n = 0$$

  - *Geometrically this can be shown by the* Instantaneous Center of Rotation (ICR)

Mobile Robot Maneuverability: Instantaneous Center of Rotation

- Ackermann Steering Bicycle





Mobile Robot Maneuverability: More on Degree of Mobility

- Robot chassis kinematics is a function of the set of *independent constraints* $rank\left[C_1(\beta_s)\right]$
  - *the greater the rank of , $C_1(\beta_s)$ the more constrained is the mobility*

- Mathematically

$$\delta_m = \dim N\left[C_1(\beta_s)\right] = 3 - rank\left[C_1(\beta_s)\right] \qquad 0 \le rank\left[C_1(\beta_s)\right] \le 3$$

  - *no standard wheels* $rank\left[C_1(\beta_s)\right] = 0$
  - *all direction constrained* $rank\left[C_1(\beta_s)\right] = 3$

- Examples:
  - *Unicycle: One single fixed standard wheel*
  - *Differential drive: Two fixed standard wheels*
    - *wheels on same axle*
    - *wheels on different axle*

Mobile Robot Maneuverability: Degree of Steerability

- Indirect degree of motion

$$\delta_s = rank\left[C_{1s}(\beta_s)\right]$$

  - *The particular orientation at any instant imposes a kinematic constraint*
  - *However, the ability to change that orientation can lead additional degree of maneuverability*
- Range of δ_s : $0 \leq \delta_s \leq 2$

- Examples:
  - *one steered wheel: Tricycle*
  - *two steered wheels: No fixed standard wheel*
  - *car (Ackermann steering):* $N_f = 2$*,* $N_s = 2$ *-> common axle*

Mobile Robot Maneuverability: Robot Maneuverability

- Degree of Maneuverability

$$\delta_M = \delta_m + \delta_s$$

> - *Two robots with same* δ_M *are not necessary equal*
> - *Example: Differential drive and Tricycle (next slide)*

> - *For any robot with* $\delta_M = 2$ *the ICR is always constrained to* lie on a line
> - *For any robot with* $\delta_M = 3$ *the ICR is not constrained an can* be set to any point on the plane

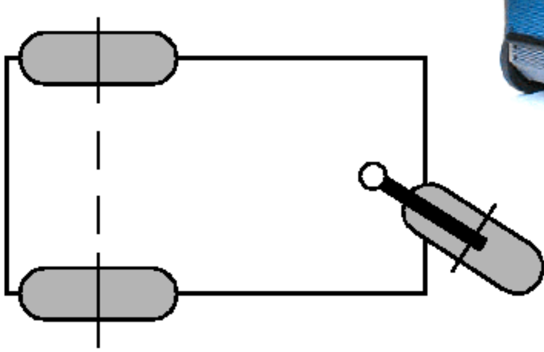
- The Synchro Drive example: $\delta_M = \delta_m + \delta_s = 1 + 1 = 2$

Mobile Robot Maneuverability: Wheel Configurations

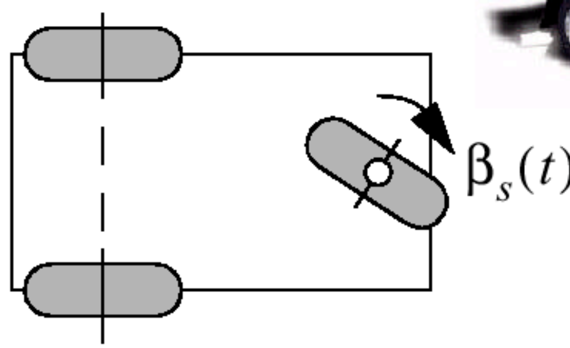
- Differential Drive

Tricycle

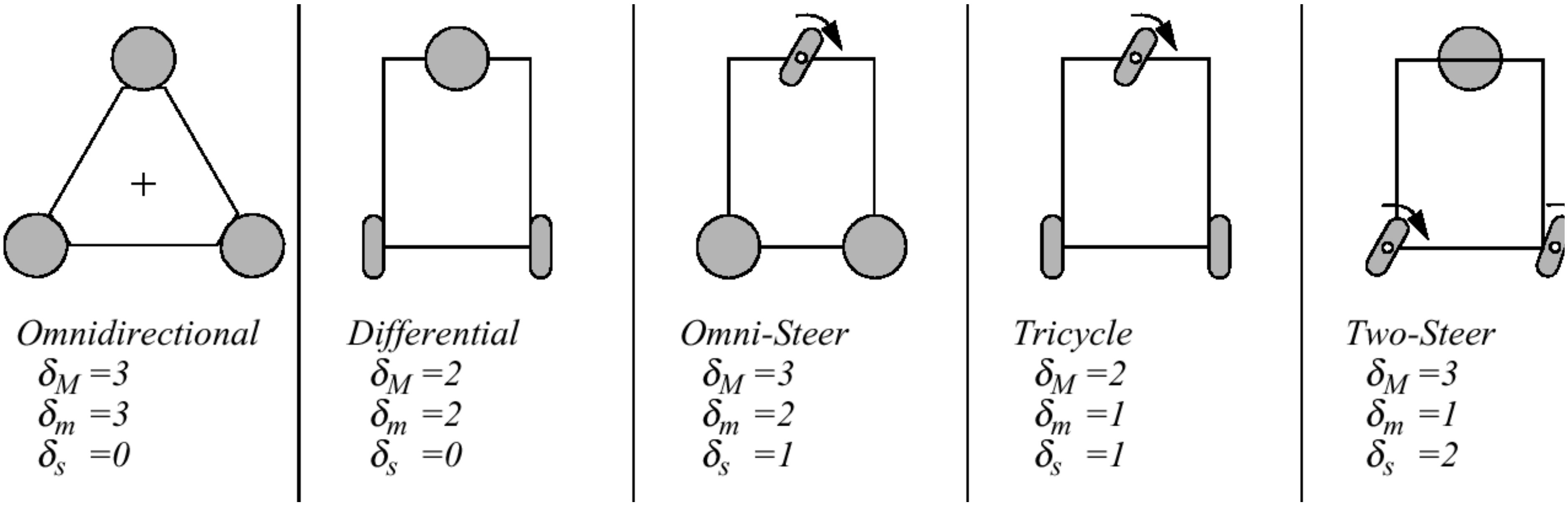
a)



