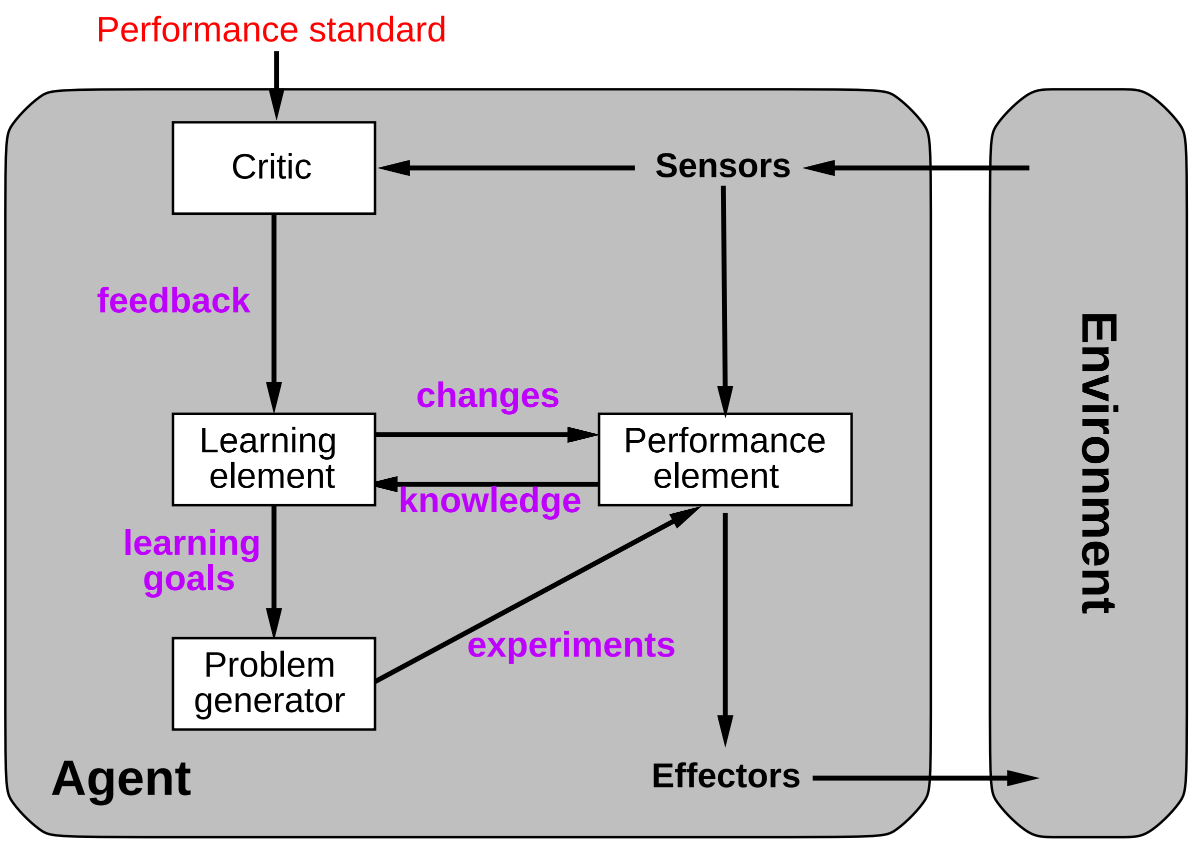
LEARNING

Overview

- Most of the time we can’t program our agents to do everything right to begin with.
- We don’t have enough information about the environment.
- So we get them to *learn* what to do.
- Different forms of learning:
  - Inductive learning; and
  - Reinforcement learning.

Learning agents



- Design of learning element is dictated by
  - what type of performance element is used
  - which functional component is to be learned
  - how that functional compoent is represented
  - what kind of feedback is available
- *Supervised learning*: correct answers for each instance.
- *Reinforcement learning*: occasional rewards.

