LECTURE 8: MAKING GROUP DECISIONS

An Introduction to Multiagent Systems

CIS 716.5, Spring 2010

Today

- We continue thinking in the same framework as last lecture:
  - multiagent encounters
  - game-like interactions
  - participants act strategically
- We will look at protocols for *group decision making*
- This is *social choice theory*.
- Agents make decisions based on their preferences, but they are aware of other agents' preferences as well.

Basic setting

- The basic setting is familiar.
- We have a set of agents:

$$Ag = \{1, \ldots, n\}$$

  we will call them *voters* and what we are looking at here we will also call *voting theory*.
- Ag is finite.
- Will usually assume that $|Ag|$ is odd, so that it reduces the likelihood of ties.

- The voters are making a decision, as a group, with respect to:

$$\Omega = \{\omega_1, \omega_2, \ldots,\}$$

  a set of *outcomes* or *candidates*.
- It is usually assumed that this is finite.
- Sometimes we will want to pick one, most preferred candidate.
- More generally, we may want to *rank*, or *order* these candidates.
- In a real election, of course, Ω would be the candidates in an election.
- (Which, as you may recall from elections that you have voted in, can often be a relatively long list.)

- A particular set of outcomes is that of a *pairwise election*:

$$|\Omega| = 2$$

- The *general* voting scenario has:

$$|\Omega| > 2$$

Preferences

- Every voter has preferences over Ω.
- We will take these to be expressed by an order over the set.
- For example, one voter, i might have preferences:

$$(\omega_2, \omega_3, \omega_1)$$

- Some notation to make writing these preference orders a bit easier:

$$\hat{\omega}_1, \hat{\omega}_2, \ldots \hat{\omega}_n$$

  indicate the preference orders of the agents in Ag, and we will write:

$$\omega \succ_i \omega'$$

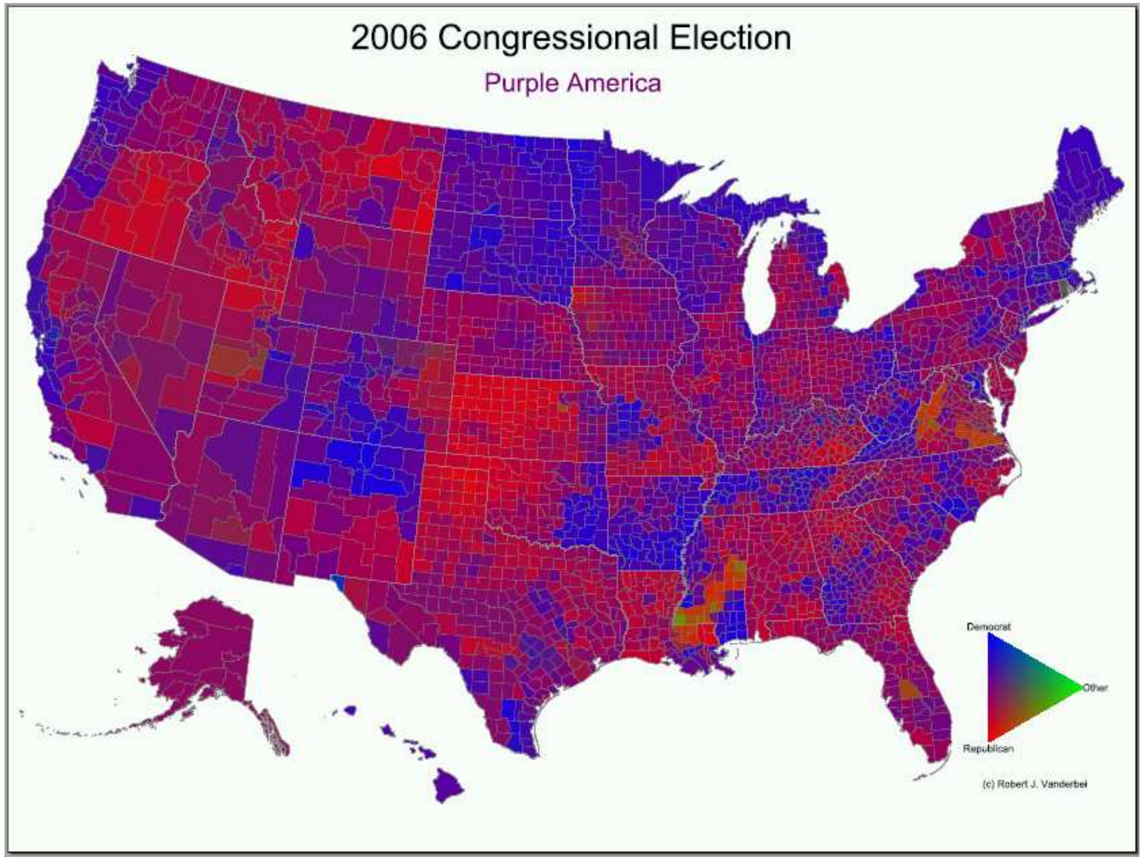
  to say that i ranks ω' above ω.
- $\Pi(\Omega)$ is the set of all preference orders over Ω.

