LOCAL SEARCH AND CONSTRAINT SATISFACTION

Introduction

- We have already looked in some detail at search techniques.
  - Both for single agent and multiagent problems.
- However, there are a couple of other topics we should look at.
  - Local search
  - Constraint satisfaction

  both of which permeate artificial intelligence.
- They are also useful techniques for all computer scientists to know.

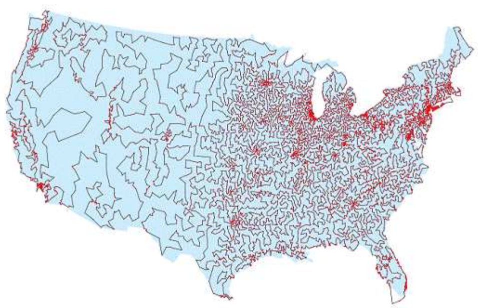
Iterative improvement

- For many problems, the *path* is irrelevant, we just want to find the goal state.
  - Optimization problems
- The state space is the set of configurations.
- We want:
  - the optimum configuration.
  - a configuration that satisfies constraints
- In these cases we can take any state and work to improve it.
  - "Local" since only keep a small part of the state space.
- Constant space.

Travelling salesman

- Problem is to visit all cities once while travelling the shortest distance.

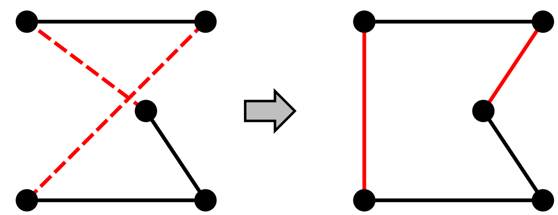
- 13,509 U.S. cities with populations of more than 500 people.



(Rice University, 2003).

Travelling salesman iteratively

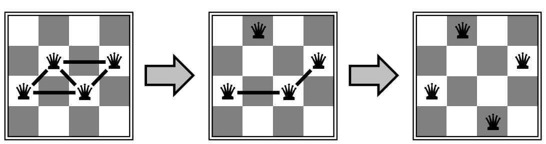
- Given a tour, do pairwise exchanges.



- Variants of this get within 1% of optimal very quickly for large numbers of points.

n-queens

- Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal
- Move a queen to reduce number of conflicts



- Almost always solves n-queens problems almost instantaneously for very large n, e.g., $n \approx 1$ *million*.

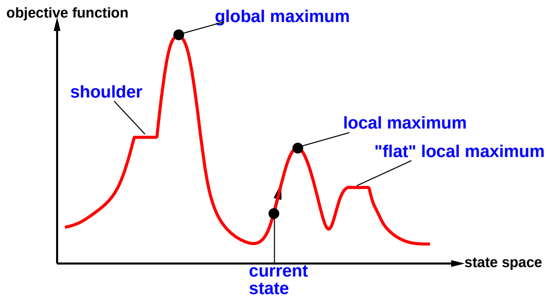
Hill-climbing

function HILL-CLIMBING(*problem*) **returns** a local maximum
  inputs: *problem*, a problem
  local variables: *current*, a node
        neighbor, a node

  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  loop do
    neighbor ← a highest-valued successor of *current*
    if VALUE[neighbor] ≤ VALUE[current]
      then return STATE[*current*]
    current ← *neighbor*
  end

- Hill climbing is also known as:
  - Gradient ascent.
  - Gradient descent.
- Like climbing a hill in the fog with amnesia.
  - All you can do is keep heading up until you get to the top.

- Useful to consider *state space landscape*



- *Random-restart hill climbing* overcomes local maxima—trivially complete.
  - Eventually you start from the bottom of every hill.
- *Random sideways moves* escapes from shoulders but loops on flat maxima

Simulated annealing

- Idea: escape local maxima by allowing some "bad" moves

  but gradually decrease their size and frequency
- The random jumping around should mean that, over time, we find the highest maximum.