- Example scenarios:

Performance element	Component	Representation	Feedback
Alpha−beta search	Eval. fn.	Weighted linear function	Win/loss
Logical agent	Transition model	Successor−state axioms	Outcome
Utility−based agent	Transition model	Dynamic Bayes net	Outcome
Simple reflex agent	Percept−action fn	Neural net	Correct action

Inductive learning

- Simplest form: learn a function from examples (*tabula rasa*)
- f is the *target function*
- An *example* is a pair x, $f(x)$:

$$\begin{array}{c|c|c} O & O & X \\ \hline & X & \\ \hline X & & \end{array}\ ,\quad +1$$

- Problem: find a(n) *hypothesis* h such that

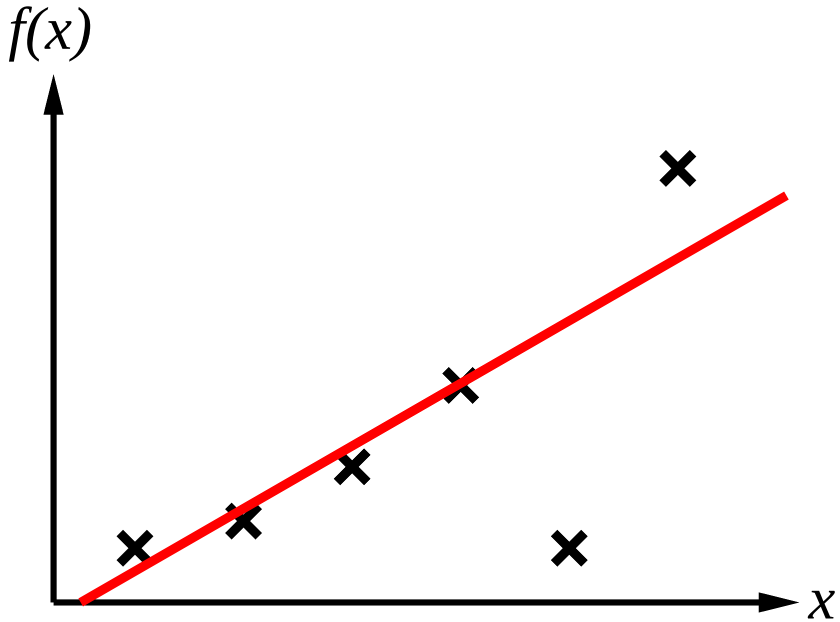
$$h \approx f$$

given a *training set* of examples

Inductive learning method

- Construct/adjust h to agree with f on training set
  (h is *consistent* if it agrees with f on all examples)

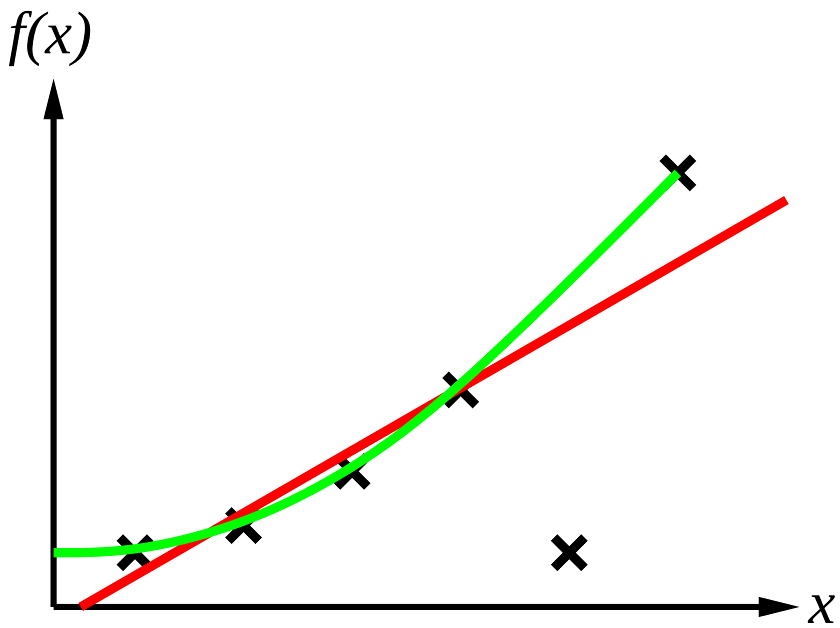
  E.g., curve fitting:



Inductive learning method II

- Construct/adjust h to agree with f on training set
  (h is *consistent* if it agrees with f on all examples)

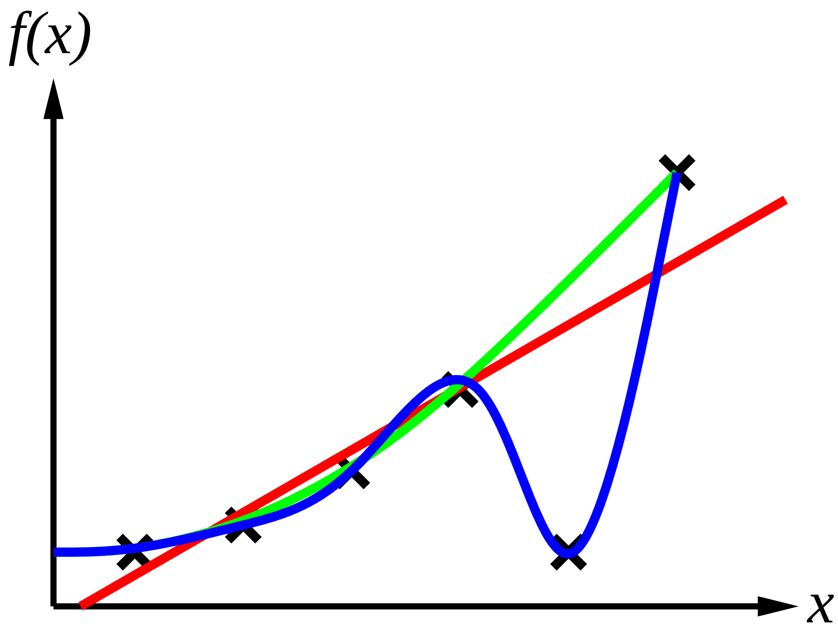
  E.g., curve fitting:



Inductive learning method III

- Construct/adjust h to agree with f on training set
  (h is *consistent* if it agrees with f on all examples)

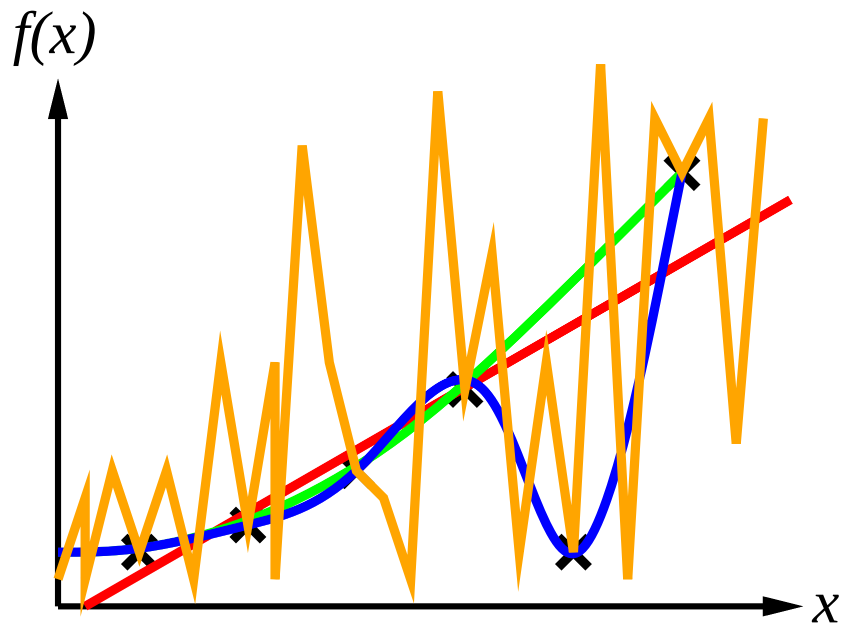
  E.g., curve fitting:



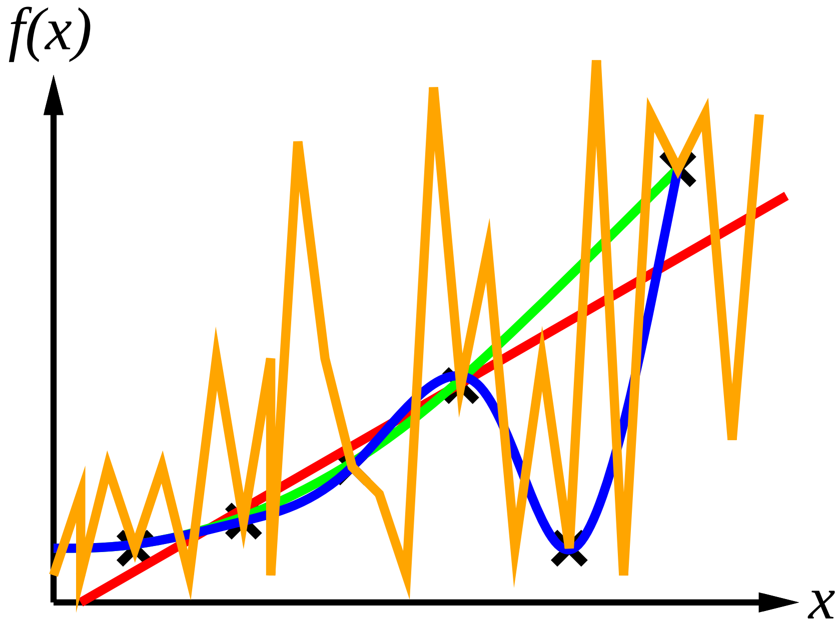
Inductive learning method IV

- Construct/adjust h to agree with f on training set
  (h is *consistent* if it agrees with f on all examples)

  E.g., curve fitting:



Inductive learning method V



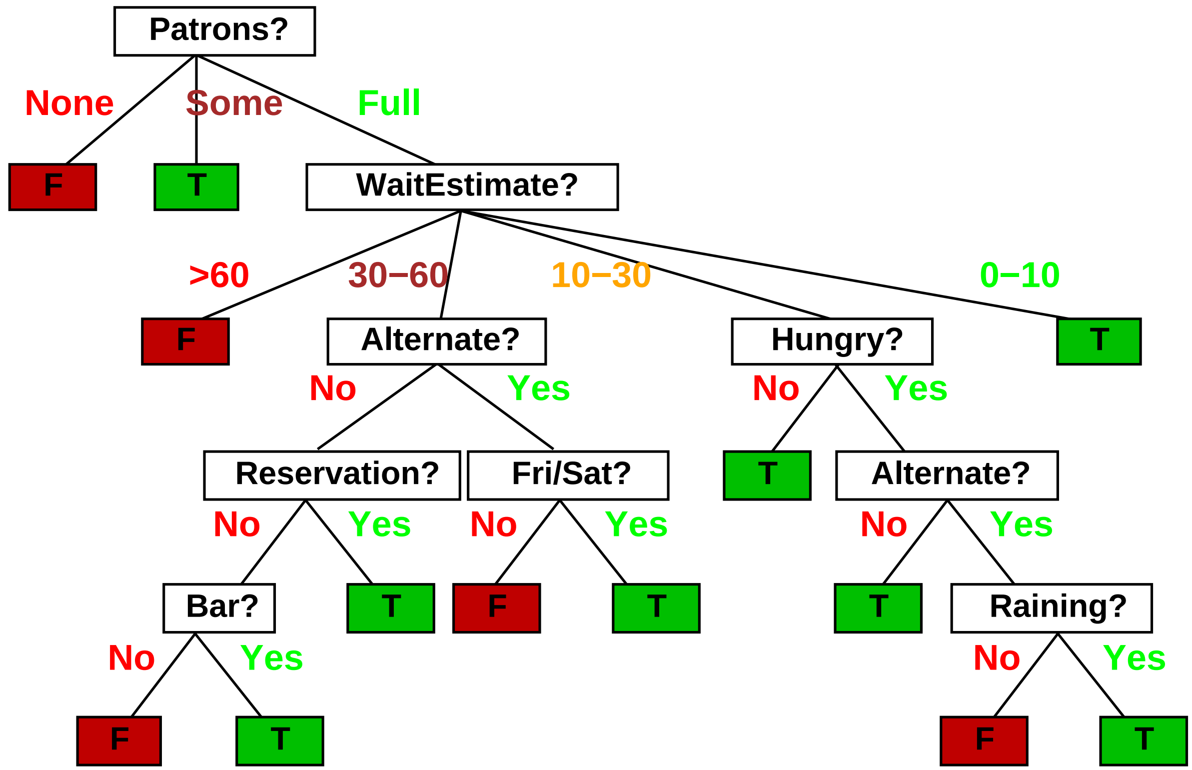
- Ockham's razor: maximize a combination of consistency and simplicity

