PROBLEM SOLVING AGENTS

Overview

Aims of the this lecture:

- introduce *problem solving*;
- introduce *goal formulation*;
- show how problems can be stated as *state space search*;
- show the importance and role of *abstraction*;
- introduce *undirected search*:
  - breadth 1st search;
  - depth 1st search.
- define main performance measures for search.

Problem Solving Agents

- Lecture 1 introduced *rational agents*.
- Now consider agents as *problem solvers*:

  Systems which set themselves *goals* and find *sequences of actions* that achieve these goals.
- What is a problem?

  A *goal* and a *means* for achieving the goal.
- The goal specifies the state of affairs we want to bring about.
- The means specifies the operations we can perform in an attempt to bring about the means.
- The difficulty is deciding what *order* to carry out the operations.

- Operation of problem solving agent:

```
/* s is sequence of actions */
repeat {
    percept = observeWorld();
    state = updateState(state, p);
    if s is empty then {
         goal = formulateGoal(state);
         prob = formulateProblem(state,p);
         s = search(prob);
    }
    action = recommendation(s);
    s = remainder(s, state);
}
until false; /* i.e., forever */
```

- Key difficulties:
  - `formulateGoal(...)`
  - `formulateProblem(...)`
  - `search(...)`
- It isn't easy to see how to tackle any of these.
- Here we will concentrate mainly on search.

