SEARCH

Overview

Aims of the this lecture:

- Introduce *problem solving*;
- Introduce *goal formulation*;
- Show how problems can be stated as *state space search*;
- Show the importance and role of *abstraction*;
- Introduce *undirected* and *heuristic* search:
  - breadth first, depth first search;
  - best first search, A*
- Define main performance measures for search.

Problem Solving Agents

- Lecture 1 introduced *rational agents* but didn't say much about how we might construct them.
- Today we make a start on understanding how to do this.
- Consider agents as *problem solvers*:

  Systems that have *goals* and find *sequences of actions* that achieve these goals.

function SIMPLE-PROBLEM-SOLVING-AGENT(*percept*) **returns** an action

  static: *seq*, an action sequence, initially empty
    state, some description of the current world state
    goal, a goal, initially null
    problem, a problem formulation

  state ← UPDATE-STATE(*state, percept*)
  if *seq* is empty **then**
    goal ← FORMULATE-GOAL(*state*)
    problem ← FORMULATE-PROBLEM(*state, goal*)
    seq ← SEARCH(*problem*)
  action ← RECOMMENDATION(*seq, state*)
  seq ← REMAINDER(*seq, state*)
  return *action*

- Key difficulties:
  - FORMULATE-GOAL(...)
  - FORMULATE-PROBLEM(...)
  - SEARCH(...)—
- It isn't easy to see how to tackle any of these.
- Here we will concentrate mainly on search but first we'll say a bit about goal formulation and problem formulation.

Goal Formulation

- Where do an agent's goals come from?
  - Agent is a *program* with a *specification*.
  - Specification is to maximise performance measure.
  - Should *adopt goal* if achievement of that goal will maximise this measure.
- But what does that mean in practice?

- As the textbook suggests, let's imagine we (or any other agent) are in Arad, Romania:

- On a given day, we might do a number of things:
  - get a suntan;
  - go sightseeing;
  - improve our spoken Romanian;
  - enjoy the nightlife;
  - avoid a hangover; and so on
- But if we have a non-refundable ticket for a flight from Bucharest the next day, then we can eliminate most of these options, and adopt the goal of getting to Bucharest.
- Anything else will clearly have a lower value.

- Goals provide a *focus* and *filter* for decision-making:
  - *focus*: need to consider how to achieve them;
  - *filter*: need not consider actions that are incompatible with goals.
- Both of these help computationally.

Problem Formulation

- What is a problem?
- Formal definition is that a problem contains 5 components:
  - Initial state;
  - Actions;
  - Transition model;
  - Goal test; and
  - Path cost.
- Let's look at each of these in detail.

Initial state

- The state that the agent starts in.
- In the Romania example the initial state might be described as:

$$In(Arad)$$

- We could obviously include a lot more detail:

$$\begin{array}{r} In(Arad) \\ Temperature(high) \\ Suntan(acceptable) \\ Romanian(rudimentary) \end{array}$$

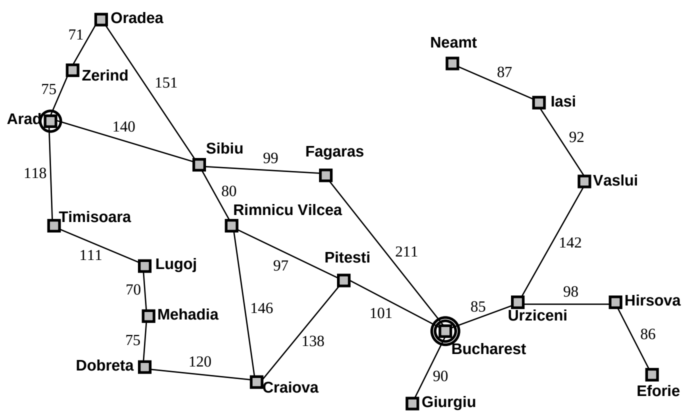
  and finding the corrected level of *abstraction* is important.
- Too much detail and (as we will see) the problem can be intractable.

Actions

- The actions that the agent can perform.
- These tend to be dependent on what state the agent is in.
- Given a particular state s, $\text{ACTIONS}(s)$ is the set of actions that are *applicable*.
- In the Romania example, in the state $In(Arad)$, the relevant actions are:

$$\{Go(Sibiu), Go(Timosoara), Go(Zerind)\}$$

- Again, abstraction is important.