Attribute-based representations

Ex	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*T*	*F*	*F*	*T*	*Some*	*$$$*	*F*	*T*	*French*	*0–10*	*T*
X_2	*T*	*F*	*F*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*30–60*	*F*
X_3	*F*	*T*	*F*	*F*	*Some*	*$*	*F*	*F*	*Burger*	*0–10*	*T*
X_4	*T*	*F*	*T*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*10–30*	*T*
X_5	*T*	*F*	*T*	*F*	*Full*	*$$$*	*F*	*T*	*French*	*>60*	*F*
X_6	*F*	*T*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Italian*	*0–10*	*T*
X_7	*F*	*T*	*F*	*F*	*None*	*$*	*T*	*F*	*Burger*	*0–10*	*F*
X_8	*F*	*F*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Thai*	*0–10*	*T*
X_9	*F*	*T*	*T*	*F*	*Full*	*$*	*T*	*F*	*Burger*	*>60*	*F*
X_{10}	*T*	*T*	*T*	*T*	*Full*	*$$$*	*F*	*T*	*Italian*	*10–30*	*F*
X_{11}	*F*	*F*	*F*	*F*	*None*	*$*	*F*	*F*	*Thai*	*0–10*	*F*
X_{12}	*T*	*T*	*T*	*T*	*Full*	*$*	*F*	*F*	*Burger*	*30–60*	*T*

Decision trees

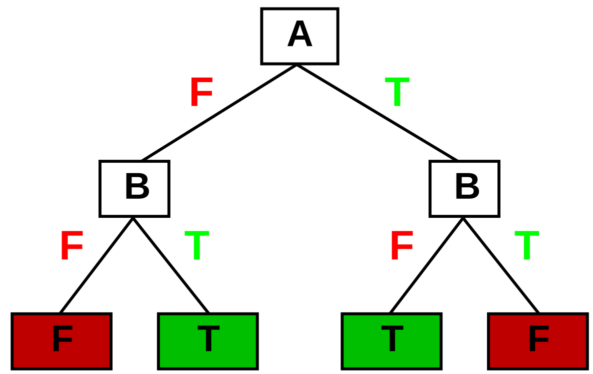
- Here is the "true" tree for deciding whether to wait:



Expressiveness

- Decision trees can express any function of the input attributes.
- For Boolean functions, truth table row $\rightarrow$ path to leaf:

A	B	A xor B
F	F	F
F	T	T
T	F	T
T	T	F



- Trivially, $\exists$ a consistent decision tree for any training set with one path to leaf for each example (unless f nondeterministic in x) but it probably won't generalize to new examples
- Prefer to find more *compact* decision trees

Hypothesis spaces

- How many distinct decision trees with n Boolean attributes?