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
inputs: *problem*, a problem
schedule, a mapping from time to "temperature"
local variables: *current*, a node *next*, a node
T, "temperature"

current ← MAKE-NODE(INITIAL-STATE[*problem*])
for $t \leftarrow 1$ **to** ∞ **do**
  $T \leftarrow$ *schedule*[t]
  if $T = 0$ **then return** *current*
  next ← a randomly selected successor of *current*
  $\Delta E \leftarrow$ VALUE[*next*] – VALUE[*current*]
  if $\Delta E > 0$ **then** *current* ← *next*
  else *current* ← *next* only with probability $e^{\Delta E/T}$

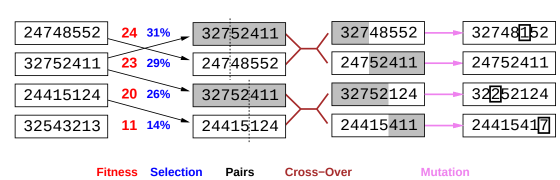
- At fixed "temperature" T, state occupation probability reaches Boltzman distribution
$$p(x) = \alpha e^{\frac{E(x)}{kT}}$$
- If T is decreased slowly enough we always reach best state x^*
  because
$$e^{\frac{E(x^*)}{kT}} / e^{\frac{E(x)}{kT}} = e^{\frac{E(x^*)-E(x)}{kT}} \gg 1$$
  for small T
- Widely used in VLSI layout, airline scheduling, etc.

Local beam search

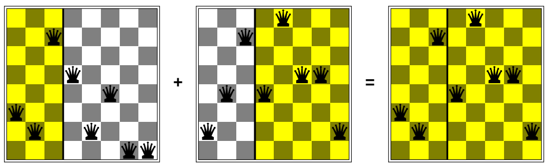
- Idea: keep k states instead of 1; choose top k of all their successors
- Not the same as k searches run in parallel!
  - Searches that find good states recruit other searches to join them.
- Problem: quite often, all k states end up on same local hill
- Idea: choose k successors randomly, biased towards good ones
- Observe the close analogy to natural selection!

Genetic algorithms

- Stochastic local beam search + generate successors from *pairs* of states



- GAs require states encoded as strings (*GPs* use *programs*)
- Crossover helps *iff substrings are meaningful components*

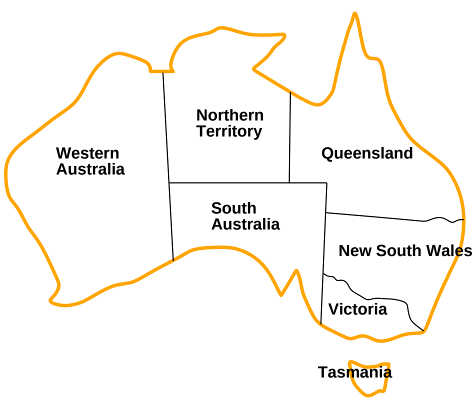


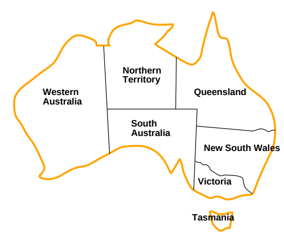
- GAs $\neq$ evolution
  - real genes encode replication machinery!

Constraint satisfaction

- Another approach to optimization.
- In standard search problems a *state* is a “black box”—any old data structure that supports goal test, eval, successor
- In CSP a *state* is defined by *variables* X_i with *values* from a *domain* D_i
- The *goal test* is a set of *constraints* specifying allowable combinations of values for subsets of variables.
- Simple example of a *formal representation language*.
- Allows useful *general-purpose* algorithms with more power than standard search algorithms

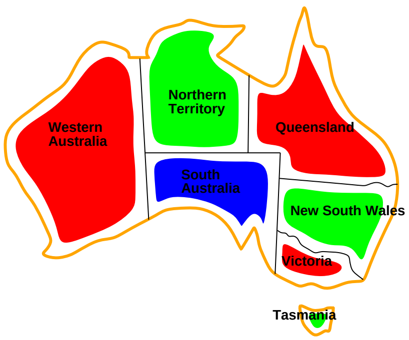
Map-Coloring





- *Variables*: WA, NT, Q, NSW, V, SA, T
- *Domains*: $D_i = \{red, green, blue\}$
- *Constraints*: adjacent regions must have different colors
  - $WA \neq NT$, or
  - $(WA, NT) \in \{(red, green), (red, blue), (green, red), (green, blue), \ldots\}$

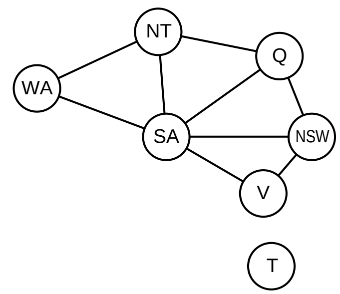
- *Solutions* are assignments satisfying all constraints,



$\{WA = red, NT = green, Q = red, NSW = green, V = red, SA = blue, T = green\}$

Constraint graph

- *Binary CSP*: each constraint relates at most two variables



- *Constraint graph*: nodes are variables, arcs show constraints

- General-purpose CSP algorithms use the graph structure to speed up search.
  - Tasmania is an independent subproblem!

