HEURISTIC SEARCH I

Recap

The last lecture introduced

- Basic problem solving techniques:
  - Breadth-first search
  - Depth-first search
- Breadth-first search is complete but expensive.
- Depth-first search is cheap but incomplete
- Can't we do better than this?
- That is what this lecture is about

Overview

Aims of this lecture:

- show how basic search (depth 1st, breadth 1st) can be improved;
- introduce:
  - *depth limited search*;
  - *iterative deepening*.
- show that even with such improvements, search is hopelessly unrealistic for real problems.

Algorithmic Improvements

- Are then any *algorithmic* improvements we can make to basic search algorithms that will improve overall performance?
- Try to get *optimality* and *completeness* of breadth 1st search with *space efficiency* of depth 1st.
- Not too much to be done about time complexity :-(

Depth Limited Search

- Depth first search has some desirable properties — space complexity.
- But if wrong branch is expanded (with no solution on it), then it won't terminate.
- Idea: introduce a *depth limit* on branches to be expanded.
- Don't expand a branch below this depth.

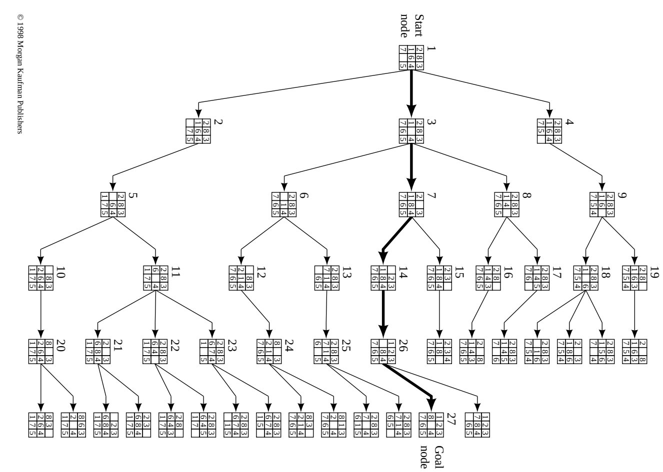
- General algorithm for depth limited search:

```
depth limit = max depth to search to;
agenda = initial state;
while agenda not empty do
  take node from front of agenda;
  new nodes = apply operations to node;
  if goal state in new nodes then {
    return solution;
  }
  if depth(node) < depth limit then {
    add new nodes to front of agenda;
  }
}
```


- For the 8-puzzle as so:

2	8	3
1	6	4
7		5

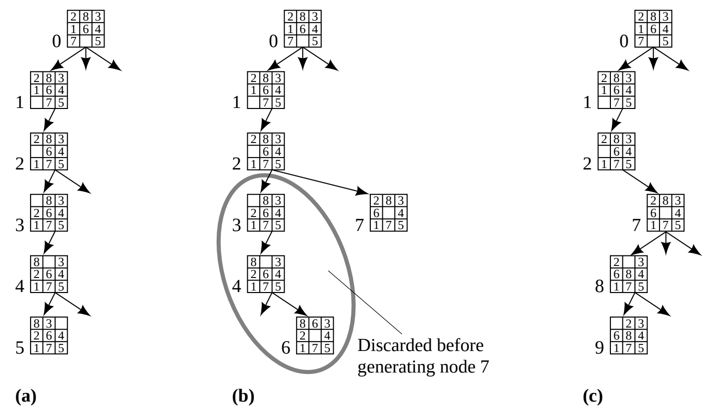
- We have the following state space:



- Given this numbering of the states, a depth limited search with depth limit of three would have an agenda that looks like

  1. 1
  2. 2, 3, 4
  3. 5, 3, 4
  4. 10, 11, 3, 4
  5. 11, 3, 4
  6. 3, 4
  7. 6, 7, 8, 4
  8. 12, 13, 7, 8, 4
  9. 13, 7, 8, 4
  10. . . .

- Let's look at the search tree in more detail:



- So, when we hit the depth bound, we don't add any more nodes to the agenda.
- Then we pick the next node off the agenda.
- This has the effect of moving the search back to the last node above depth limit that that is "partly expanded".
- This is known as *chronological backtracking*.
- The effect of the depth limit is to force the search of the whole state space down to the limit.
- We get the completeness of breadth-first (down to the limit), with the space cost of depth first.

Iterative Deepening

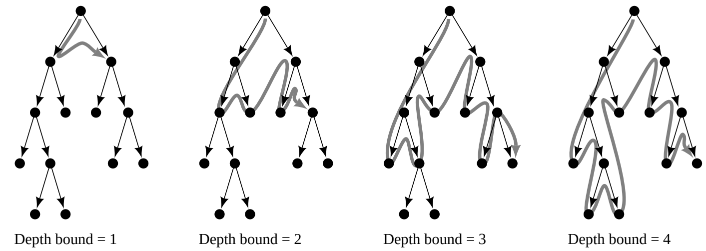
- Unfortunately, if we choose a max depth for d.l.s. such that shortest solution is longer, d.l.s. is not complete.
- Iterative deepening an ingenious *complete* version of it.
- Basic idea is:
  - do d.l.s. for depth 1; if solution found, return it;
  - otherwise do d.l.s. for depth n; if solution found, return it;
  - otherwise, . . .
- So we *repeat* d.l.s. for all depths until solution found.