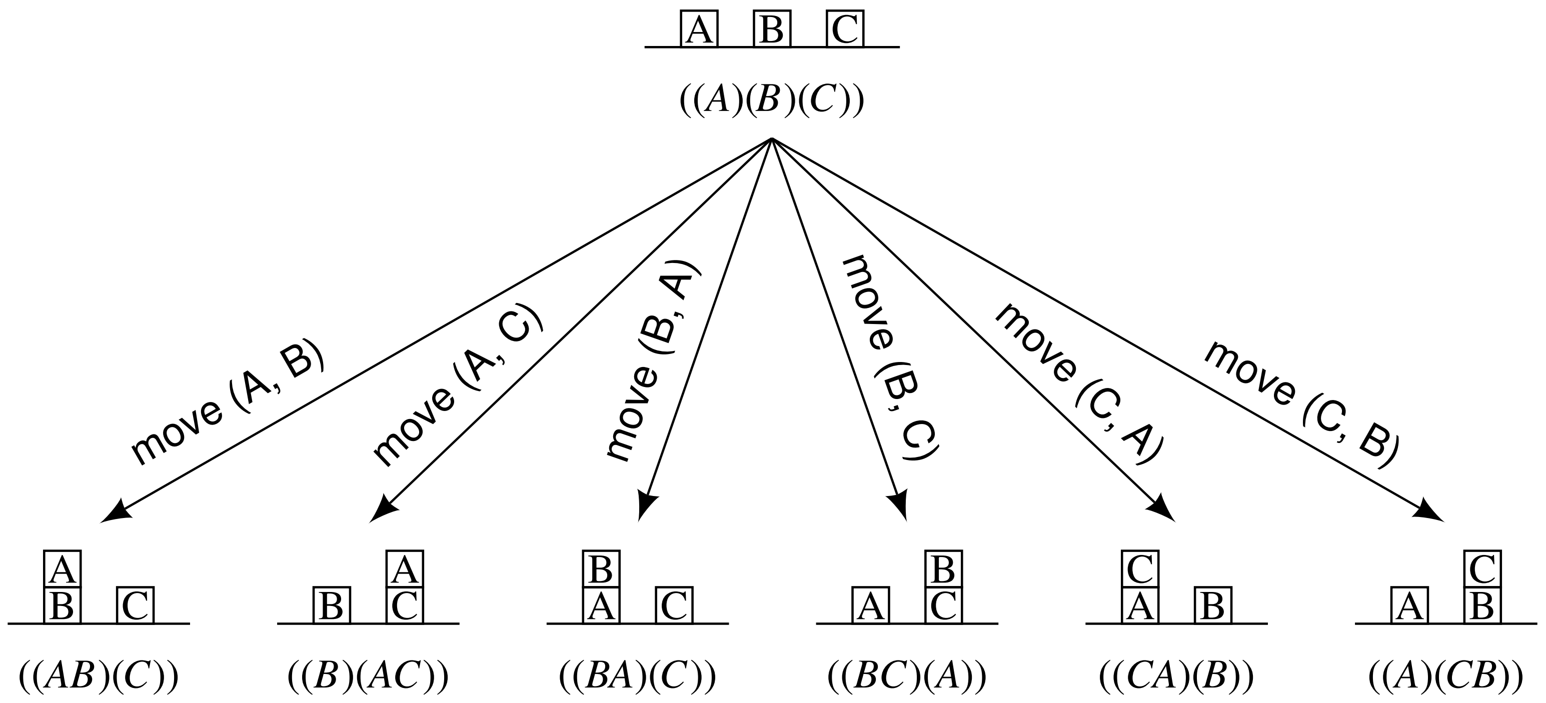
Goal Formulation

- Where do an agent's goals come from?
  - Agent is a *program* with a *specification*.
  - Specification is to maximise performance measure.
  - Should *adopt goal* if achievement of that goal will maximise this measure.
- Goals provide a *focus* and *filter* for decision-making:
  - *focus*: need to consider how to achieve them;
  - *filter*: need not consider actions that are incompatible with goals.