b)

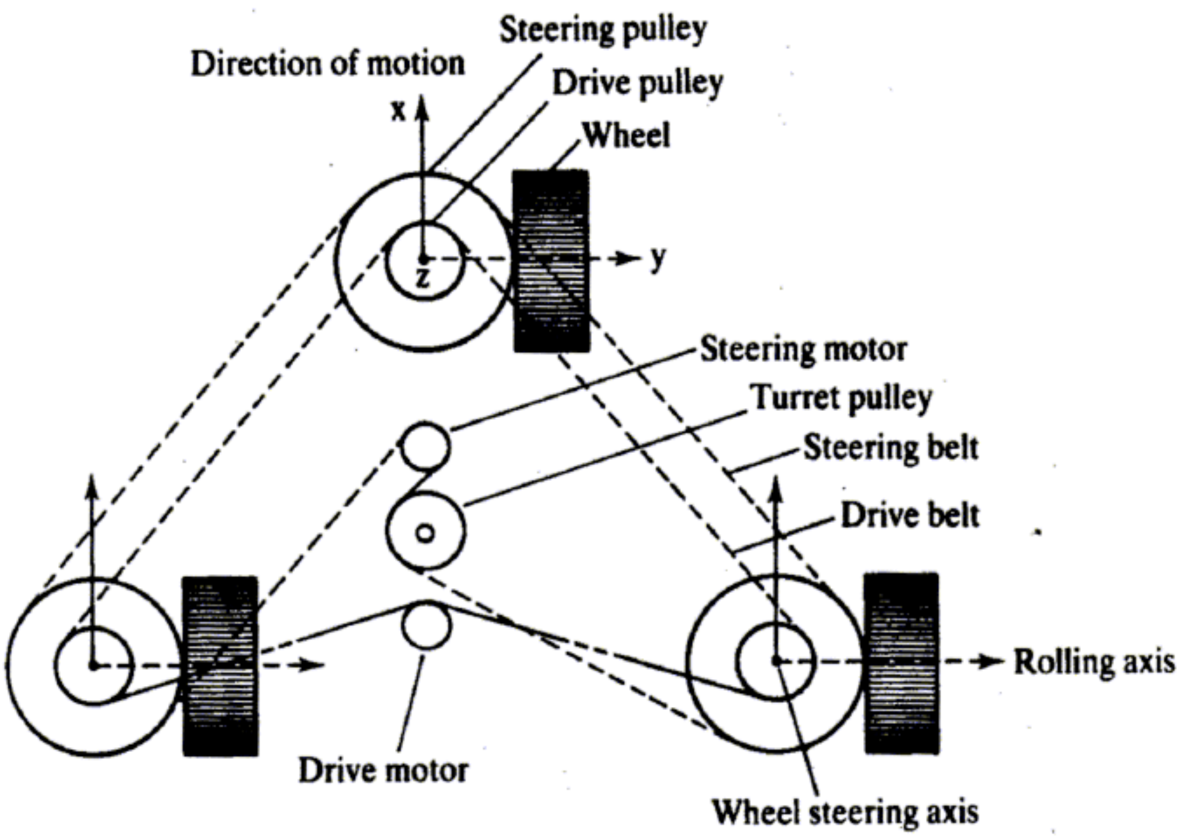


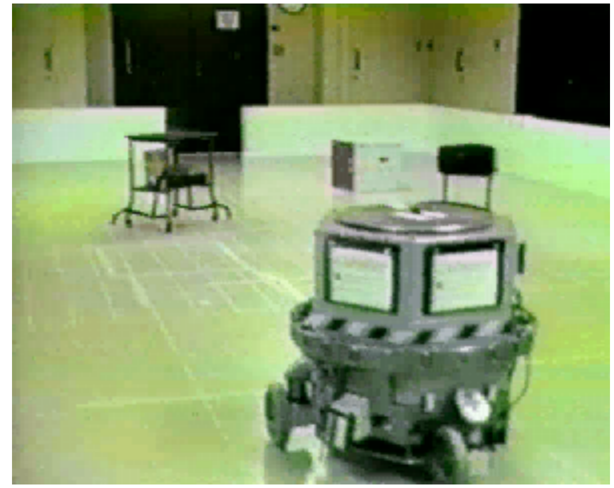
Five Basic Types of Three-Wheel Configurations