Social choice

- Now, the basic problem that social choice theory deals with is:

  Different voters typically have different preference orders.

- Given that, we need a way to combine these opinions into on overall decision.
- What social choice theory is about is finding a way to do this.



Map by Robert J. Vanderbei. Shade indicates proportion of vote.

- A *social welfare function* takes voter preferences and constructs a *social preference order*.
- That is it merges voter opinions and comes up with an order over the candidates.

$$f : \underbrace{\Pi(\Omega) \times \cdots \times \Pi(\Omega)}_{n \text{ times}} \mapsto \Pi(\Omega)$$

- We write $\succ^*$ to indicate the order that emerges from a social welfare function:

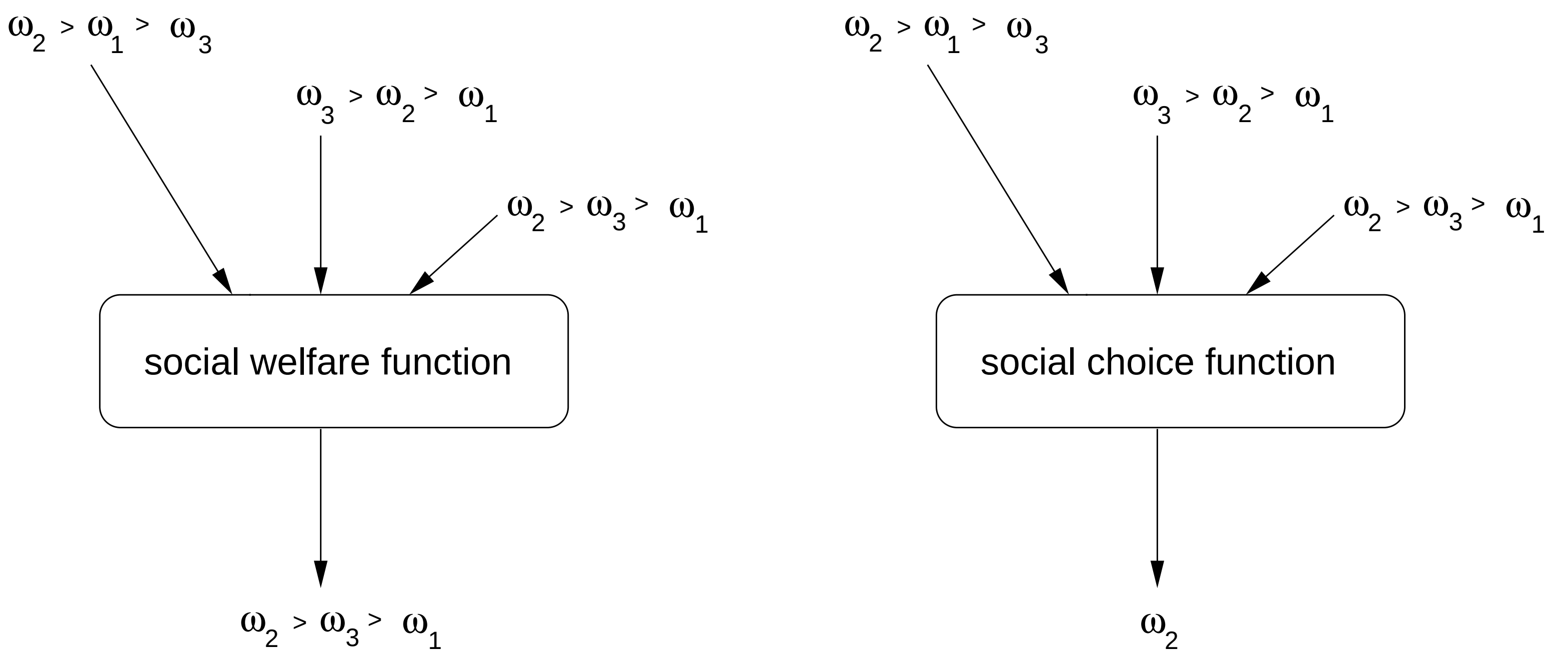
$$\omega \succ^* \omega'$$

indicates that ω is ranked above ω' in the social ordering.