Hypothesis spaces II

- How many distinct decision trees with n Boolean attributes?

  = number of Boolean functions

Hypothesis spaces III

- How many distinct decision trees with n Boolean attributes?

  = number of Boolean functions

  = number of distinct truth tables with 2^n rows

Hypothesis spaces IV

- How many distinct decision trees with n Boolean attributes?

  = number of Boolean functions

  = number of distinct truth tables with 2^n rows = 2^{2^n}

Hypothesis spaces V

- How many distinct decision trees with n Boolean attributes?

  = number of Boolean functions

  = number of distinct truth tables with 2^n rows = 2^{2^n}

  6 Boolean attributes means 18,446,744,073,709,551,616 trees

Hypothesis spaces VI

- How many distinct decision trees with n Boolean attributes?

  = number of Boolean functions

  = number of distinct truth tables with 2^n rows = 2^{2^n}

  6 Boolean attributes means 18,446,744,073,709,551,616 trees

- How many purely conjunctive hypotheses ($Hungry \wedge \neg Rain$)?

Hypothesis spaces VII

- How many distinct decision trees with n Boolean attributes?

  = number of Boolean functions

  = number of distinct truth tables with 2^n rows = 2^{2^n}

  6 Boolean attributes means 18,446,744,073,709,551,616 trees

- How many purely conjunctive hypotheses ($Hungry \wedge \neg Rain$)?
- Each attribute can be in (positive), in (negative), or out $\Rightarrow 3^n$ distinct conjunctive hypotheses
- More expressive hypothesis space
  - increases chance that target function can be expressed
  - increases number of hypotheses consistent with training set $\Rightarrow$ may get worse predictions

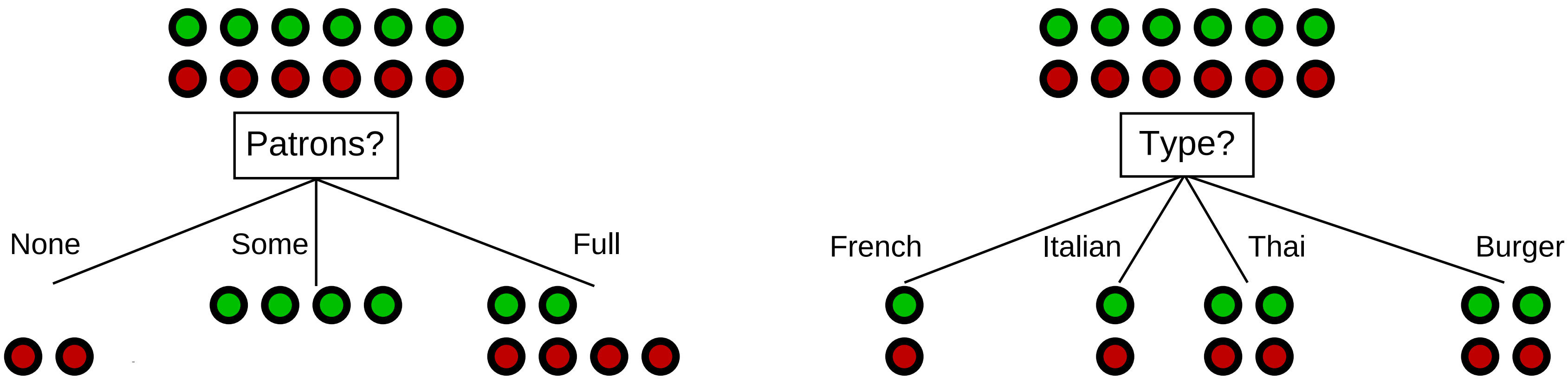
Decision tree learning

- Aim: find a small tree consistent with the training examples
- Idea: (recursively) choose "most significant" attribute as root of (sub)tree

```
function DTL(examples, attributes, default) returns a decision tree
  if examples is empty then return default
  else if all examples have the same classification then return the classification
  else if attributes is empty then return MODE(examples)
  else
     best ← CHOOSE-ATTRIBUTE(attributes, examples)
     tree ← a new decision tree with root test best
     for each value v_i of best do
        examples_i ← {elements of examples with best = v_i}
        šubtree ← DTL(examples_i, attributes − best, MODE(examples))
        add a branch to tree with label v_i and subtree subtree
     return tree
```