Transition model

- The transition model describes what each action does.
- Formally we have a function $\textsc{Result}(s, a)$ which defines the state the agent gets to when it executes action a in state s.
  We will call the state we get to a *successor state*.
- In the Romania example:

$$\textsc{Result}(In(Arad), Go(Zerind)) = In(Zerind)$$

- For now we will deal with deterministic environments, so that a state only has a single successor.

- The combination of initial state, actions, and transitions define what we call the *state space*.
- This is the set of all states that we can get to from the intitial state.
- The state space can be pictured as a directed graph in which nodes are states and links are actions.
- In the Romania example, the map can be thought of as a picture of the state space.
- A *path* in a state space is a sequence of actions and states.
- A path through the state space from initial state to goal state is a *plan* to get to the goal.

Goal test

- Determines whether a given state is the goal state.
- In the Romania example:

$$\{In(Bucharest)\}$$

  is the goal.
- So a possible goal test would be:

$$Equal(state, In(Bucharest))$$

Path cost

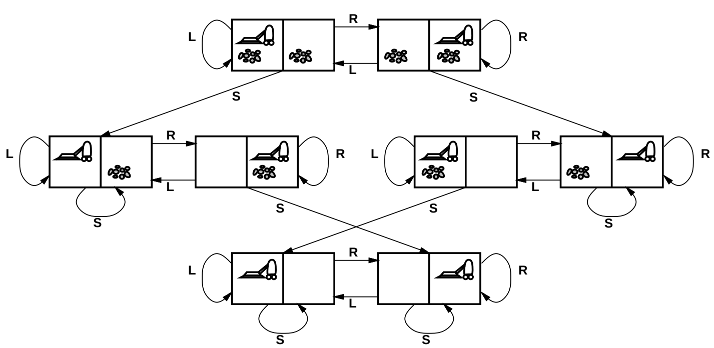
- Function that assigns a numeric cost to each path.
- What we use as a path cost depends on the problem we are solving.
- In the Romania example it makes sense to use distance as a cost function since the agent is in a hurry.
- A more leisurely agent might want to use the price of taking the bus on each leg as the cost function.
- We will often assume that the path cost can be computed as the sum of the costs along a path.
- The *step cost* of taking action a in state s to reach state s' is written as $c(s, a, s')$.

Problem

- Together these elements define a problem.
- A *solution* is an action sequence (plan) that leads from the initial state to the goal.
- The quality of a solution is measured by the path cost.
- The *optimal* solution is the one with the lowest path cost.
- Since we can define the path cost in different ways:
  - Distance
  - Time
  - Monetary cost
  - ...

  there is no loss of generality in equating optimal with the lowest path cost.

Example problem: Vacuum world



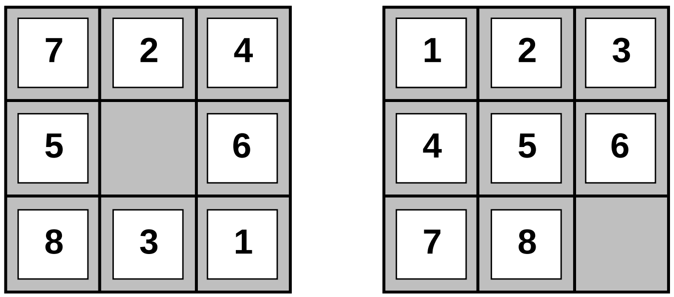
- States: There are two locations, each of which may contain dirt, and the agent can be in either.

  That leads to 8 possible states.

  We might consider any of these to be the initial state.
- Actions: *Left*, *Right*, *Suck*.
- Transition model: The actions work as their names suggest, except that *Left* and *Right* have no effect in (respectively) the leftmost and rightmost positions.

  Suck has no effect in a clean square.
- Goal test: Checks if both squares are clean.
- Path cost: Each step costs 1.

Example problem: 8 puzzle



Start State **Goal State**

- States: Each state specifies the location of each tile and the blank. Any of these can be the initial state.
- Actions: Simplest way to specify actions is to say what happens to the blank — *Left*, *Right*, *Up* and *Down*.