- A *social choice function* is a simpler mapping, which just picks the most preferred candidate taking into account what all the voters think:

$$f : \underbrace{\Pi(\Omega) \times \cdots \times \Pi(\Omega)}_{n \text{ times}} \mapsto \Omega$$

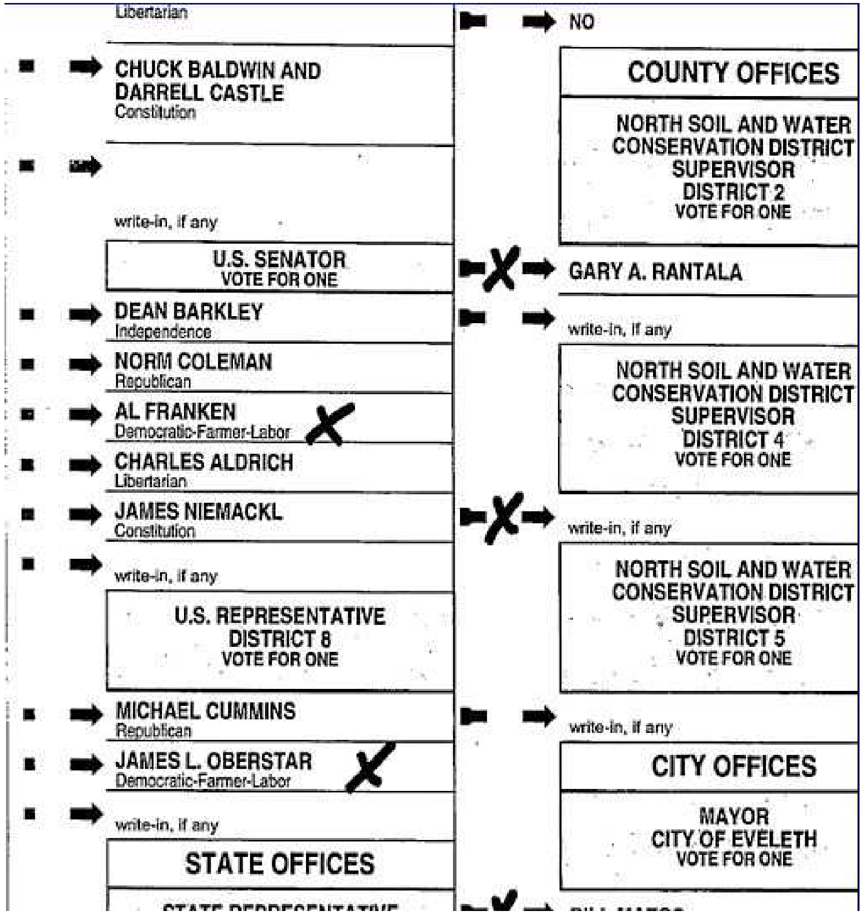
- In other words, we don't get an ordering out of a social choice function but, as its name suggests, we get a single choice.
- Of course, if we have a social welfare function, we also have a social choice function.
- For the rest of this lecture we'll call both social choice and social welfare functions *voting procedures*.



$\omega_2 > \omega_1 > \omega_3$
$\omega_3 > \omega_2 > \omega_1$
$\omega_2 > \omega_3 > \omega_1$
social welfare function
$\omega_2 > \omega_3 > \omega_1$
$\omega_2 > \omega_1 > \omega_3$
$\omega_3 > \omega_2 > \omega_1$
$\omega_2 > \omega_3 > \omega_1$
social choice function
ω_2

Plurality

- Simplest and best known voting procedure.
- Every voter submits their preference order.
- We add up the number of first place votes for each candidate.
- The candidate with the highest number of votes wins.
- Often, since we only use the highest place vote, that is the only thing we ask voters for.