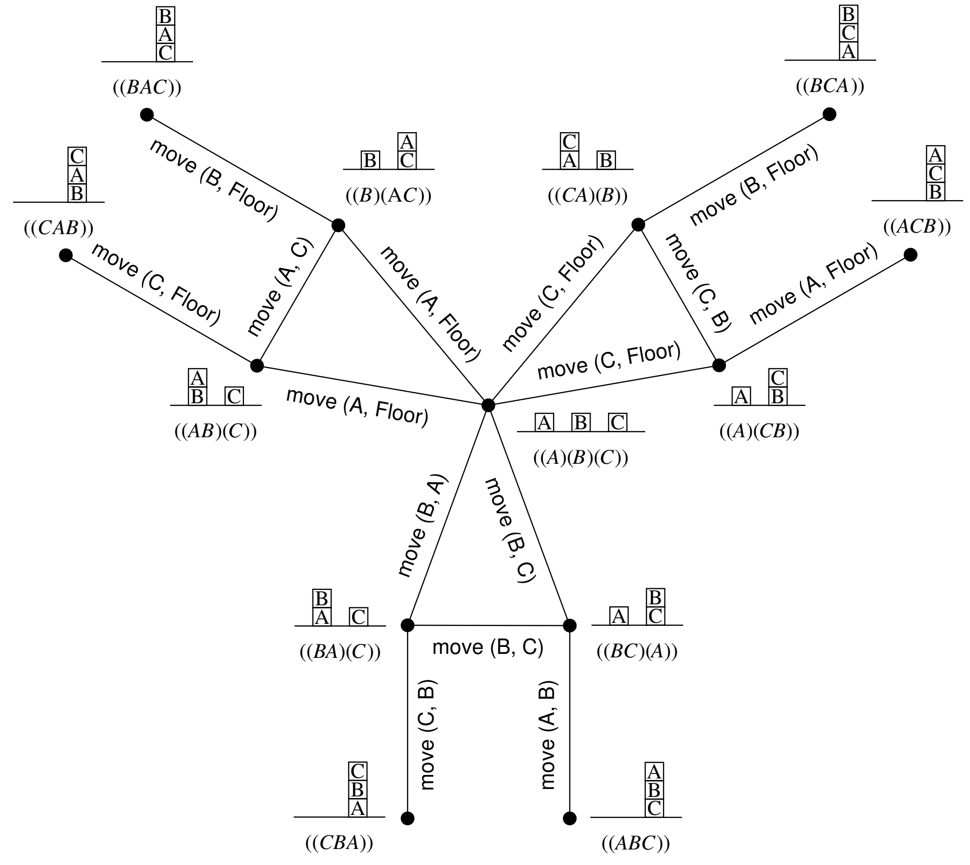
Problem Formulation

- Once goal is determined, formulate the problem to be solved.
- First determine set of possible states S of the problem.
- Then problem has:
  - *initial state* — the starting point, s_0;
  - *operations* — the actions that can be performed, $\{o_1, \ldots, o_n\}$.
  - *goal* — what you are aiming at — subset of S.
- The initial state together with operations determines *state space* of problem.
- Operations cause *changes* in state.