Synchro Drive

$$\delta_M = \delta_m + \delta_s = 1 + 1 = 2$$





Mobile Robot Workspace: Degrees of Freedom

- Maneuverability is equivalent to the vehicle's degree of freedom (DOF)
- But what is the degree of vehicle's freedom in its environment?
  - *Car example*
- Workspace
  - *how the vehicle is able to move between different configuration in its workspace?*
- The robot's independently achievable velocities
  - = differentiable degrees of freedom (DDOF) = δ_m
  - *Bicycle:* $\delta_M = \delta_m + \delta_s = 1 + 1$ *DDOF = 1; DOF=3*
  - *Omni Drive:* $\delta_M = \delta_m + \delta_s = 1 + 1$ *DDOF=3; DOF=3*

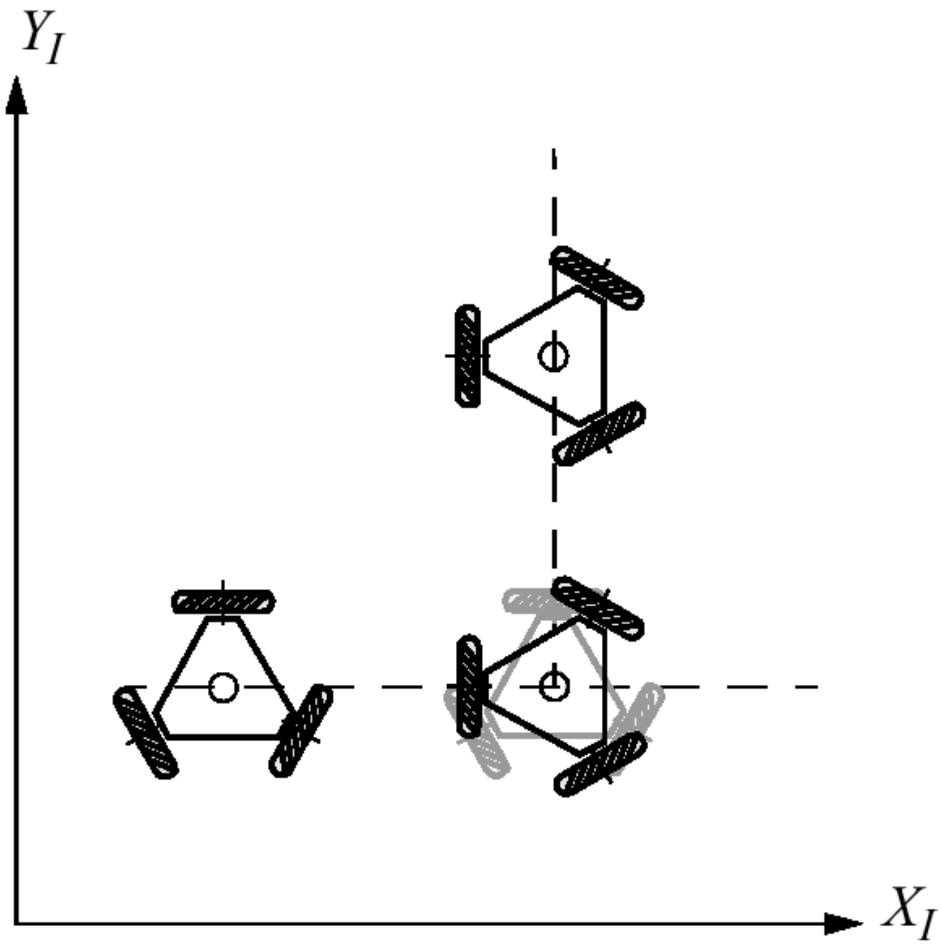
Mobile Robot Workspace: Degrees of Freedom, Holonomy

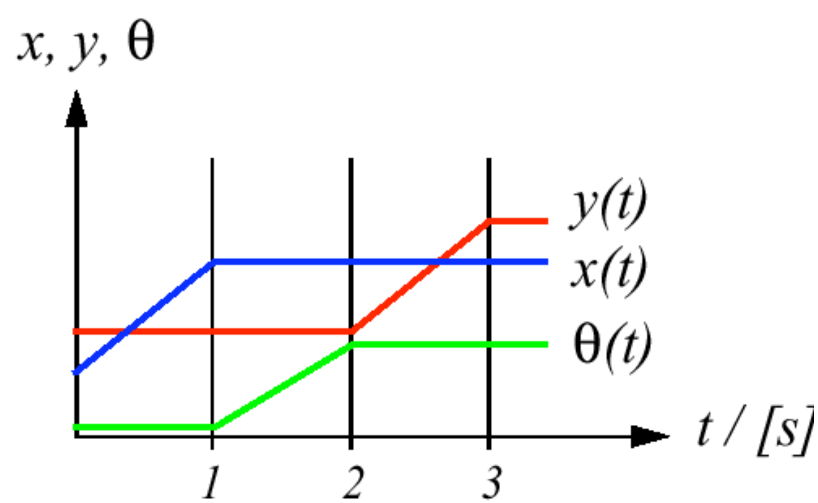
- DOF *degrees of freedom*:
  - *Robots ability to achieve various poses*
- DDOF *differentiable degrees of freedom*:
  - *Robots ability to achieve various path*