Libertarian

CHUCK BALDWIN AND
DARRELL CASTLE
Constitution

write-in, if any

U.S. SENATOR
VOTE FOR ONE

DEAN BARKLEY
Independence

NORM COLEMAN
Republican

AL FRANKEN
Democratic-Farmer-Labor

CHARLES ALDRICH
Libertarian

JAMES NIEMACKL
Constitution

write-in, if any

U.S. REPRESENTATIVE
DISTRICT 8
VOTE FOR ONE

MICHAEL CUMMINS
Republican

JAMES L. OBERSTAR
Democratic-Farmer-Labor

write-in, if any

STATE OFFICES

NO

COUNTY OFFICES

NORTH SOIL AND WATER
CONSERVATION DISTRICT
SUPERVISOR
DISTRICT 2
VOTE FOR ONE

GARY A. RANTALA

write-in, if any

NORTH SOIL AND WATER
CONSERVATION DISTRICT
SUPERVISOR
DISTRICT 4
VOTE FOR ONE

write-in, if any

NORTH SOIL AND WATER
CONSERVATION DISTRICT
SUPERVISOR
DISTRICT 5
VOTE FOR ONE

write-in, if any

CITY OFFICES

MAYOR
CITY OF EVELETH
VOTE FOR ONE

- The advantage of plurality is that it is simple.
- Easy to understand.
- If we have just two candidates it is *simple majority voting*.
- Which is what most US presidential elections would be like if it weren't for the electoral college.
- Where there are more than 3 candidates, some strange effects can take place.
- Let's look at an example of this.

Elections in the UK

Whigs have gathered inside the inn and the Tories are demonstrating on the street. Liberty and Loyalty is written on the Whig banner, Liberty can be read on the Tory banner. In this ”loyal and liberal” atmosphere the one party is hurling bricks while the other party is answering with the content of a chamber pot. The man in the foreground has his head injured in the battle but has succeeded in capturing a banner of the opposing party. The inscription Give us our eleven days refers to adopting the Gregorian calendar in 1752 due to the efforts of the President of the Royal Society being the father of the Whig candidate. Obviously the jump straight from September 3 to September 14 was an eloquent argument con the Whigs who had ”stolen” eleven days of Tories’ lives. On the contrary, the luxurious treat (the Mayor seems to have collapsed from a surfeit of oysters!), the huge pot of punch, and the kisses of the candidates are the best arguments pro.

- In the UK there are three main parties.

  Labour, Liberal Democratic, Conservative.

- In theory Labour is the most left wing, the LibDems are a center party, and the Conservatives are more right wing.
- (In practice, the LibDems are to the left of Labour on many issues.).
- In an election, imagine have three types of voter:
  - Left of center;
  - Center; and
  - Right of center.

  with the center voters being about 12% of the total, with the remainder split roughly between left (43%) and right (45%).

- (While historically this is the case, ans it serves the example nicely, at the time of writing, April 2010, the vote in the UK is much more evenly split.)
- Preferences might be:

  Left of center: $\omega_L \succ \omega_D \succ \omega_C$

  Center: $\omega_D \succ \omega_L \succ \omega_C$

  Right of center: $\omega_C \succ \omega_D \succ \omega_L$
- And the result of the plurality vote might be that the Conservative candidate wins sinc ethey have the most first place votes.
- That seems reasonable until you recall that 55% of the voters rated the winning candidate *last*.

- In fact the situation in the UK can be worse that this.

- The difference in votes between Labour and Conservative is usually small, but one party invariably ends up with a large majority.

- And despite getting close to 20% of the vote at times, the LibDems typically have only a handful of seats.
- The result is *tactical voting*.
- You vote not for your preferred candidate, but for the candidate of the third party that you think might beat the party you think will otherwise win.
- So you misrepresent your preferences.
- Which is what I would do if I was to vote in the UK.

- Let's look at another version of this problem.
- Imagine a situation with three outcomes:

$$\Omega = \{\omega_1, \omega_2, \omega_3\}$$

and three voters with preferences:

$$\omega_1 \succ_1 \omega_2 \succ_1 \omega_3$$
$$\omega_3 \succ_2 \omega_1 \succ_2 \omega_2$$
$$\omega_2 \succ_3 \omega_3 \succ_3 \omega_1$$

- Plurality voting says that this is a tie. Each candidate gets one third of the votes.

- In many systems, we'd settle this with some tie-breaker.
- (In the UK, the candidates might draw a card, or throw a die).
- But there isn't an outcome that we can argue is good.
  - If we pick ω_1, well two thirds of the voters prefer ω_3 to ω_1.
  - If we pick ω_3, then two thirds of the voters prefer ω_2.
  - If we pick ω_2, it is still the case that two thirds of the voters prefer a different cancdidate, in this case ω_1 to the candidate we picked.
- This is an example of *Condorcet's paradox*.

Condorcet's paradox

- In a democracy, it seems inevitable that we can't choose an outcome that will make everyone happy.
- Condorcet's paradox tells us that in some situations, no matter which outcome we choose, a *majority* of voters will be unhappy with the outcome.