  Not all of these will be applicable in all locations of the blank.
- Transition model: Gives the resulting state of each action. For example *Left* in the initial state above switches the 5 and the blank.
- Goal test: Checks if the goal configuration has been reached.
- Path cost: Each step costs 1.

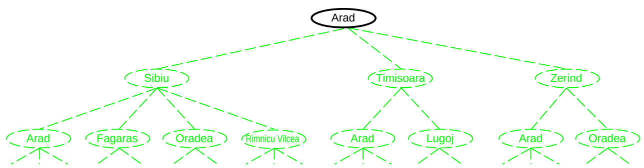
Problem Solving as Search

- As with the Romania example, we can think of the state-space of a problem as a graph.
- Systematically generate a *search tree*
- The tree is built by taking the initial state and identifying some states that can be obtained by applying a single operator.
- These new states become the *children* of the initial state in the tree.
- These new states are then examined to see if they are the goal state.
- If not, the process is repeated on the new states.
- We can formalise this description by giving an algorithm for it.

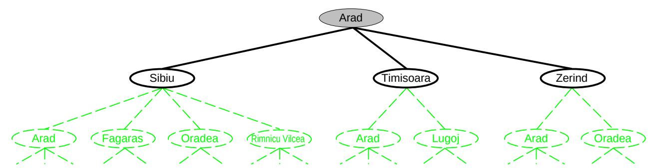
function TREE-SEARCH(*problem*, *strategy*) **returns** a solution, or failure
  initialize the search tree using the initial state of *problem*
  loop do
    if there are no candidates for expansion **then return** failure
    choose a leaf node for expansion according to *strategy*
    if the node contains a goal state **then return**
      the corresponding solution
    else expand the node and add the resulting nodes to
      the search tree
  end

- Note that we call "candidates for expansion" both *fringe* and *frontier*.

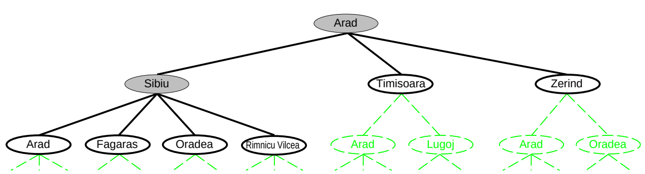
- Initial state



- Successor states of the initial state.



- Successors of the sucessors



- Note how Arad reappears

- Note the difference between *state space* and *search tree*.
- State space is every possible state and the relationships between them.
  - It is *static*.
- Search tree the set of states the agent has looked at (is looking at) and some of the relationships between them.
  - It is *dynamic*.
- Now, about those states that pop up more than once.

function GRAPH-SEARCH(*problem*, *fringe*) **returns** a solution, or failure

```
closed ← an empty set
fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE[node]) then return node
    if STATE[node] is not in closed then
        add STATE[node] to closed
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
end
```


Search strategies

- Question: How to pick states for expansion?
- A range of possibilities:
  - Breadth-first
  - Depth-first
  - Iterative deepening
  - Best-first
  - A*
  - D*, D*-Lite, ...

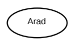
Breadth First Search

- Start by *expanding* initial state — gives tree of depth 1.
- Then expand *all* nodes that resulted from previous step — gives tree of depth 2.
- Then expand *all* nodes that resulted from previous step, and so on.
- Expand nodes at depth n before level $n+1$.

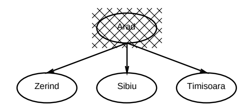
function BREADTH-FIRST-SEARCH(*problem*, *fringe*) **returns** a solution, or failure

```
closed ← an empty set
fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE[node]) then return node
    if STATE[node] is not in closed then
        add STATE[node] to closed
        fringe ← ADDTOBACK(EXPAND(node, problem), fringe)
end
```

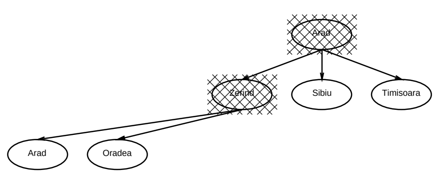
- Add the node representing the initial state into the fringe.