$$DDOF \leq \delta_m \leq DOF$$

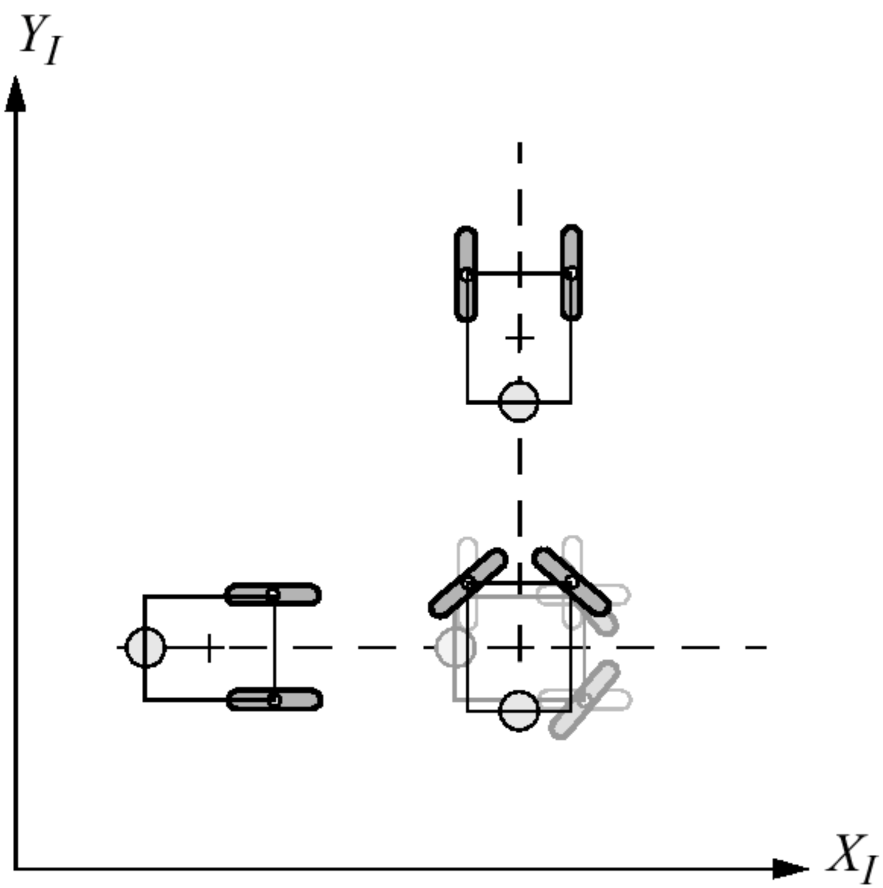
- Holonomic Robots
  - *A holonomic kinematic constraint can be expressed a an explicit function of position variables only*
  - *A non-holonomic constraint requires a different relationship, such as the derivative of a position variable*
  - Fixed and steered standard wheels impose non-holonomic constraints

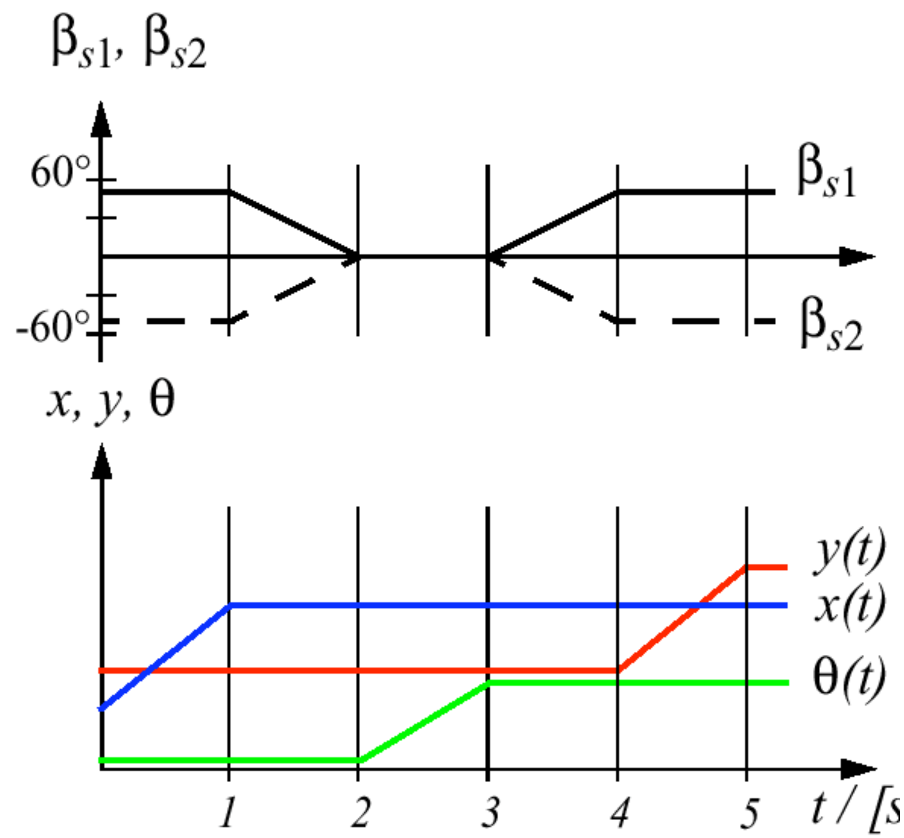
Path / Trajectory Considerations: Omnidirectional Drive





