Proof rules for Propositional Logic

This includes all the rules in the lecture notes and some additional ones — together they make a complete set of rules for propositional logic.

$$\frac{}{\vdash S} \; \text{TAUT} \quad \text{if } S \text{ is a tautology}$$

$$\frac{\vdash S_1 \land S_2}{\vdash S_1; \vdash S_2} \; \land\text{-E} \qquad\qquad \frac{\vdash S_1; \vdash S_2}{\vdash S_1 \land S_2} \; \land\text{-I}$$

$$\frac{\begin{array}{c}\vdash S_1 \lor \cdots \lor S_n; \\ S_1 \vdash C; \\ \cdots; \\ S_n \vdash C\end{array}}{\vdash C} \; \lor\text{-E} \qquad\qquad \frac{\vdash S_1}{\vdash S_1 \lor \cdots \lor S_n} \; \lor\text{-I}$$

$$\frac{\vdash S_1 \Rightarrow S_2; \vdash S_1}{\vdash S_2} \; \Rightarrow\text{-E} \qquad\qquad \frac{S_1 \vdash S_2}{S_1 \Rightarrow S_2} \; \Rightarrow\text{-I}$$

$$\frac{\vdash S_1 \Leftrightarrow S_2}{\vdash S_1 \Rightarrow S_2; \vdash S_2 \Rightarrow S_1} \; \Leftrightarrow\text{-E} \qquad\qquad \frac{\vdash S_1 \Rightarrow S_2; \vdash S_2 \Rightarrow S_1}{\vdash S_1 \Leftrightarrow S_2} \; \Leftrightarrow\text{-I}$$

$$\frac{\vdash \neg\neg S_1}{\vdash S_1} \; \neg\text{-E} \qquad\qquad \frac{S_1 \vdash \bot}{\vdash \neg S_1} \; \neg\text{-I}$$

For the last of these rules, remember that $\bot$ stands for any formula which is inconsistent (for example $S_1 \land \neg S_1$).