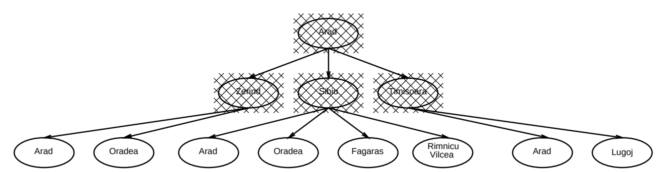
- Remove the first node in the fringe and add its children
- The queue is FIFO.



- Remove the first node in the fringe and add its children — they are added to the back of the queue.



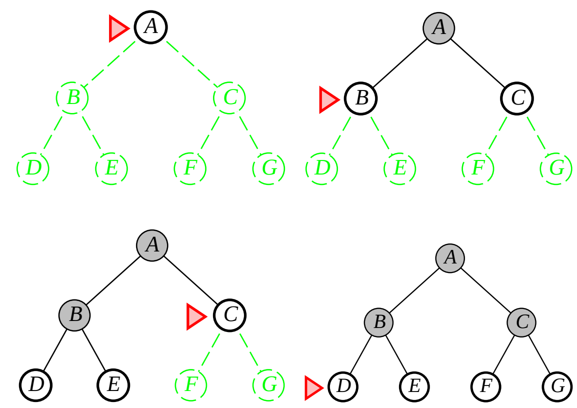
- Repeat twice more.



- Advantage: *guaranteed* to reach a solution if one exists.
- If all solutions occur at depth n, then this is a good approach.
- Disadvantage: time taken to reach solution!
- Let b be *branching factor* — average number of operations that may be performed from any level.
- If solution occurs at depth d, then we will look at

$$1 + b + b^2 + \cdots + b^d$$

nodes before reaching solution — *exponential*.



- Time for breadth first search, $b = 10$, 1 million nodes per second, each node needs 1000 bytes of storage.

Depth	Nodes	Time	Memory
2	110	.11 msec	107 kilobytes
4	11,110	11 msecs	10.6 megabytes
6	10^6	1.1 secs	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
20	10^{20}	350 years	10 exabytes

- *Combinatorial explosion!*

Performance Measures for Search

- *Completeness*:
  Is the search technique *guaranteed* to find a solution if one exists?
- *Time complexity*:
  How many computations are required to find solution?
- *Space complexity*:
  How much memory space is required?
- *Optimality*:
  How good is a solution going to be w.r.t. the path cost function.

- Time and space complexity are measured in terms of:
  - b —maximum branching factor of the search tree.
  - d —depth of the least-cost solution.
  - m —maximum depth of the state space (may be ∞)

- How does breadth-first search measure up?

Uniform-cost search

- Expand least-cost unexpanded node.
- We think of this as having an *evaluation function*:

$$g(n)$$

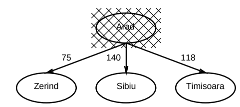
  which returns the path cost to a node n.
- *fringe* = queue ordered by evaluation function, lowest first
- Equivalent to breadth-first if step costs all equal
- Complete and optimal.
- Time and space complexity are as bad as for breadth-first search.

function UNIFORM-COST-SEARCH(*problem*, *fringe*) **returns** a solution, or failure

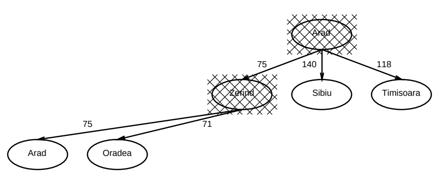
```
closed ← an empty set
fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE[node]) then return node
    if STATE[node] is not in closed then
        add STATE[node] to closed
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
        fringe ← SORTBYGVALUE(fringe)
end
```

- Add the node representing the initial state into the fringe.

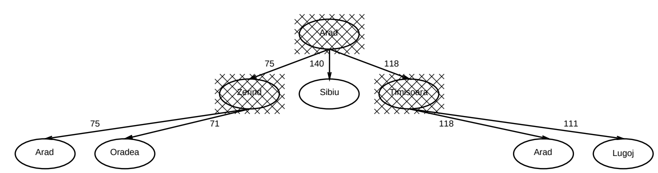
- Remove the first node in the fringe and add its children
- The queue is ordered with the cheapest first.



- Remove the first node in the fringe and add its children — they are added in priority order.