- Discrete variables

  Finite domains; size $d \Rightarrow O(d^n)$ complete assignments
  - Boolean CSPs, including Boolean satisfiability (NP-complete)

  Infinite domains (integers, strings, etc.)
  - job scheduling, variables are start/end days for each job
  - need a *constraint language*, e.g., $StartJob_1 + 5 \leq StartJob_3$
  - *linear* constraints solvable, *nonlinear* undecidable

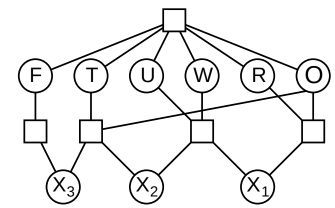
  Continuous variables
  - start/end times for Hubble Telescope observations
  - linear constraints solvable in polynomial time by LP methods

- *Unary* constraints involve a single variable
  - $SA \neq green$
- *Binary* constraints involve pairs of variables
  - $SA \neq WA$
- *Higher-order* constraints involve 3 or more variables
  - cryptarithmetic column constraints
- *Preferences* (soft constraints)
  - *red* is better than *green*

  often representable by a cost for each variable assignment → constrained optimization problems

Cryptarithmetic

```
  T W O
+ T W O
F O U R
```



- Variables: $F\ T\ U\ W\ R\ O\ X_1\ X_2\ X_3$
- Domains: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- Constraints: $alldiff(F, T, U, W, R, O)$, $O + O = R + 10 \cdot X_1$, etc.

Real-world CSPs

- Assignment problems
  - who teaches what class
- Timetabling problems
  - which class is offered when and where?
- Hardware configuration
- Spreadsheets
- Transportation scheduling
- Factory scheduling
- Floorplanning

Notice that many real-world problems involve real-valued variables

Standard search formulation

- Let's start with the straightforward, dumb approach, then fix it
- States are defined by the values assigned so far
  - Initial state: the empty assignment, $\emptyset$
  - Successor function: assign a value to an unassigned variable that does not conflict with current assignment. ⇒ fail if no legal assignments (not fixable!)
  - Goal test: the current assignment is complete
- This is the same for all CSPs!
- Every solution appears at depth n with n variables ⇒ use depth-first search
- Path is irrelevant, so can also use complete-state formulation
- $b = (n - \ell)d$ at depth ℓ, hence $n!d^n$ leaves!

Backtracking search

- Variable assignments are *commutative*, i.e., [$WA = red$ then $NT = green$] same as [$NT = green$ then $WA = red$]
- Only need to consider assignments to a single variable at each node so $b = d$ and there are d^n leaves.
- Depth-first search for CSPs with single-variable assignments is called *backtracking* search
- Backtracking search is the basic uninformed algorithm for CSPs
- Can solve n-queens for $n \approx 25$.