- Solution is a sequence of actions such that when applied to initial state s_0, we have goal state.
- Pictorially part of the state space is:



- The whole space for this problem is:

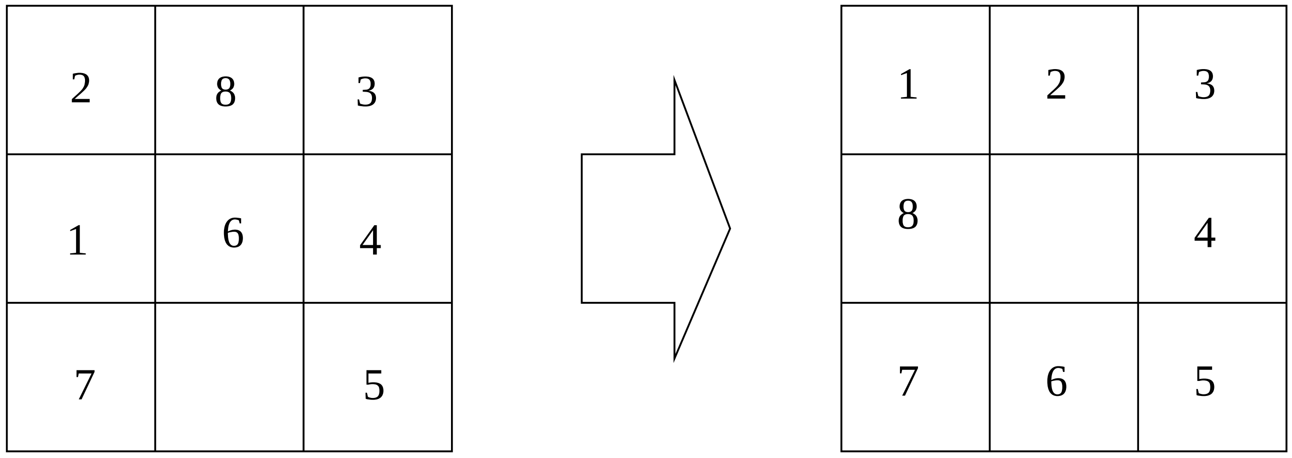


© 1998 Morgan Kaufman Publishers

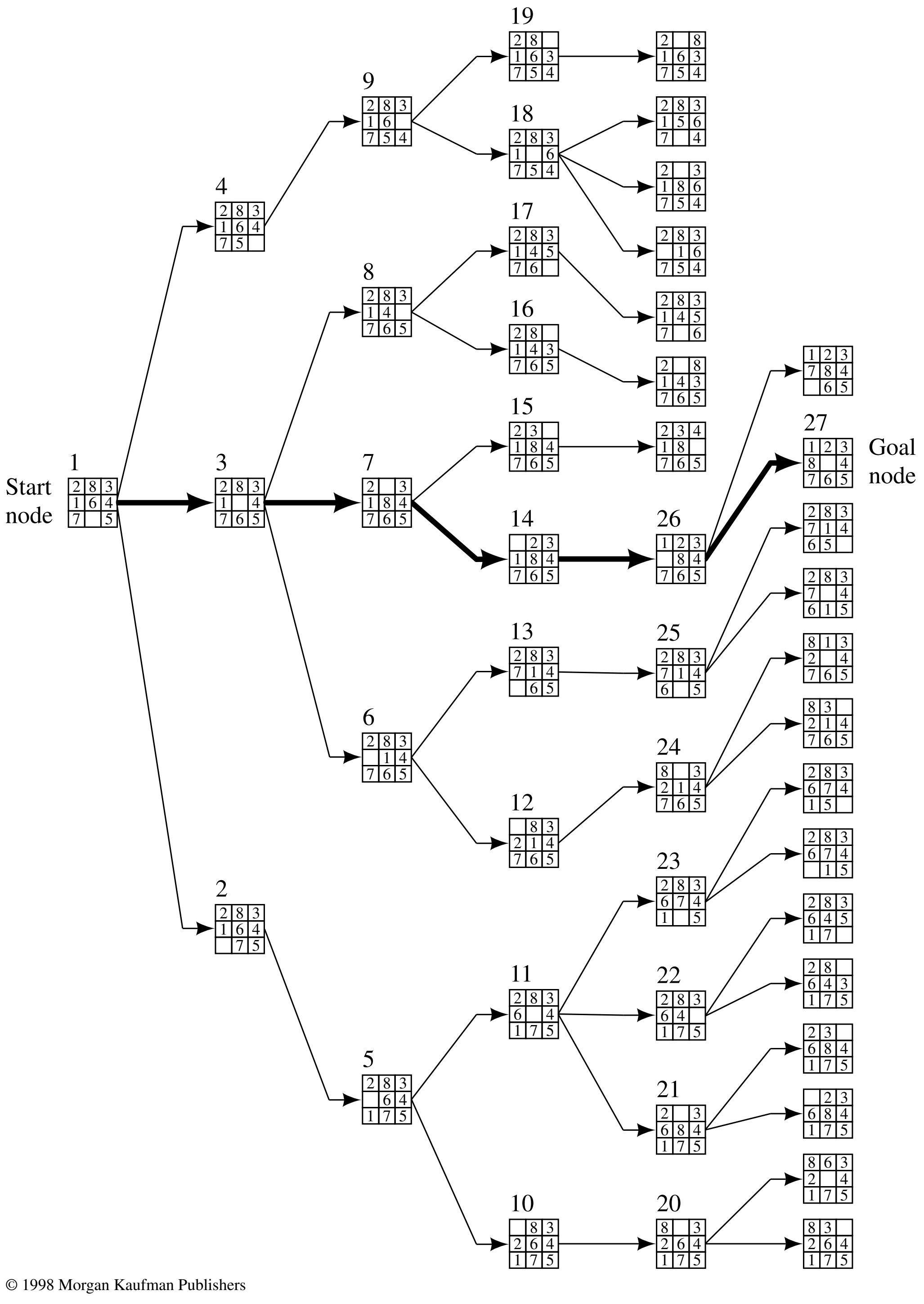
Examples of Toy Problems

- *Example 1*: The 8 puzzle.

  Do the following transformation, moving tile from occupied space to filled space.



- Initial state as shown above.
- Goal state as shown below.
- Operations:
  - o_1: move any tile to left of empty square to right;
  - o_2: move any tile to right of empty square to left;
  - o_3: move any tile above empty square down; and
  - o_4: move any tile below empty square up.
- This defines the following state space:


Start node
Goal node
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27

- Example 2: The n queens problem from chess.
- Place n queens on chess board so that no queen can be taken by another.
- Initial state: empty chess board.
- Goal state: n queens on chess board, one occupying each space, so that none can take others.
- Operations: place queen in empty square.

Solution Cost

- For most problems, some solutions are better than others:
  - in 8 puzzle, number of moves to get to solution;
  - number of moves to checkmate;
  - length of distance to travel.
- Mechanism for determining *cost* of solution is *path cost function*.
- This is the length of the path through the state-space from the initial state to the goal state.

- As an example, consider the following state in the 8-puzzle:

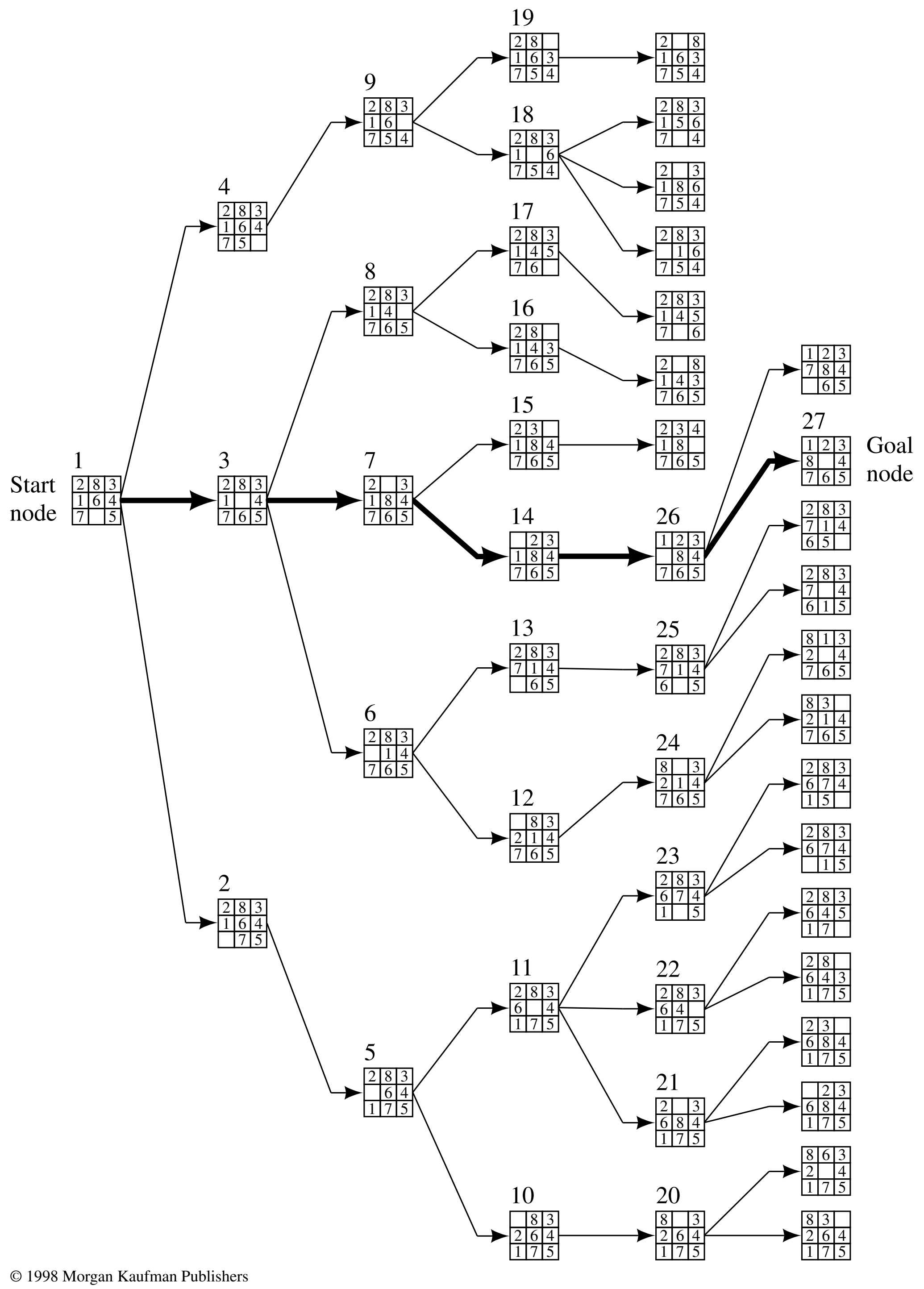
2	8	3
1	6	4
7		5

- How many moves are there to the solution?

- Obviously :-) there are five moves:

  1. o_3
  2. o_3
  3. o_1
  4. o_4
  5. o_2

- And the path through the solution space looks like:



Start node
Goal node
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27

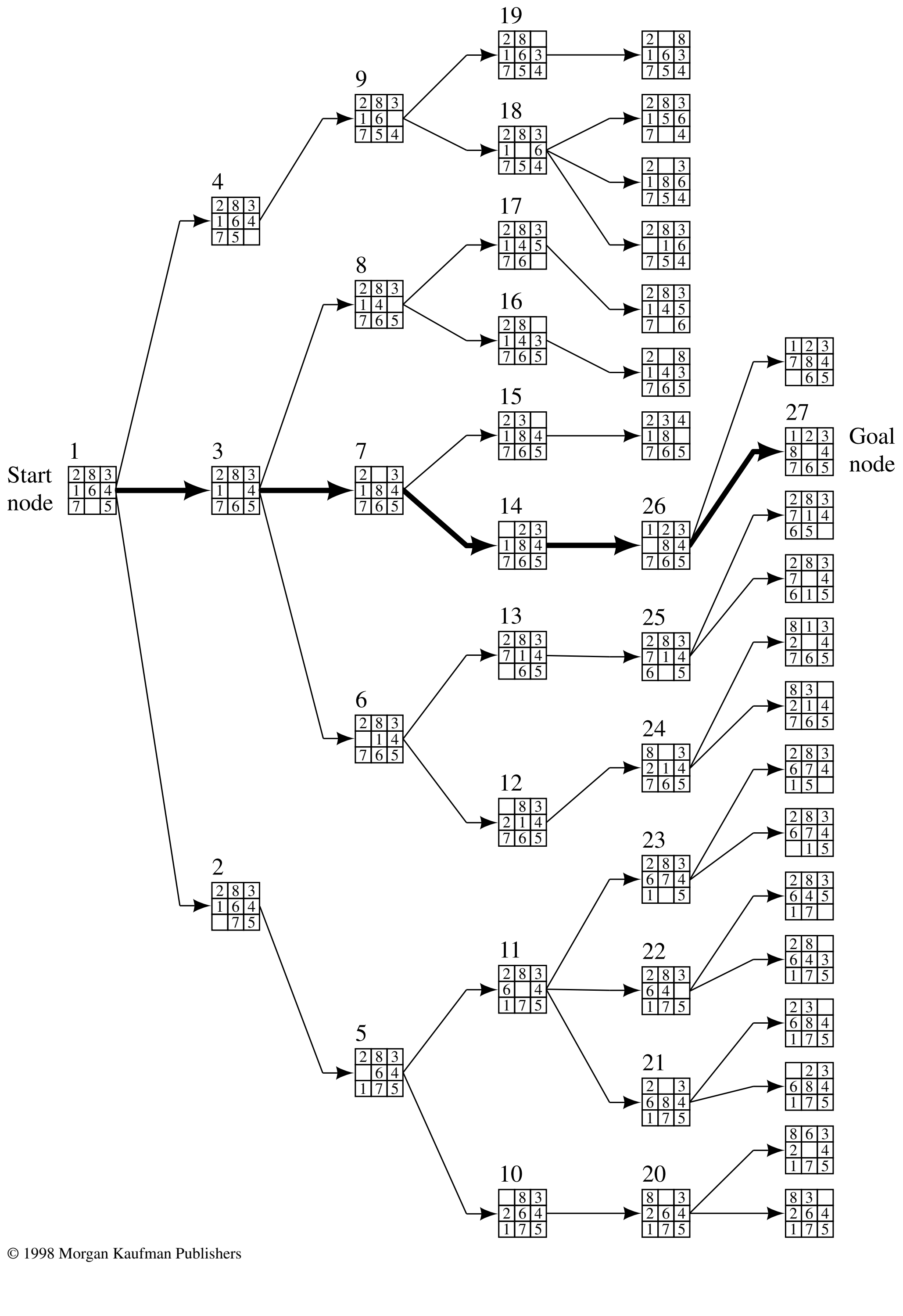
Problem Solving as Search

- In the state space view of the world, finding a solution is finding a path through the state space.
- When we solve a problem like the 8-puzzle we have some idea of what constitutes the next best move.
- It is hard to program this kind of approach.
- Instead we start by programming the kind of repetitive task that computers are good at.
- A *brute force* approach to problem solving involves *exhaustively searching* through the space of *all possible* action sequences to find one that achieves goal.

- Systematically generate a *search tree* (which is just the state space we saw already).
- For the 8-puzzle setup as:

2	8	3
1	6	4
7		5

- The search tree is:


Start node
Goal node
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27

- The tree is built by taking the initial state and identifying some states that can be obtained by applying a single operator.
- These new states become the *children* of the initial state in the tree.
- These new states are then examined to see if they are the goal state.
- If not, the process is repeated on the new states.
- We can formalise this description by giving an algorithm for it.