Path / Trajectory Considerations: Two-Steer



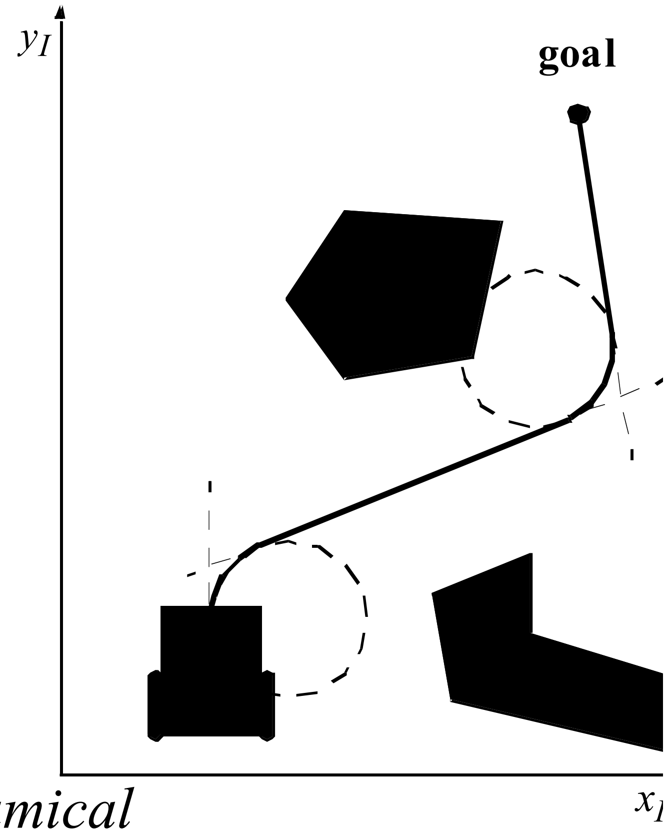


Motion Control (kinematic control)

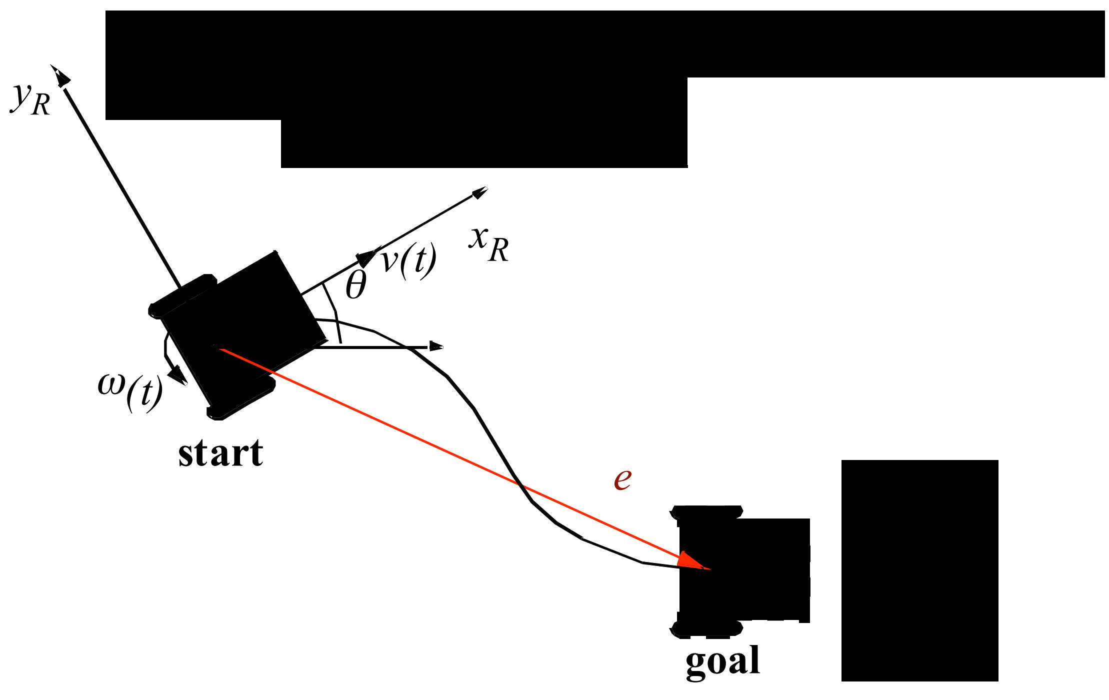
- The objective of a kinematic controller is to follow a trajectory described by its position and/or velocity profiles as function of time.
- Motion control is not straight forward because mobile robots are non-holonomic systems.
- However, it has been studied by various research groups and some adequate solutions for (kinematic) motion control of a mobile robot system are available.
- Most controllers are not considering the dynamics of the system