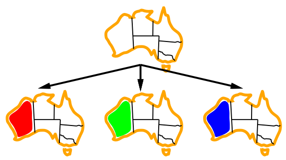

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
  return RECURSIVE-BACKTRACKING({ }, csp)
```



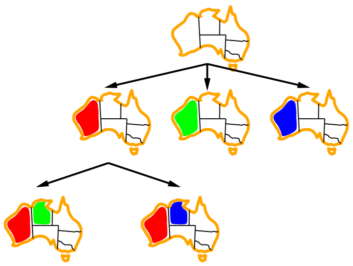
```
function RECURSIVE-BACKTRACKING(assignment, csp) returns
    soln/failure
  if assignment is complete then return assignment
  var ← SELECT-UNASSIGNED-VARIABLE
  (VARIABLES[csp], assignment, csp)
  for each value in ORDER-DOMAIN-VALUES(var, assignment, csp)
  do
    if value is consistent with assignment given CONSTRAINTS[csp]
     then
        add {var = value} to assignment
        result ← RECURSIVE-BACKTRACKING(assignment, csp)
        if result ≠ failure then return result
        remove {var = value} from assignment
  return failure
```


- No variables assigned values.

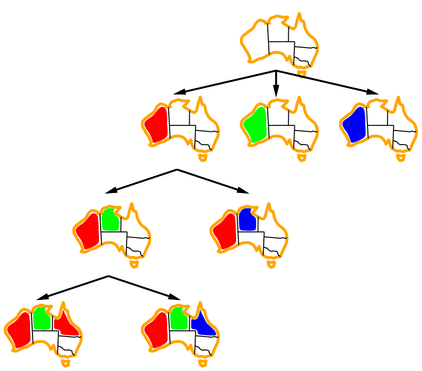
- Assign one variable each of the possible values.



- Then take one of those proto-solutions and assign another variable each possible value.



- And so on, until you get a solution, or a failure.



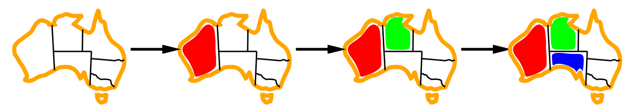
- The search has the name *backtracking* because of what happens when the solution fails.
- Search jumps back to the most recent branch point.
  - The "back track"
- Does this method of searching remind you of anything we have seen already?

Improving efficiency

- *General-purpose* methods can give huge gains in speed:
  1. Which variable should be assigned next?
  2. In what order should its values be tried?
  3. Can we detect inevitable failure early?
  4. Can we take advantage of problem structure?

Minimum remaining values (MRV)

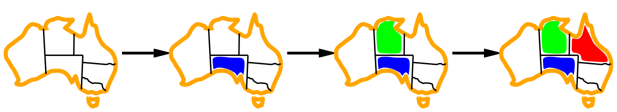
- Choose the variable with the fewest legal values



- Reduces the number of states explored before failure/solution.

Degree heuristic

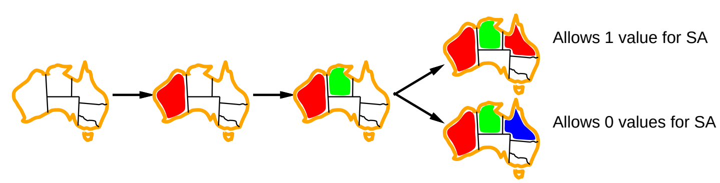
- Tie-breaker among MRV variables
- Choose the variable with the most constraints on remaining variables



- Again, reduces the amount of branching below each choice point.

Least constraining value

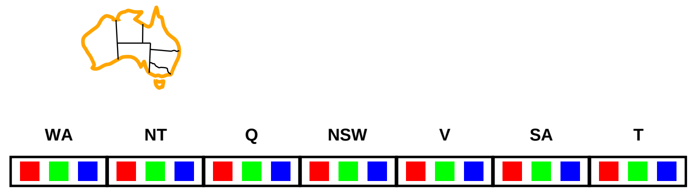
- When there are several values to choose from apply this heuristic.
- Given a variable, choose the least constraining value — the one that rules out the fewest values in the remaining variables