Choosing an attribute

- Idea: a good attribute splits the examples into subsets that are (ideally) "all positive" or "all negative"



- *Patrons?* is a better choice—gives *information* about the classification

Information

- Information answers questions
- The more clueless I am about the answer initially, the more information is contained in the answer
- Scale: 1 bit = answer to Boolean question with prior $\langle 0.5, 0.5\rangle$
- Information in an answer when prior is $\langle P_1, \ldots, P_n\rangle$ is

$$H(\langle P_1, \ldots, P_n\rangle) = \sum_{i=1}^{n} -P_i \log_2 P_i$$

(also called *entropy* of the prior)

Information II

- Suppose we have p positive and n negative examples at the root
  $\Rightarrow H(\langle p/(p+n), n/(p+n)\rangle)$ bits needed to classify a new example.
- For 12 restaurant examples, $p = n = 6$ so we need 1 bit
- An attribute splits the examples E into subsets E_i, each of which (we hope) needs less information to complete the classification

Information III

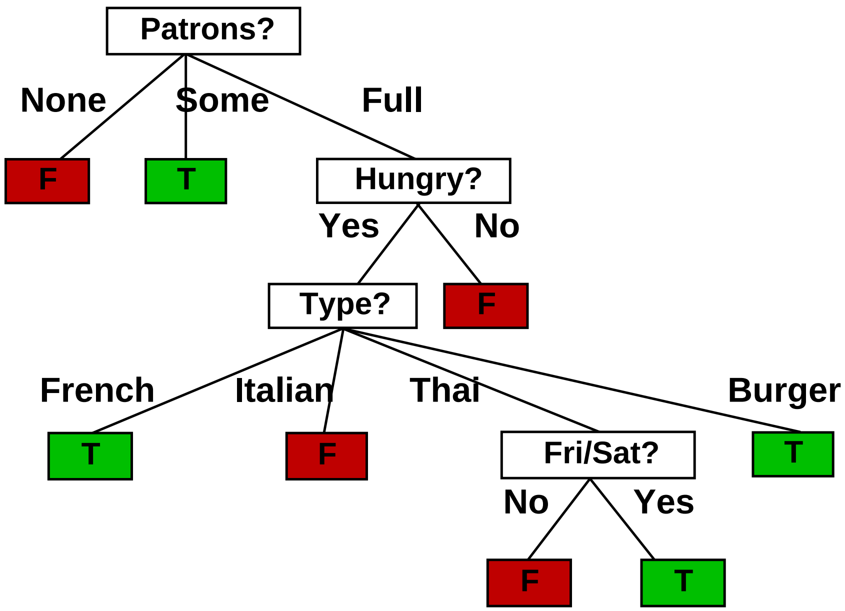
- Let E_i have p_i positive and n_i negative examples
  $\Rightarrow H(\langle p_i/(p_i + n_i), n_i/(p_i + n_i)\rangle)$ bits needed to classify a new example
  $\Rightarrow$ *expected* number of bits per example over all branches is

$$\sum_i \frac{p_i + n_i}{p + n} H(\langle p_i/(p_i + n_i), n_i/(p_i + n_i)\rangle)$$

- For $Patrons?$, this is 0.459 bits, for $Type$ this is (still) 1 bit
  $\Rightarrow$ choose the attribute that minimizes the remaining information needed