Motion Control: Open Loop Control

- trajectory (path) divided in motion segments of clearly defined shape:
  - *straight lines and segments of a circle.*
- control problem:
  - *pre-compute a smooth trajectory based on line and circle segments*
- Disadvantages:
  - *It is not at all an easy task to pre-compute a feasible trajectory*
  - *limitations and constraints of the robots velocities and accelerations*
  - *does not adapt or correct the trajectory if dynamical changes of the environment occur.*
  - *The resulting trajectories are usually not smooth*



Motion Control: Feedback Control, Problem Statement



- Find a control matrix *K*, if exists

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix}$$

with $k_{ij}=k(t,e)$

- such that the control of $v(t)$ and $\omega(t)$

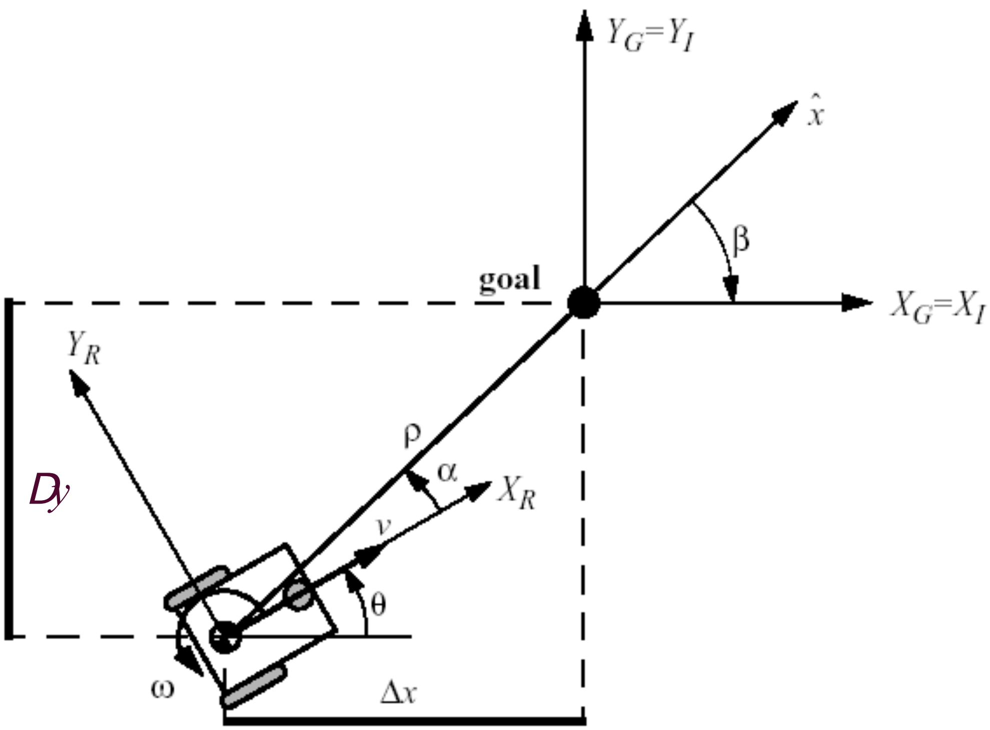
$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = K \cdot e = K \cdot {}^{R}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

- drives the error e to zero.

$$\lim_{t \to \infty} e(t) = 0$$

Motion Control:

Kinematic Position Control



The kinematic of a differential drive mobile robot described in the initial frame $\{x_I, y_I, q\}$ is given by,

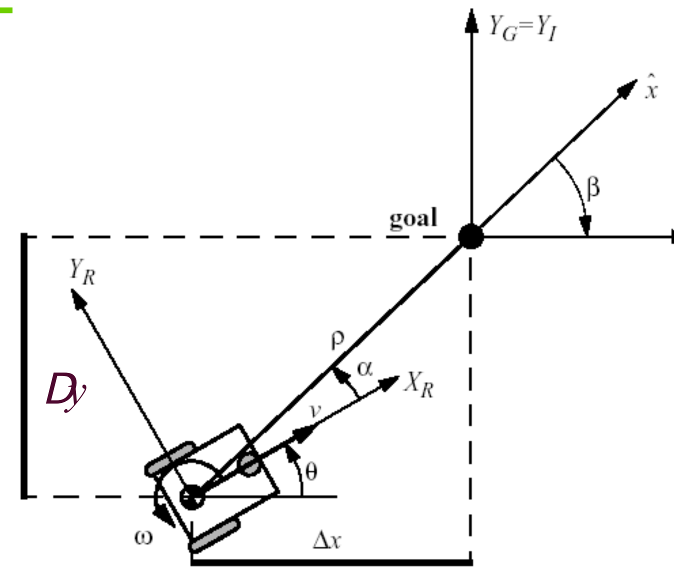
$$^{I}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix}$$

where and are the linear velocities in the direction of the x_I and y_I of the initial frame.