- Repeat.



- What will be the next node to be expanded?

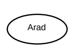
Depth First Search

- Start by expanding initial state.
- Pick one of nodes resulting from 1st step, and expand it.
- Pick one of nodes resulting from 2nd step, and expand it, and so on.
- Always expand *deepest* node — make *fringe* a LIFO queue.
- Follow one "branch" of search tree.

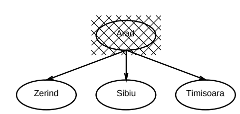
function DEPTH-FIRST-SEARCH(*problem*, *fringe*) **returns** a solution, or failure

closed ← an empty set
fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
loop do
  if *fringe* is empty **then return** failure
  node ← REMOVE-FRONT(*fringe*)
  if GOAL-TEST(*problem*, STATE[*node*]) **then return** *node*
  if STATE[*node*] is not in *closed* **then**
    add STATE[*node*] to *closed*
    fringe ← ADDTOFRONT(EXPAND(*node*, *problem*), *fringe*)
end

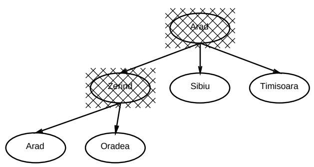
- Depth-first search on the Romania example — we start with the initial state in the frontier.



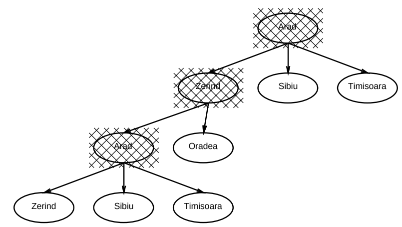
- Now we delete that node, and add its children.



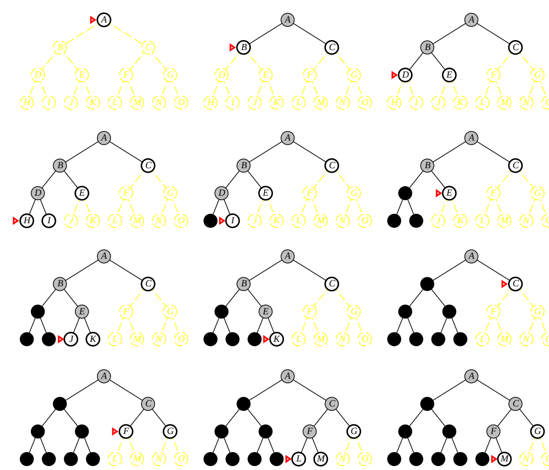
- Now pick a child and add its children



- and repeat.



- Depth first search is *not* guaranteed to find a solution if one exists.
- However, if it *does* find one, amount of time taken is much less than breadth first search.
- *Memory requirement* is much less than breadth first search.
- Solution found is *not* guaranteed to be the best.



Algorithmic Improvements

- Are then any *algorithmic* improvements we can make to basic search algorithms that will improve overall performance?
- Try to get:
  - *optimality* and *completeness*

  of breadth 1st search with:
  - *space efficiency*

  of depth 1st.
- Not too much to be done about time complexity :-(

Depth-limited Search

- Depth first search has some desirable properties — space complexity.
- But if wrong branch is expanded (with no solution on it), then it won't terminate.
- Idea: introduce a *depth limit* on branches to be expanded.
  - Don't expand a branch below this depth.
- Obviously this can be a source of incompleteness,

  BUT knowledge of the problem can help to set a sensible limit.

function DEPTH-LIMITED-SEARCH(*problem*, *limit*) **returns** soln/fail/cutoff
RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[*problem*]), *problem*, *limit*)

function RECURSIVE-DLS(*node*, *problem*, *limit*) **returns** soln/fail/cutoff
  cutoff-occurred? ← false
  if GOAL-TEST(*problem*, STATE[*node*]) **then return** *node*
  else if DEPTH[*node*] = *limit* **then return** *cutoff*
  else for each *successor* **in** EXPAND(*node*, *problem*) **do**
    result ← RECURSIVE-DLS(*successor*, *problem*, *limit*)
    if *result* = *cutoff* **then** *cutoff-occurred?* ← true
    else if *result* $\neq$ *failure* **then return** *result*
  if *cutoff-occurred?* **then return** *cutoff* **else return** *failure*