Nicolas de Caritat, marquis de Condorcet

Sequential majority elections

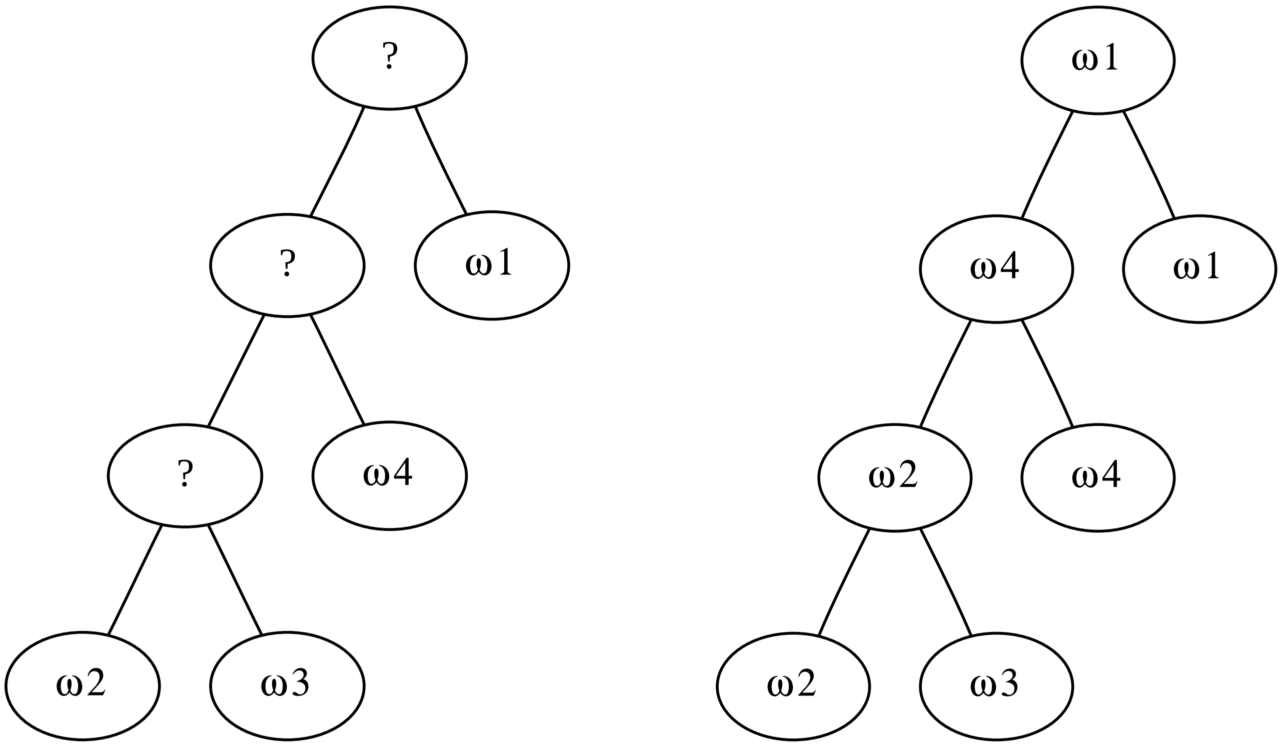
- One way to improve on plurality voting is to reduce a general voting scenario to a series of pairwise voting scenarios.
- We can do this in a number of ways.
- One *agenda* for the election between