Back to the example

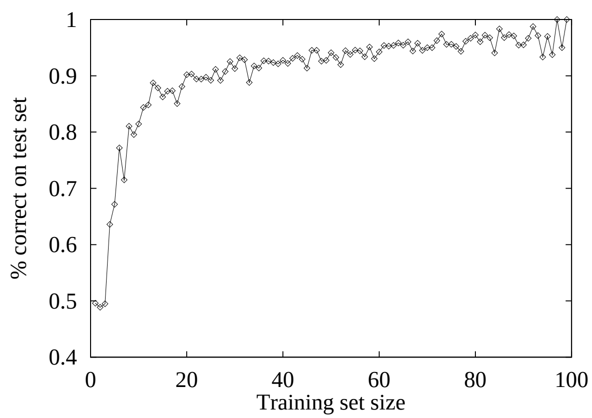
- Decision tree learned from the 12 examples:



- Substantially simpler than "true" tree—a more complex hypothesis isn't justified by small amount of data

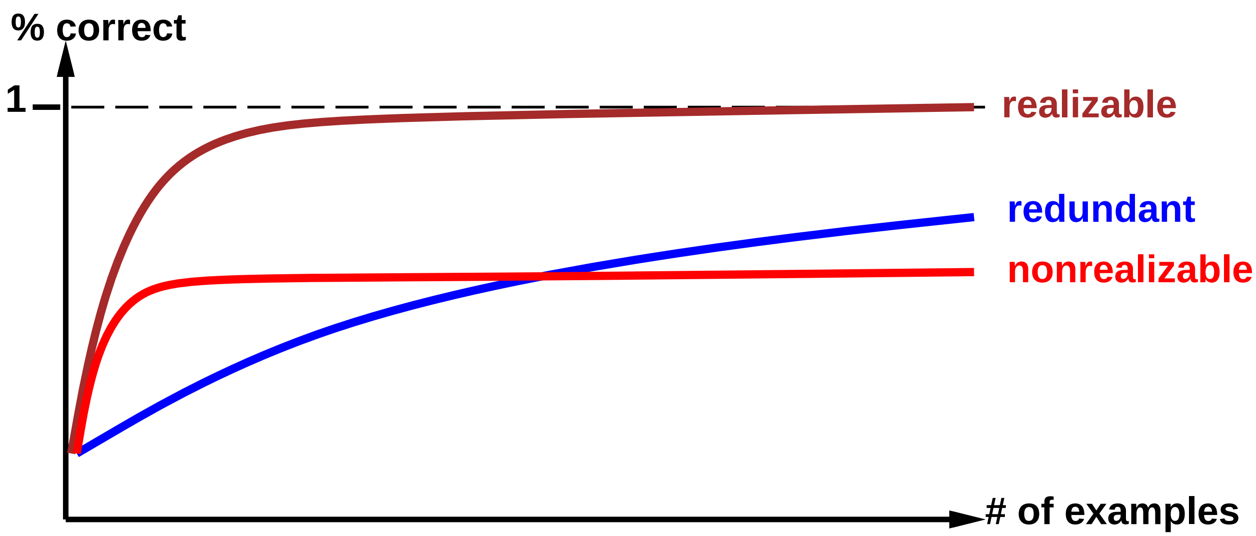
Performance measurement

- How do we know that $h \approx f$?
  1. Use theorems of computational/statistical learning theory
  2. Try h on a new *test set* of examples
     (use *same distribution over example space* as training set)
- *Learning curve* = % correct on test set as a function of training set size



Performance measurement II

- Learning curve depends on
  - *realizable* (can express target function) vs. *non-realizable* non-realizability can be due to missing attributes or restricted hypothesis class
  - redundant expressiveness (e.g., loads of irrelevant attributes)



Summary

- Learning needed for unknown environments, lazy designers
- Learning agent = performance element + learning element
- Learning method depends on type of performance element, available feedback, type of component to be improved, and its representation
- For supervised learning, the aim is to find a simple hypothesis approximately consistent with training examples
- Decision tree learning using information gain
- Learning performance = prediction accuracy measured on test set