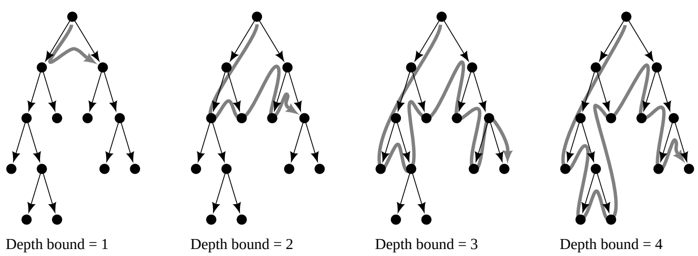
Iterative Deepening

- Unfortunately, if we choose a max depth for DLS such that shortest solution is longer, DLS is not complete.
- Iterative deepening an ingenious *complete* version of it.
- Basic idea is:
  - do DLS for depth 1; if solution found, return it;
  - otherwise do DLS for depth n; if solution found, return it;
  - otherwise, …
- So we *repeat* DLS for all depths until solution found.

function ITERATIVE-DEEPENING-SEARCH(*problem*) **returns** a solution
    inputs: *problem*, a problem
    for *depth* $\leftarrow$ 0 **to** ∞ **do**
        result $\leftarrow$ DEPTH-LIMITED-SEARCH(*problem*, *depth*)
        if *result* $\neq$ cutoff **then return** *result*
    end

- Calls DLS as subroutine.

- The search covers the whole state space down to the depth limit.



- The order it searches the nodes changes for each depth limit.

- Note that in iterative deepening, we *re-generate nodes on the fly.* Each time we do call on depth limited search for depth d, we need to regenerate the tree to depth $d-1$.
- Isn't this inefficient?
- Tradeoff *time* for *memory*.
- In general we might take a *little* more time, but we save a *lot* of memory.
- Now for breadth-first search to level d:

$$\begin{aligned} N_{bf} &= 1 + b + b^2 + \ldots b^d \\ &= \frac{b^{d+1} - 1}{b - 1} \end{aligned}$$

- In contrast a complete depth-limited search to level j:

$$N^j_{df} = \frac{b^{j+1} - 1}{b - 1}$$

- (This is just a breadth-first search to depth j.)
- In the worst case, then we have to do this to depth d, so expanding:

$$\begin{aligned} N_{id} &= \sum_{j=0}^{d} \frac{b^{j+1} - 1}{b - 1} \\ &\vdots \\ &= \frac{b^{d+2} - 2b - bd + d + 1}{(b-1)^2} \end{aligned}$$

- For large d:

$$\frac{N_{id}}{N_{bf}} = \frac{b}{b - 1}$$

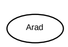
- So for high branching and relatively deep goals we do a small amount more work.
- Example: Suppose $b = 10$ and $d = 5$.

  Breadth first search would require examining $111,111$ nodes, with memory requirement of $100,000$ nodes.

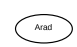
  Iterative deepening for same problem: $123,456$ nodes to be searched, with memory requirement only 50 nodes.

  Takes 11% longer in this case.

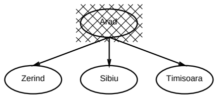
- On the Romania example we start with the initial state, expand one node, and fail to find the goal.



- For the next iteration we start over

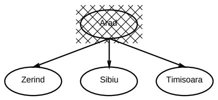


- This time we push down another level before failing.

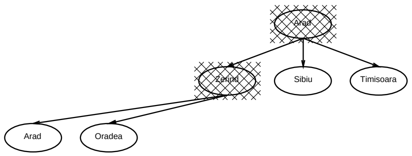


- Then we start a third time

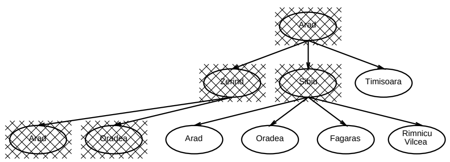
- And when we get here, we push down another level



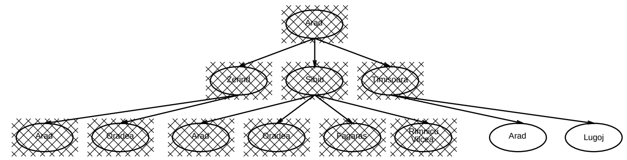
- Expanding the first child node on the second level.