$$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$$

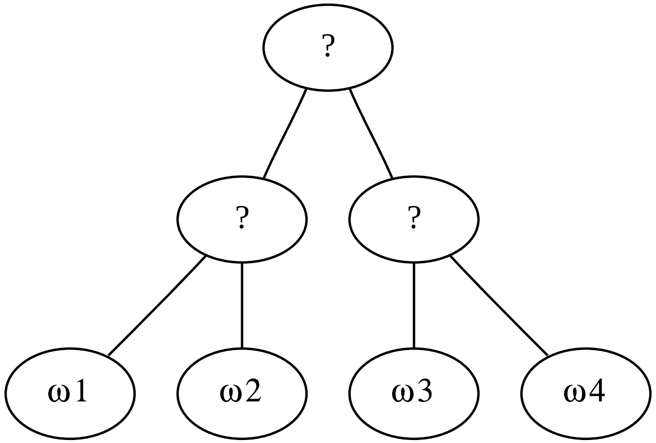
is

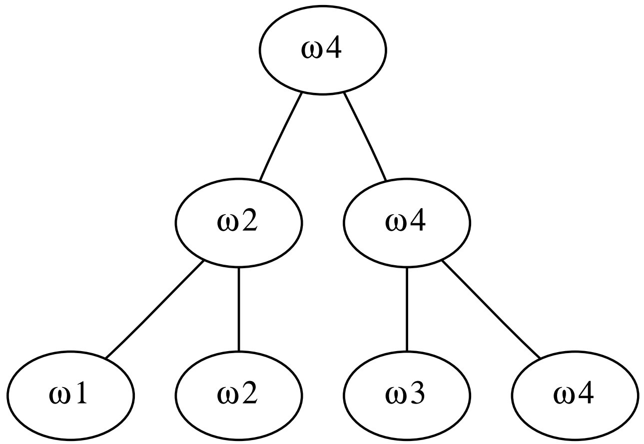
$$\omega_2, \omega_3, \omega_4, \omega_1$$

- First we have an election between ω_2 and ω_3.
- The winner enters an election with ω_4.
- The winner of that faces ω_1.


?
?
ω1
?
ω4
ω2
ω3
ω1
ω4
ω1
ω2
ω4
ω2
ω3

- We can also organise this as a *balanced binary tree*.
  - An election between ω_1 and ω_2.
  - An election between ω_3 and ω_4.
  - An election between the two winners.
- Rather like the Final Four.


?
?
?
ω1
ω2
ω3
ω4


ω4
ω2
ω4
ω1
ω2
ω3
ω4

- In fact, during this year's March madness, `jezebel.com` held a sequential majority election to find the best dessert.
- The election followed the format of the basketball tournament:

Cheesecake won.

- Since there is scope for fixing the outcome when choosing match-ups, the agenda for a sequential majority election opens up the possibility of *manipulation*.
- We can analyse this.

Majority graph

- A *majority graph* is a directed graph.
- Nodes are outcomes in Ω.
- A directed edge from node ω to ω' if a mjority of voters rank ω above ω'

  If ω would beat ω' in a direct competition.
- With an odd number of voters (no ties) the majorty graph is such that:
  - The graph is *complete*.
  - The graph is *asymmetric*.
  - The graph is *irreflexive*.
- Such a graph is called a *tournament*, a nice summarization of information about voter preferences.

- lets go back to this situation:

$$\Omega = \{\omega_1, \omega_2, \omega_3\}$$

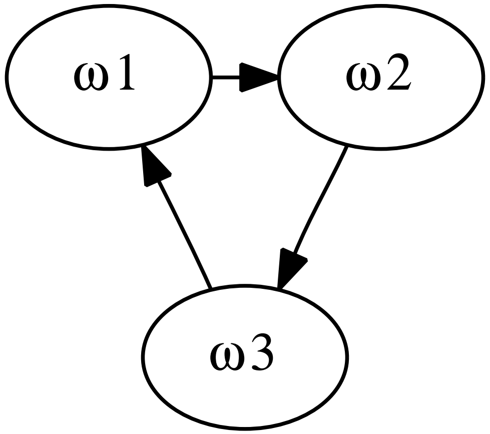
and three sets of voters with preferences:

$\omega_1 \succ_1 \omega_2 \succ_1 \omega_3$

$\omega_3 \succ_2 \omega_1 \succ_2 \omega_2$

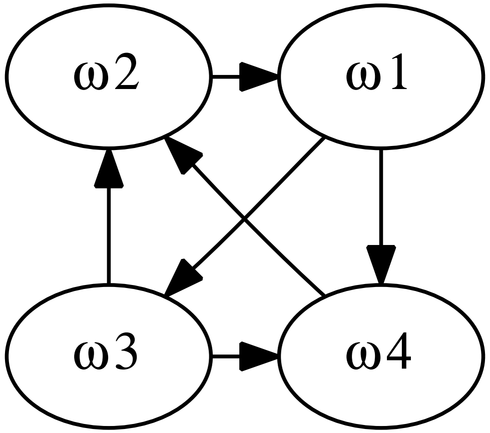
$\omega_2 \succ_3 \omega_3 \succ_3 \omega_1$

- This has the following majority graph.



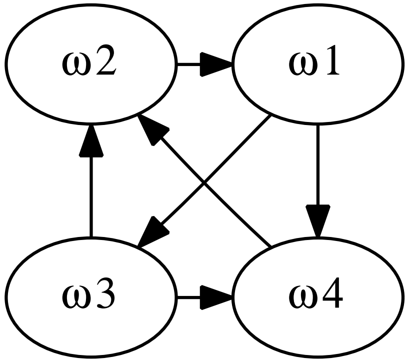
- Since the graph contains a cycle, it turns out that we can fix whatever result we want.
- All we have to do is to pick the right order of the elections.

