HEURISTIC SEARCH II

Recap

The last lecture introduced

- More advanced problem solving techniques:
  - Depth limited search
  - Iterative deepening
  - Bidirectional search
- These improved on basic techniques like breadth-first and depth-first search.
- However, they still aren't powerful enough to give solutions for realistic problems.
- Are there more improvements we can make?

Overview

Aims of this lecture:

- To show how applying some knowledge of the problem can help.
- Introduce *heuristics* — rules of thumb.
- Introduce *heuristic search*: guided by rules of thumb which help to decide which node to expand:
  - *uniform-cost search*;
  - *greedy search*;
  - *A* search*.

Heuristic (Informed) Search

- Whatever search technique we use, *exponential time complexity*.
- Tweaks to the algorithm will not reduce this to polynomial.
- We need *problem specific knowledge to guide the search*.
- Simplest form of problem specific knowledge is *heuristic*.
- Usual implementation in search is via an *evaluation function* which indicates desirability of expanding node.

Uniform Cost Search

- Recall we have a *path cost function*,

$$g : Nodes \rightarrow R$$

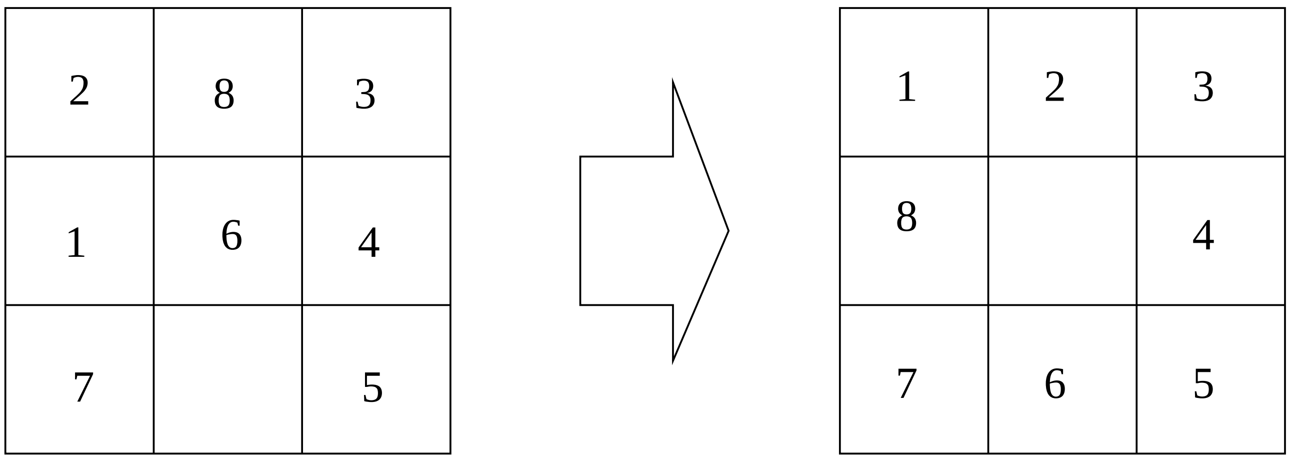
  which gives cost to each path.
- Why not expand the *cheapest* path first?
- Intuition: cheapest is likely to be best!

- General algorithm for uniform search:

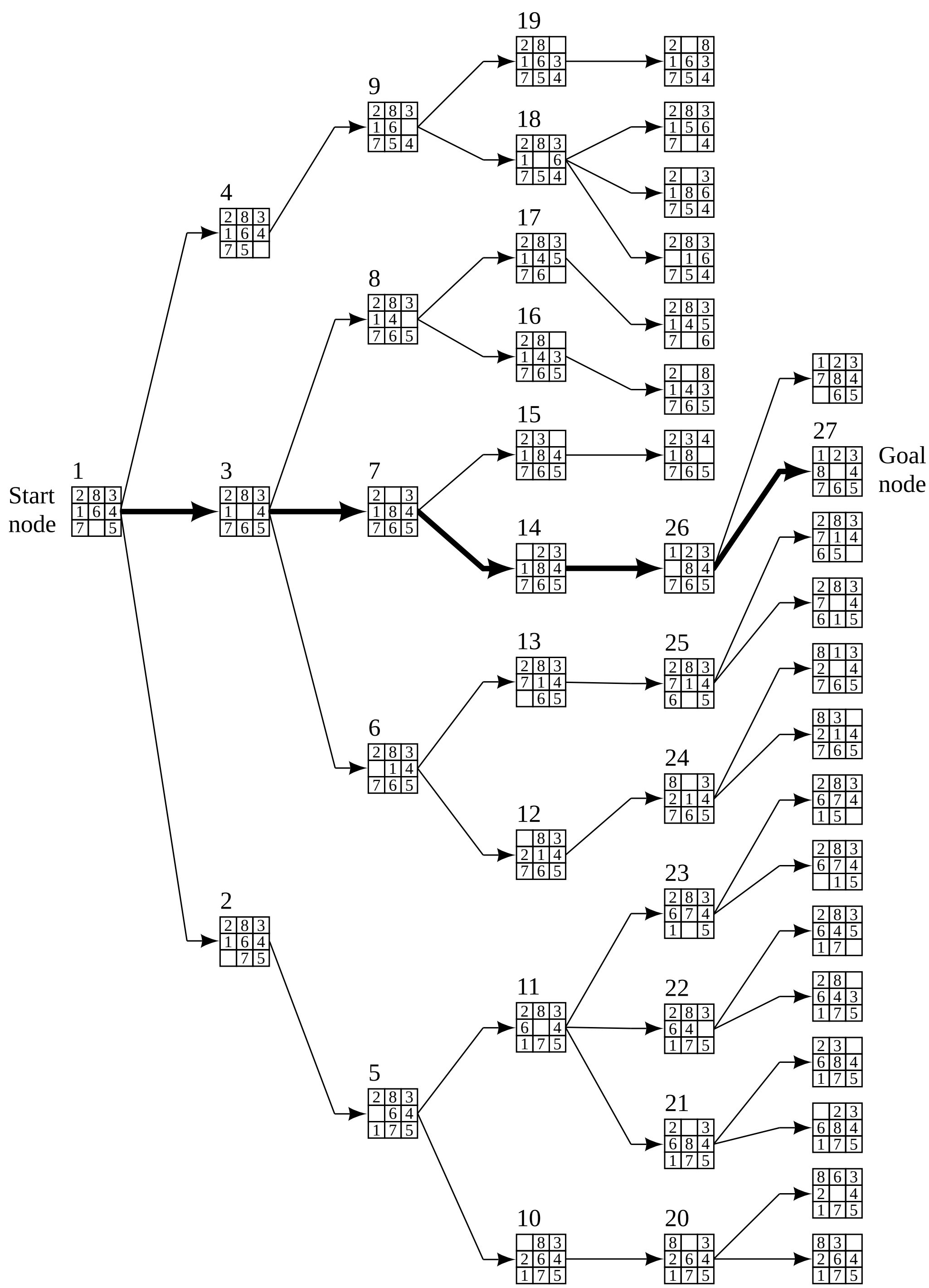
```
agenda = initial state;
while agenda not empty do
{
  take node from agenda such that
    g(node) = min { g(n) | n in agenda}
  new nodes = apply operations to node;
  if goal state in new nodes then {
    return solution;
  }
  else add new nodes to agenda
}
```

- Uniform cost search guaranteed to find cheapest solution *assuming path costs grow monotonically*.
- In other words, adding another step to the solution makes it more costly.
- If path costs *don't* grow monotonically, then exhaustive search is required.