Let a denote the angle between the x_R axis of the robots reference frame and the vector connecting the center of the axle of the wheels with the final position.

Kinematic Position Control: Coordinates Transformation

Coordinates transformation into polar coordinates with its origin at goal position:



$$\rho = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\alpha = -\theta + \operatorname{atan2}(\Delta y, \Delta x)$$

$$\beta = -\theta - \alpha$$

System description, in the new polar coordinates

$$\begin{bmatrix} \dot\rho \\ \dot\alpha \\ \dot\beta \end{bmatrix} = \begin{bmatrix} -\cos\alpha & 0 \\ \frac{\sin\alpha}{\rho} & -1 \\ -\frac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

for $I_1 = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\begin{bmatrix} \dot\rho \\ \dot\alpha \\ \dot\beta \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 \\ -\frac{\sin\alpha}{\rho} & 1 \\ \frac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

for $I_2 = (-\pi, -\pi/2] \cup (\pi/2, \pi]$

Kinematic Position Control: Remarks

- The coordinates transformation is not defined at $x = y = 0$; as in such a point the determinant of the Jacobian matrix of the transformation is not defined, i.e. it is unbounded

- For $\alpha \in I_1$ the forward direction of the robot points toward the goal, for $\alpha \in I_2$ it is the backward direction.

- By properly defining the forward direction of the robot at its initial configuration, it is always possible to have $\alpha \in I_1$ at t=0. However this does not mean that a remains in I_1 for all time t.

Kinematic Position Control: The Control Law

- It can be shown, that with

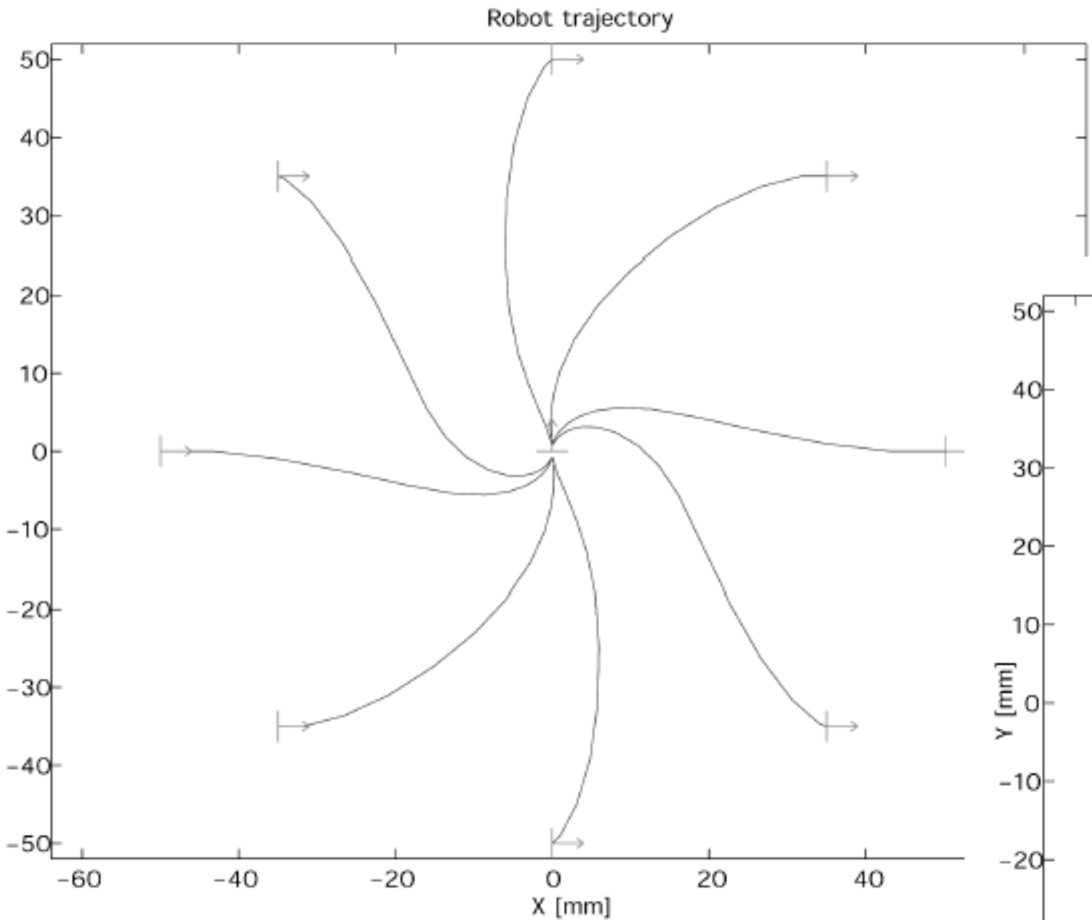
$$v = k_\rho \rho \qquad \omega = k_\alpha \alpha + k_\beta \beta$$