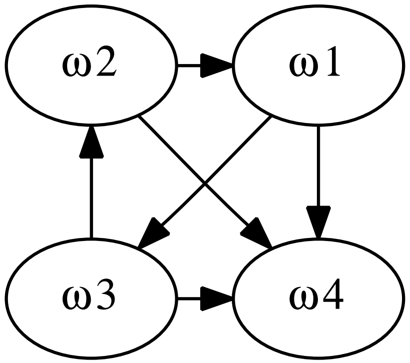
- Here's another example where any candidate can win:



- Now, we say that a result is a *possible winner* if there is an agenda that will result in it winning overall.
- An outcome is a *Condorcet winner* if it is the overall winner in all agendas.
- The majority graph helps us determine this.
- To determine if ω_i is a possible winner, we have to find, for every other ω_j, if there is a path from ω_i to ω_j in the majority graph.
- This is computationally easy to do.
- Checking for a Condorcet winner is also easy. To determine if ω_i is a Condorcet winner, we have to find, if there is an edge from ω_i to every other node in the majority graph.

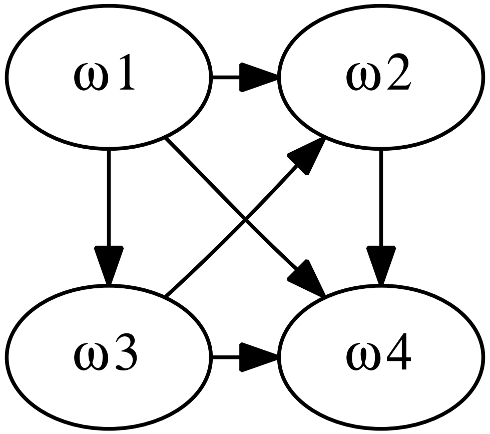
- Clearly in the top majority graph, every candidate is a possible winner.





- Whereas in the bottom graph ω_4 is not a possible winner.

- Finally, here is a majority graph.



- Where ω_1 is a Condorcet winner.

The Borda count

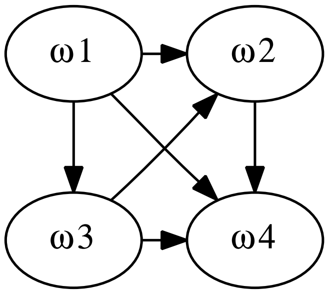
- The procedures we have looked at so far only consider the top-ranked candidates.
- That seems to ignore a lot of information.
- The *Borda count* takes into all the preference information.
- With k candidates, for each candidate we have a number, the strength of opinion in favor of the candidate — you can think of it as the number of votes.
- This is the Borda count.
- If an outcome ω_i appears first in the perference order of some voter, it gets a count of $k - 1$ added to its Borda count.
- If it appears second in the preference order of some voter, its count gets increased by $k - 2$, and so on.
- The final ordering just puts the candidates in order of their count.

The Slater ranking

- The Slater ranking looks to construct a social ranking that is acyclic but is as close to the majority graph as possible.
- A majority graph represents a preference order over a set of outcomes.
- Any two graphs for the same set of outcomes can be transformed into one another by flipping edges (which corresponds to flipping preferences between candidates).
- If we count the flips we have a notion of distance between the two graphs or their corresponding preference orders.

- Thus, if we have the result of a social welfare function, we can compare it with the majority graph that summarises the set of voter preferences.
- And we can, in theory, look through all the possible social welfare functions to find the one that is as close as possible to the majority graph.
- This closest ranking is the *Slater ranking*.

- Looking back at our last example:



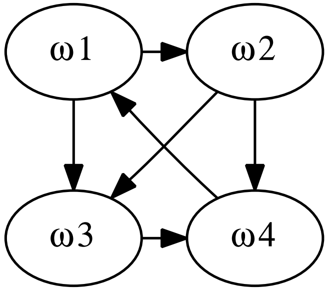
we see that the social choice:

$$\omega_1 \succ \omega_3 \succ \omega_2 \succ \omega_4$$

would be acceptable.

- Every entry appears before an entry that it would beat in a pairwise election.

- Now consider this next example.



- Since the graph has a cycle, any order will disagree with the majority graph in sense of ranking one outcome above the other despite the fact it would lose in a pairwise election.
- Consider:

$$\omega_1 \succ \omega_2 \succ \omega_3 \succ \omega_4$$

- Here ω_1 appears before ω_4 despite what the graph says.