- Once again we can illustrate this on the 8-puzzle:



- For this set up, the search of the space:



Start node
Goal node
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27

- Will happen in the following way.
- States would be expanded in the order:

1. 1
2. 2, 3, 4
3. 5, 6, 7, 8, 9
4. 10, 11, 12, 13, 14, 15, 16, 17, 18, 19
5. …

- Note that this is just like breadth first search (because the path costs are just the same).

- Instead, assume up/down moves cost 2 and left/right moves cost 1.
- States would be expanded in the order:

1. 1
2. 2, 3, 4
3. 5
4. 9
5. 6, 7, 8
6. …

Greedy Search

- Most heuristics *estimate cost of cheapest path from node to solution*.
- We have a *heuristic function*,

$$h : Nodes \longrightarrow R$$

  which estimates the distance from the node to the goal.
- Example: In route finding, heuristic might be straight line distance from node to destination.
- Heuristic is said to be *admissible* if it *never overestimates* cheapest solution.

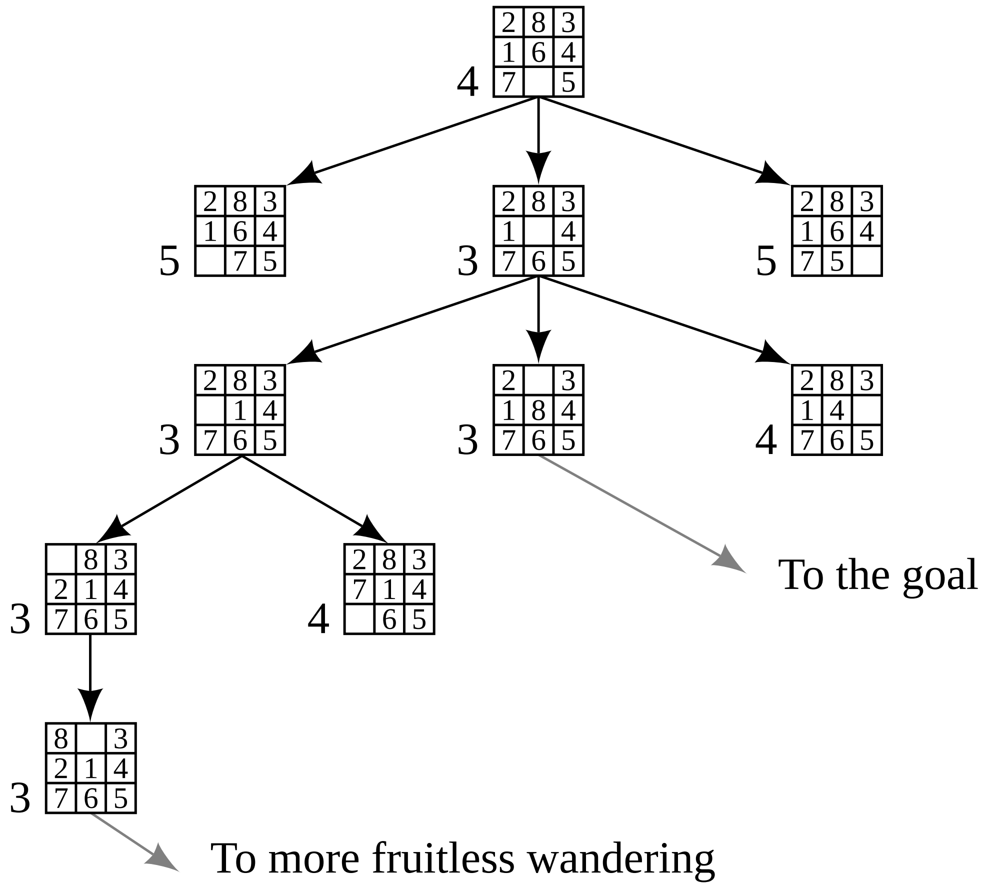
  Admissible = optimistic.
- Greedy search involves *expanding node with cheapest expected cost to solution*.