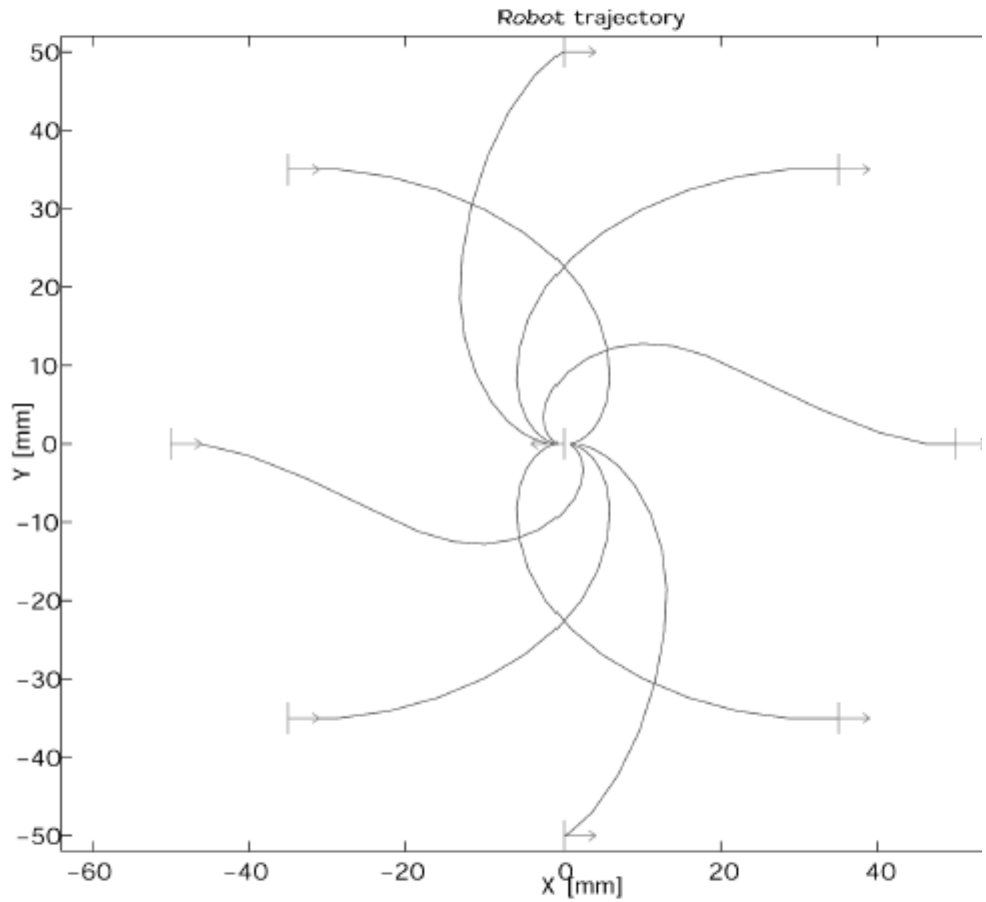
the feedback controlled system

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_\rho \rho \cos\alpha \\ k_\rho \sin\alpha - k_\alpha \alpha - k_\beta \beta \\ -k_\rho \sin\alpha \end{bmatrix}$$

- will drive the robot to $(\rho, \alpha, \beta) = (0,0,0)$
- The control signal v has always constant sign,
  - *the direction of movement is kept positive or negative during movemen*
  - *parking maneuver is performed always in the most natural way and without ever inverting its motion.*

Kinematic Position Control: Resulting Path





Kinematic Position Control: Stability Issue

- It can further be shown, that the closed loop control system is locally exponentially stable if

$$k_\rho > 0 \quad ; \quad k_\beta < 0 \quad ; \quad k_\alpha - k_\rho > 0$$

- Proof:
  for small x −> $\cos x = 1$, $\sin x = x$

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_\rho & 0 & 0 \\ 0 & -(k_\alpha - k_\rho) & -k_\beta \\ 0 & -k_\rho & 0 \end{bmatrix} \begin{bmatrix} \rho \\ \alpha \\ \beta \end{bmatrix} \qquad A = \begin{bmatrix} -k_\rho & 0 & 0 \\ 0 & -(k_\alpha - k_\rho) & -k_\beta \\ 0 & -k_\rho & 0 \end{bmatrix}$$

  and the characteristic polynomial of the matrix A of all roots

$$(\lambda + k_\rho)(\lambda^2 + \lambda(k_\alpha - k_\rho) - k_\rho k_\beta)$$

  have negative real parts.

Summary

- This lecture looked at robot kinematics
- The point of discussing kinematic models is:
  - *To allow us to build models of how a robot will move given particular actuator outputs*
  - *A robot can then figure out what it should have done ☺*
- We use this:
  - *To establish the correct control regime for a specific task; or*
  - *A input to the localization process*

    (if I know where I was, and how I have moved, then I have some idea where I am now).