CSc 74010 Fall 2011, Homework 2

1. Which, if any, of the following sentences is a proposition? In each case, justify your answer.

- (a) Twice two is four.
- (b) The square root of ten is three.
- (c) 5 > 6
- (d) Please write a specimen of your signature in the space provided.
- (e) Would you believe Lady Gaga once stood in exactly that spot?
- (f) Yohan Blake, who won at the world championship after Usain Bolt was disqualified on a false start, ran a personal-best 9.82 seconds Thursday.

(20 points)

- 2. Using the following propositions:
- p Its is raining
- q I have an umbrella
- r I get wet

Formulate the following expressions in words:

(a)
$$(p \land q)$$

(b) $(p \land \neg q) \land r$
(c) $\neg p \land \neg r$
(d) $p \land (q \lor r)$
(e) $\neg p \lor r$
(f) $(q \land \neg r) \lor (\neg q \land r)$
(15 points)

3. For each of the propositions in the previous question, write out the truth table.

(15 points)

- For each of the propositions in question 2, say whether it is: consistent, inconsistent and/or a tautology. (15 points)
- 5. Use the truth table method to decide whether the following statements are true:
 - (a) $(p \land q) \models \neg (p \lor q)$ (b) $p, (p \Leftrightarrow q) \models p \land q$ (c) $(p \Rightarrow q) \models ((p \Rightarrow r) \Rightarrow (p \Rightarrow r))$ (15 points)
- 6. There is a list of natural deduction proof rules for propositional logic on the class web page. Using these, try to prove the following:

$$\begin{array}{ll} \textbf{(a)} & (p,p \Rightarrow (q \land r)) \vdash (p \land r) \\ \textbf{(b)} & (p,p \Rightarrow (q \land r)) \vdash (s \lor r) \\ \textbf{(c)} & (p \land (p \Rightarrow (q \land r))) \vdash (p \Rightarrow r) \\ \textbf{(d)} & (p \Rightarrow (q \land r)) \vdash (p \Rightarrow r) \\ \textbf{(e)} & (\neg q,p \Leftrightarrow (q \land r)) \vdash \neg p \end{array}$$

(20 points)