- When that fails to produce a solution, we expand the second node on the second level.



- And finally the third node on that level.



Heuristic search

- We now turn to informed search — where the search uses problem specific information to guide the search.
- Whatever search technique we use, *exponential time complexity*.
- We want to search less, by having an idea where the goal is.
- Simplest form of problem specific knowledge is *heuristic*.
- Usual implementation in search is via an *evaluation function* which indicates desirability of a given node.

$$f(n)$$

- We are already familiar with this idea from uniform cost search where

$$f(n) = g(n)$$

Greedy Search

- Most heuristics estimate cost of *cheapest path* from node to solution.
- We have a *heuristic function*,

$$h : Nodes \rightarrow R$$

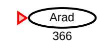
  which estimates the distance from the node to the goal.
- Example: In the Romania example, heuristic might be straight line distance from node to Bucharest.
- Heuristic is said to be *admissible* if it *never overestimates* cheapest solution.

  Admissible = optimistic.
- Greedy search involves expanding node with cheapest expected cost to solution.

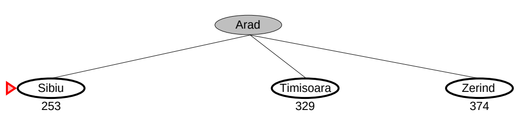
function GREEDY-SEARCH(*problem*, *fringe*) **returns** a solution, or failure

  closed ← an empty set
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
  loop do
    if *fringe* is empty **then return** failure
    node ← REMOVE-FRONT(*fringe*)
    if GOAL-TEST(*problem*, STATE[*node*]) **then return** *node*
    if STATE[*node*] is not in *closed* **then**
      add STATE[*node*] to *closed*
      fringe ← INSERTALL(EXPAND(*node*, *problem*), *fringe*)
      fringe ← SORTBYHVALUE(*fringe*)
  end

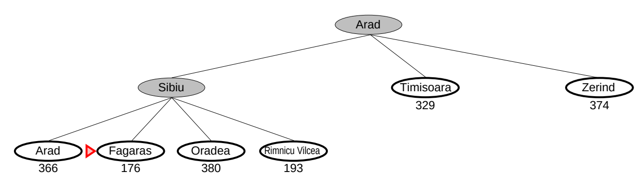
- As ever we start with the initial node. Note the heuristic value.



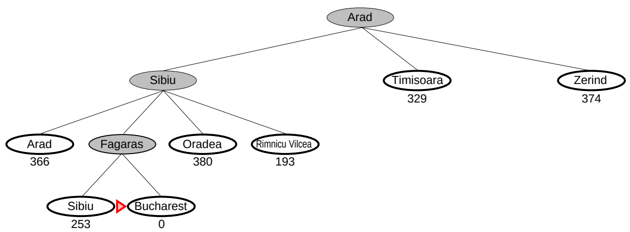
- When then expand the child node with the lowest heuristic value



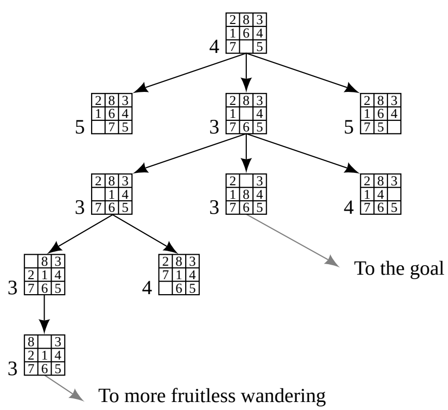
- And then we repeat.



- In the next level we find the goal.



- Greedy search finds solutions quickly.
- It doesn't always find the best solution where there is more than one.
- Susceptible to false starts.
  - Chases good looking options that turn out to be bad.
- Only looks at *current* node. Ignores past!
- Also *myopic* (shortsighted).



- For the 8-puzzle one good heuristic is:
  - count tiles out of place.
- Another is:
  - *Manhattan blocks' distance*
- The latter works for other problems as well:
  - Robot navigation.

A^* Search

- A^* is very efficient search strategy.
- Basic idea is to *combine*

uniform cost search
and
greedy search.