- General algorithm for iterative deepening search:

```
depth limit = 1;
repeat {
  result = depth_limited_search(
    max depth = depth limit;
    agenda = initial node;
  );
  if result contains goal then {
    return result;
  }
  depth limit = depth limit + 1;
} until false; /* i.e., forever */
```

- Calls d.l.s. as subroutine.

- The search covers the whole state space down to the depth limit.



- The order it searches the nodes changes for each depth limit.

- Note that in iterative deepening, we *re-generate nodes on the fly*. Each time we do call on depth limited search for depth d, we need to regenerate the tree to depth $d-1$.
- Isn't this inefficient?
- Tradeoff *time* for *memory*.
- In general we might take a *little* more time, but we save a *lot* of memory.
- Now for breadth-first search to level d:

$$
\begin{aligned}
N_{bf} &= 1 + b + b^2 + \ldots b^d & (1)\\
&= \frac{b^{d+1}-1}{b-1} & (2)
\end{aligned}
$$

- In contrast a complete depth-limited search to level j:

$$N^j_{df} = \frac{b^{j+1}-1}{b-1} \qquad (3)$$

- (This is just a breadth-first search to depth j.)
- In the worst case, then we have to do this to depth d, so expanding:

$$
\begin{aligned}
N_{id} &= \sum_{j=0}^{d} \frac{b^{j+1}-1}{b-1} & (4)\\
&\vdots & (5)\\
&= \frac{b^{d+2}-2b-bd+d+1}{(b-1)^2} & (6)
\end{aligned}
$$

- For large d:

$$\frac{N_{id}}{N_{bf}} = \frac{b}{b-1} \tag{7}$$

- So for high branching and relatively deep goals we do a small amount more work.
- Example: Suppose $b = 10$ and $d = 5$.

  Breadth first search would require examining $111,111$ nodes, with memory requirement of $100,000$ nodes.

  Iterative deepening for same problem: $123,456$ nodes to be searched, with memory requirement only 50 nodes.

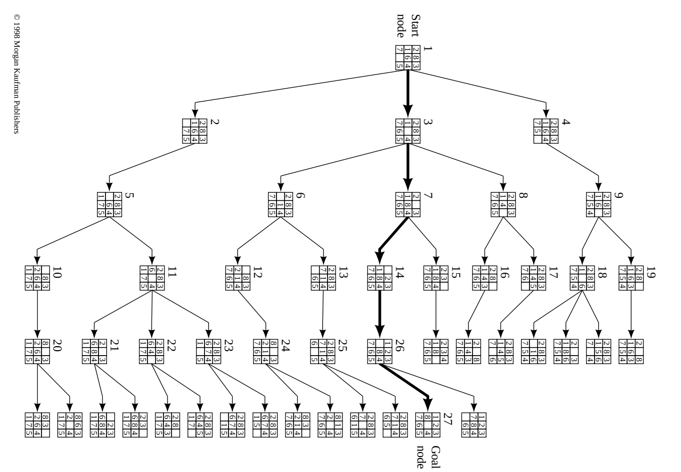
  Takes 11% longer in this case.

- For the 8-puzzle setup as:

2	8	3
1	6	4
7		5

1	2	3
8		4
7	6	5

- What would iterative deepening search look like?
- Well, it would explore the state space:



- In the following way.
- States would be expanded in the order:
  1. 1
  2. 1, 2, 3, 4
  3. 1, 2, 5, 3, 6, 7, 8, 4, 9.
  4. 1, 2, 5, 10, 11, 3, 6, 12, 13, 7, 14, 15, 8, 16, 17, 4, 9, 18, 19.
  5. ...
- Note that these are the states *visited*, not the nodes on the agenda.

Bi-directional Search

- Suppose we search from *the goal state backwards* as well as from *initial state forwards*.
- Involves determining *predecessor* nodes to goal, and then looking at predecessor nodes to this, . . .
- Rather than doing one search of b^d, we do *two* $b^{d/2}$ searches.
- *Much* more efficient.

- Example:
  Suppose $b = 10$, $d = 6$.
  Breadth first search will examine nodes.
  Bidirectional search will examine nodes.
- Can combine different search strategies in different directions.
- For large d, is still impractical!

Summary

- This lecture has looked at some more efficient techniques than breadth first and depth first search.
  - depth-limited search;
  - iterative-deepening search; and
  - bidirectional search.
- These all improve on depth-first and breadth-first search.
- However, all fail for big enough problems (too large state space).
- Next lecture, we will look at approaches that cut down the size of the state-space that is searched.