- General algorithm for greedy search:

```
agenda = initial state;
while agenda not empty do
{
  take node from agenda such that
    h(node) = min { h(n) | n in agenda}
  new nodes = apply operations to node;
  if goal state in new nodes then {
    return solution;
  }
  else add new nodes to agenda
}
```

- Greedy search finds solutions quickly.
- Doesn't always find best.
- Susceptible to false starts.
  - Chases good looking options that turn out to be bad.
- Only looks at *current* node. Ignores past!
- Also *myopic* (shortsighted).

- For the 8-puzzle one good heuristic is:
  - count tiles out of place.
- Another is:
  - *Manhattan blocks' distance*
- The latter works for other problems as well:
  - Robot navigation.


4
2 8 3
1 6 4
7 5
5
2 8 3
1 6 4
7 5
3
2 8 3
1 4
7 6 5
5
2 8 3
1 6 4
7 5
3
2 8 3
1 4
7 6 5
3
2 3
1 8 4
7 6 5
4
2 8 3
1 4
7 6 5
3
8 3
2 1 4
7 6 5
4
2 8 3
7 1 4
6 5
To the goal
3
8 3
2 1 4
7 6 5
To more fruitless wandering

A* Search

- A* is very efficient search strategy.
- Basic idea is to *combine*

uniform cost search
and
greedy search.

- We look at the *cost so far* and the *estimated cost to goal*.
- Gives heuristic f:

$$f(n) = g(n) + h(n)$$

where

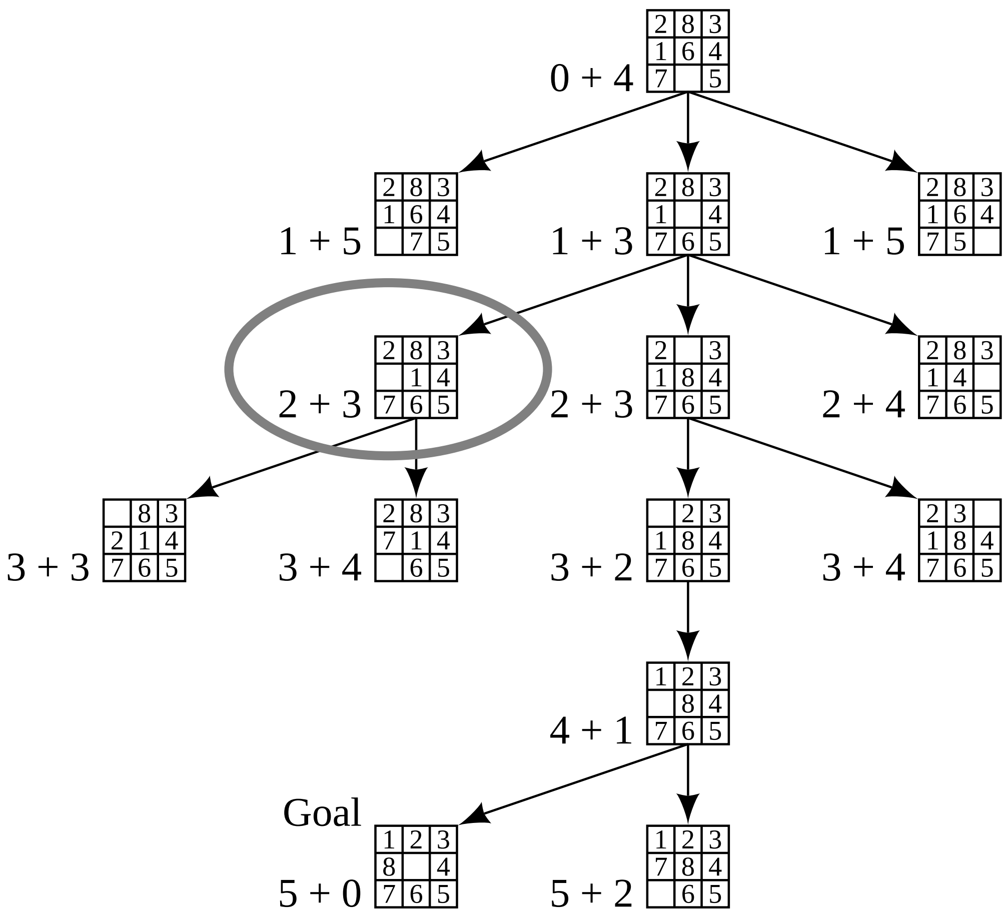
  - $g(n)$ is path cost of n;
  - $h(n)$ is expected cost of cheapest solution from n.