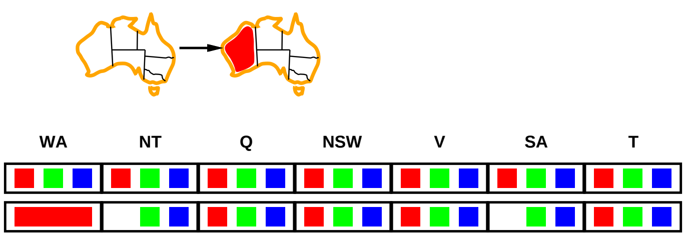


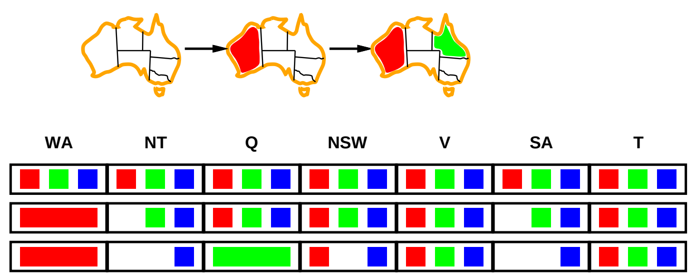
- Combining these heuristics makes 1000 queens feasible

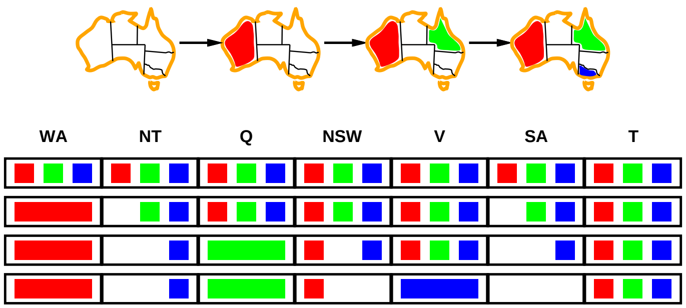
Forward-checking

- Keep track of remaining legal values for unassigned variables
- Terminate search when any variable has no legal values
- This is a form of *inference*.
  - We figure out the effect of the choice of variable value before we get to the relevant point in the search.



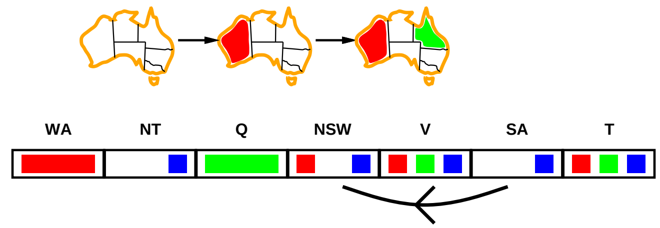




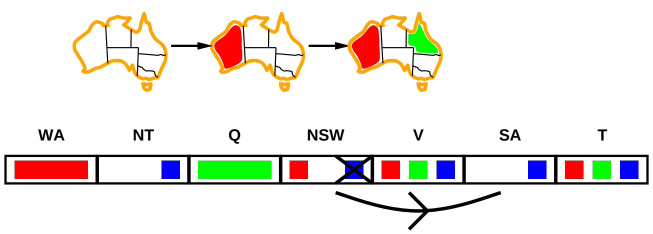


Arc-consistency

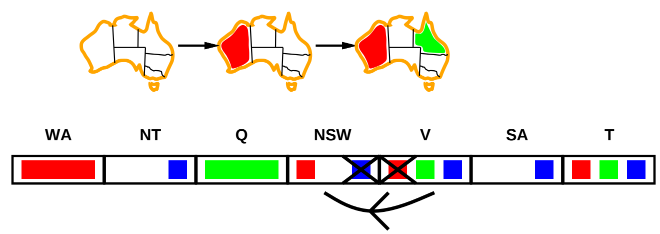
- Simplest form of propagation makes each arc *consistent*
- $X \to Y$ is consistent iff for *every* value x of X there is *some* allowed y that Y can take.



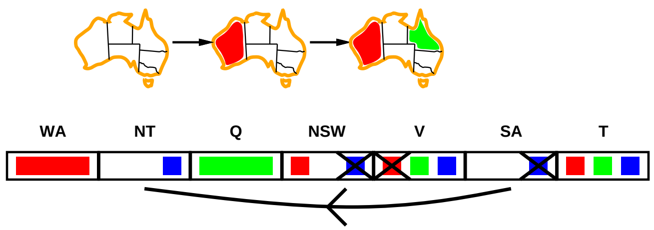
- SA is consitent with NSW



- But NSW is *not* consistent with SA.



- IF X loses a value, then its neighbors need to be rechecked.



- Arc consistency detects failure earlier than forward checking because of this propagation.
- Run it after each new assignment of values.

function AC-3(*csp*) **returns**
the CSP, possibly with reduced domains
inputs: *csp*, a binary CSP with variables $\{X_1, X_2, \ldots, X_n\}$
local variables: *queue*, a queue of arcs, initially all the arcs in *csp*

```
while queue is not empty do
   (X_i, X_j) ← REMOVE-FIRST(queue)
   if REMOVE-INCONSISTENT-VALUES(X_i, X_j) then
      for each X_k in NEIGHBORS[X_i] do
         add (X_k, X_i) to queue
```


function REMOVE-INCONSISTENT-VALUES(X_i, X_j) **returns** true iff succeeds

```
   removed ← false
   for each x in DOMAIN[X_i] do
      if no value y in DOMAIN[X_j] allows (x,y)
            to satisfy the constraint X_i ↔ X_j
         then delete x from DOMAIN[X_i]; removed ← true
   return removed
```


Summary

- We have looked at some variations of search that work when we are only interested in the solution, *not* the path.
- We looked at local search:
  - Iterative improvement
  - Hill-climbing
  - Simulated annealing
  - Genetic algorithms
- Then we looked at constraint propagation.
- We only scratched the surface of all of these topics — the textbook covers much more on both topics.