- General algorithm for search:

```
agenda = initial state;
while agenda not empty do{
   pick node from agenda;
   new nodes = apply operations to state;
   if goal state in new nodes
   then {
                 return solution;
      }
   add new nodes to agenda;
}
```

- Question: How to pick states for expansion?
- Two obvious solutions:
  - depth first search;
  - breadth first search.

Breadth First Search

- Start by *expanding* initial state — gives tree of depth 1.
- Then expand *all* nodes that resulted from previous step — gives tree of depth 2.
- Then expand *all* nodes that resulted from previous step, and so on.
- Expand nodes at depth n before level $n + 1$.

```
/* Breadth first search */

agenda = initial state;

while agenda not empty do
{
    pick node from front of agenda;
    new nodes = apply operations to state;
    if goal state in new nodes then
    {
         return solution;
    }

    APPEND new nodes to END of agenda;
 }
```

- Advantage: *guaranteed* to reach a solution if one exists.
- If all solutions occur at depth n, then this is good approach.
- Disadvantage: time taken to reach solution!
- Let b be *branching factor* — average number of operations that may be performed from any level.
- If solution occurs at depth d, then we will look at

  $$1 + b + b^2 + \cdots + b^d$$

  nodes before reaching solution — *exponential*.

- Time for breadth first search:

Depth	Nodes	Time
0	1	1 msec
1	11	.01 sec
2	111	.1 sec
4	11,111	11 secs
6	10^6	18 mins
8	10^8	31 hours
10	10^{10}	128 days
12	10^{12}	35 years
14	10^{14}	2500 years
20	10^{20}	3^{15} years

- *Combinatorial explosion!*

Importance of ABSTRACTION

- When formulating a problem, it is crucial to pick the right level of *abstraction*.
- Example: Given the task of driving from New York to Boston.
- Some possible actions…
  - depress clutch;
  - turn steering wheel right 10 degrees;

  … inappropriate level of *abstraction*.

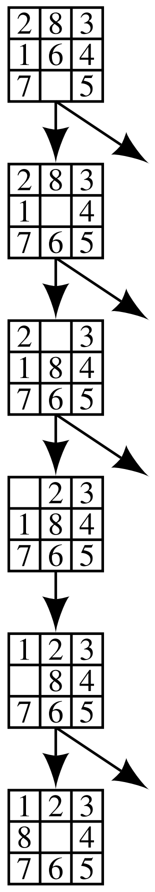
  Too much *irrelevant detail*.

- Better level of abstraction:
  - Take the Henry Hudson Parkway north
  - Take the Cross County turnoff

  … and so on.
- Getting abstraction level right lets you focus on the specifics of problem and is one way to combat the combinatorial explosion.
- (Tell that to Mapquest).

Depth First Search

- Start by expanding initial state.
- Pick one of nodes resulting from 1st step, and expand it.
- Pick one of nodes resulting from 1nd step, and expand it, and so on.
- Always expand *deepest* node.
- Follow one “branch” of search tree.



Goal node

```
/* Depth first search */

agenda = initial state;

while agenda not empty do
{
    pick node from front of agenda;
    new nodes = apply operations to state;
    if goal state in new nodes then
    {
         return solution;
    }

put new nodes on FRONT of agenda;
}
```

- Depth first search is *not* guaranteed to find a solution if one exists.
- However, if it *does* find one, amount of time taken is much less than breadth first search.
- *Memory requirement* is much less than breadth first search.
- Solution found is *not* guaranteed to be the best.

Performance Measures for Search

- *Completeness*:
  Is the search technique *guaranteed* to find a solution if one exists?
- *Time complexity*:
  How many computations are required to find solution?
- *Space complexity*:
  How much memory space is required?
- *Optimality*:
  How good is a solution going to be w.r.t. the path cost function.

Summary

- This lecture introduced the basics of problem solving.
- In particular it discussed *state space* models and looked at the basic techniques for solving them.
  - Search for the goal.
  - Path through state space is the solution.
- We also looked at two techniques for search:
  - Breadth first.
  - Depth first.