- Aims to mimimise *overall cost*.

- General algorithm for A* search:

```
agenda = initial state;
while agenda not empty do
{
  take node from agenda such that
    f(node) = min { f(n) | n in agenda}
    where f(n) = g(n) + h(n)
  new nodes = apply operations to node;
  if goal state in new nodes then {
    return solution;
  }
  else add new nodes to agenda
}
```

- Considering the 8-puzzle (for the last time :-):
- We combine:
  - Path cost function:
    - number of moves.
  - Heuristic function:
    - tiles out of places.
- This gives the following search.



0 + 4
1 + 5
1 + 3
1 + 5
2 + 3
2 + 3
2 + 4
3 + 3
3 + 4
3 + 2
3 + 4
4 + 1
Goal
5 + 0
5 + 2

The optimality of A*

- A* is optimal in precise sense—it is guaranteed to find a minimum cost path to the goal.
- There are a set of conditions under which A* will find such a path:
  1. Each node in the graph has a finite number of children.
  2. All arcs have a cost greater than some positive ϵ.
  3. For all nodes in the graph $h(n)$ always underestimates the true distance to the goal.
- The key here is the third bullet — the notion of *admissibility*.
- We will express this by saying a heuristic $h(\cdot)$ is admissible if

$$h(n) \leq h_T(n)$$

More informed search

- IF two versions of A*, A_1^* and A_2^* use different functions h_1 and h_2,
- AND
$$h_1(n) < h_2(n)$$
for all non-goal nodes,
- THEN we say that A_2^* is *more informed* than A_1^*.
- The better informed A* is, the less nodes it has to expand to find the minimum cost path.

- As an example of ”more informed” consider the 8-puzzle:
  - tiles out of place; and
  - Manhattan blocks distance.
- We need $h(n)$ to underestimate $h_T(n)$ to ensure admissibility.
- But, the closer the estimate, the easier it is to reject nodes which are not on the optimal path.
- This means less nodes need to be searched.

Iterative deepening A*

- When we do heuristic search, we search some portion of the full search space.
- ”Focussed breadth first search”.
- So we can still hit intractability.
- Adapting iterative deepening can help us.
- Instead of a depth limit, we impose a cost limit, and do a depth first search until it is exceeded.
- Then we backtrack, and extend the limit if we don’t find the goal.

- The initial cost cut off is set to $f(n_0)$.
- This is just the estimated cost of finding a solution $h(n_0)$.
- This will never overestimate the cost, so is a good start point.
- If this cost-limit does not provide a solution, what is the next cost limit.
- Well, if the heuristic is a good one, the cost of the cheapest path to the goal will be the lowest $f(n)$ of an unexpanded node.
- So we set the new cost bound to this.
- This, then is iterative deepening A* (IDA*).

Summary

- This lecture has looked at some techniques for refining the search space:
  - uniform cost search;
  - greedy search; and
  - A* search.
- When these work they explore just the relevant part of the search space.
- There are also techniques that go further than those we have studied.

- These techniques include:
  - Focussed Dynamic A* (called D*)
  - D* Lite
  - Delayed D*
  - Life-long planning A* (called LPA*)
  - PAO*
- There are three directions we will take from here:
  - Adversarial search
  - Learning the state space.
  - Adding in more knowledge about the domain.