

## Proof rules for Propositional Logic

This includes all the rules in the lecture notes and some additional ones — together they make a complete set of rules for propositional logic.

$$\frac{}{\vdash S} \text{TAUT if } S \text{ is a tautology}$$

$$\frac{\vdash S_1 \wedge S_2}{\vdash S_1; \vdash S_2} \wedge\text{-E}$$

$$\frac{\vdash S_1; \vdash S_2}{\vdash S_1 \wedge S_2} \wedge\text{-I}$$

$$\frac{\begin{array}{l} \vdash S_1 \vee \dots \vee S_n; \\ S_1 \vdash C; \\ \dots; \\ S_n \vdash C \end{array}}{\vdash C} \vee\text{-E}$$

$$\frac{\vdash S_1}{\vdash S_1 \vee \dots \vee S_n} \vee\text{-I}$$

$$\frac{\vdash S_1 \Rightarrow S_2; \vdash S_1}{\vdash S_2} \Rightarrow\text{-E}$$

$$\frac{S_1 \vdash S_2}{S_1 \Rightarrow S_2} \Rightarrow\text{-I}$$

$$\frac{\vdash S_1 \Leftrightarrow S_2}{\vdash S_1 \Rightarrow S_2; \vdash S_2 \Rightarrow S_1} \Leftrightarrow\text{-E}$$

$$\frac{\vdash S_1 \Rightarrow S_2; \vdash S_2 \Rightarrow S_1}{\vdash S_1 \Leftrightarrow S_2} \Leftrightarrow\text{-I}$$

$$\frac{\vdash \neg\neg S_1}{\vdash S_1} \neg\text{-E}$$

$$\frac{S_1 \vdash \perp}{\vdash \neg S_1} \neg\text{-I}$$

For the last of these rules, remember that  $\perp$  stands for any formula which is inconsistent (for example  $S_1 \wedge \neg S_1$ ).