- We look at the *cost so far* and the *estimated cost to goal*.
- Gives heuristic f:

$$f(n) = g(n) + h(n)$$

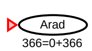
where

  - $g(n)$ is path cost of n;
  - $h(n)$ is expected cost of cheapest solution from n.

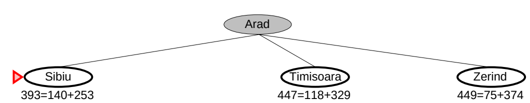
- Aims to mimimise *overall cost*.


```
function A-STAR-SEARCH(problem, fringe) returns a solution, or failure
   closed ← an empty set
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
      if fringe is empty then return failure
      node ← REMOVE-FRONT(fringe)
      if GOAL-TEST(problem, STATE[node]) then return node
      if STATE[node] is not in closed then
         add STATE[node] to closed
         fringe ← INSERTALL(EXPAND(node, problem), fringe)
         fringe ← SORTBYFVALUE(fringe)
   end
```

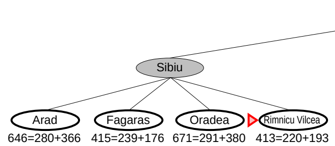

- Start with the initial node, this is the one we expand next.



- At the next level down, Sibiu has the lowest $f(\cdot)$ value.

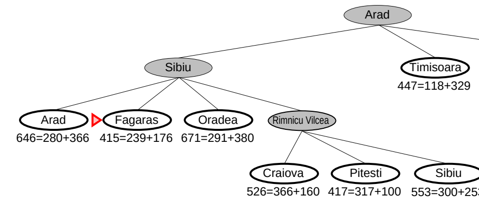


- At the next level, Rimnicu Vicea is the best-looking option.

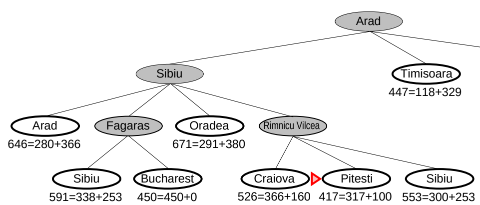


- Though it is further from the start than, for example, Timisoara, it is also closer to Bucharest.

- However, it is a false start, once we expand its children, they are worse options than Fagaras.



- And when we look at Fagaras' children, they include Bucharest.



The optimality of A^*

- A^* is optimal in precise sense—it is guaranteed to find a minimum cost path to the goal.
- There are a set of conditions under which A* will find such a path:
  1. Each node in the graph has a finite number of children.
  2. All arcs have a cost greater than some positive ϵ.
  3. For all nodes in the graph $h(n)$ always *underestimates* the true distance to the goal.
- The key here is the third bullet — the notion of *admissibility*.
- We will express this by saying a heuristic $h(\cdot)$ is admissible if

$$h(n) \leq h_T(n)$$

More informed search

- IF two versions of A^*, A_1^* and A_2^* use different functions h_1 and h_2,
- AND
$$h_1(n) < h_2(n)$$
for all non-goal nodes,
- THEN we say that A_2^* is *more informed* than A_1^*.
- As an example of "more informed" consider the 8-puzzle:
  - tiles out of place; and
  - Manhattan blocks distance.

- Why is "more informed" better?
- We need $h(n)$ to underestimate $h_T(n)$ to ensure admissibility.
- But, the closer the estimate, the easier it is to reject nodes which are not on the optimal path.
- This means less nodes need to be searched.

- There are techniques that go further than those we have studied:
  - Iterative deepening A^* (IDA^*)
  - Focussed Dynamic A^* (called D^*)
  - D^* Lite
  - Delayed D^*
  - Life-long planning A^* (called LPA^*)
  - PAO^*
- There are four directions we will take from here:
  - Local search
  - Adversarial search
  - Learning the state space.
  - Adding in more knowledge about the domain.

Summary

- This lecture introduced the basics of problem solving.
- In particular it discussed *state space* models and looked at some techniques for solving them.
  - Search for the goal.
  - Path through state space is the solution.
- We also looked at some techniques for search:
  - Breadth first.
  - Uniform cost
  - Depth first.
  - Iterative deepening
  - Best-first search
  - A^* search