- To make the ordering consistent with the graph we'd have to flip the edge (ω_1, ω_4)
- This is the only change we'd have to make, so the cost of the order is 1.
- In comparison, the cost of:

$$\omega_1 \succ \omega_2 \succ \omega_4 \succ \omega_3$$

is 2.

- The Slater ranking is helpful because it allows us to get a grip on how good an ordering is.
- Sadly computing the Slater ranking is NP-hard.

(Desirable) properties of voting procedures

- So far we have looked at specific examples of voting procedures.
- There's a more systematic approach — figure out what we would like from a social choice function and then derive one that meets these conditions.
- Conditions like:
  - The Pareto condition
  - The Condorcet winner condition
  - Independence of irrelevant alternatives
  - Dictatorships

The Pareto condition

- Recall the notion of Pareto efficiency from the previous lecture.
- An outcome is Pareto efficient if there is no other outome that makes one agent better off withot mking another worse off.
- In voting terms, if every voter ranks ω_i above ω_j then $\omega_i \succ^* \omega_j$.
- Satisfied by plurality and Borda but not by sequential majority.

The Condorcet winner condition

- Recall that the Condorcet winner is an outcome that would beat every other outcome in a pairwise election.
  - A Condorcet winner is a strongly perferred outcome.
- The Condorcet winner condition says that if there is a Condorcet winner, then it should be ranked first.
- Seems obvious.
- However, of the ones we've seen, only sequential majority satisfies it.

Independence of irrelevant alternatives

- Suppose there are a number of candidates including ω_i and ω_j. and voter preferences make $\omega_i \succ^* \omega_j$.
- Now assume one voter k changes perferences, but still ranks $\omega_i \succ_k \omega_j$
- The independence of irrelevant alternative (IIA) condition says that however $\succ^*$ changes, $\omega_i \succ^* \omega_j$ still.
- In other words, the social ranking of ω_i and ω_j should depend only on the way they are ranked in the $\succ$ relations of the voters.
- Plurality, Borda and sequential majority do not satisfy IIA.

Dictatorships

- Not a desriable property, but a useful notion to define.
- A social welfare function f is a dictatorship if for some i:

$$f(\hat{\omega}_1, \hat{\omega}_2, \ldots \hat{\omega}_n) = \hat{\omega}_i$$

- In other words the output is exactly the preference order of i.
- Plurality, the Slater ranking and the Borda count are not dictatorships.
- But, dictatorships satisfy the Pareto condition and IIA.

Arrow's theorem

- Given the list of properties of social choice functions we can ask:

  Is there a voting procedure which satisfies these conditions?

- Answer: Not really! Arrow's theorem proves this.

Kenneth Arrow

- Assume we are not in a pairwise election.
- Assume we have a voting procedure that satisfies the Pareto condition and the independence of irrelevant alternatives.
- What can this procedure be:

  Dictatorship is the only option

- Which is not really a “good” voting procedure.

- What does this mean in practice?
- Well, it doesn't seem to mean that we have to settle for dictatorship :-)
- What it does mean is that we have to compromise in terms of what we accept as a voting procedure.
- Whatever more democratic procedure than dictatorship we opt for, it will have problems.

E.M. Forster

- "So two cheers for Democracy: one because it admits variety and two because it permits criticism."

Winston Churchill

- "It has been said that democracy is the worst form of government except all the others that have been tried."

Strategic manipulation

- We have seen informally how misrepresenting preferences might lead to a preferred outcome than being truthful.
- Can we find voting mechanisms that are not manipulable?
- The Gibbard-Satterthwaite theorem says no.
- If we have more than two outcomes, the only non-manipulable voting protocol that satisfies the Pareto condition is

  Dictatorship
- Argh!

- However, what we can find are mechanisms that are hard (in a computational sense) to manipulate.
- In theory they are able to be manipulated, but in practice they are difficult to manipulate.
- This gives us some guarantee that the mechanism won't be manipulated

  For example, because there is not enough time to decide how to manipulate it.
- (Though we only have worst-case guarantees on manipulability).

Summary

- In this lecture we have looked at mechanisms for group decision making.
- This has been a bit stylised — we looked at how, if a group of agents ranks a set of outcomes, we might create a consensus ranking.
- This does have a real application in voting systems.
- We looked at the behavior of some existing voting systems.
- We also looked at theoretical results for voting systems in general.
- Sadly most of these results were pretty negative.