

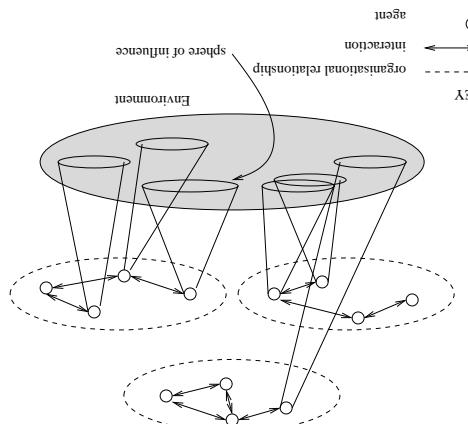
$$\begin{aligned} w \succ^i w' &\text{ means } u^i(w) < u^i(w') \\ w \geq^i w' &\text{ means } u^i(w) \geq u^i(w') \end{aligned}$$

- Utility functions lead to **preference orderings** over outcomes:

$$\begin{aligned} u^i : \mathcal{O} &\rightarrow I\!\!R \\ u^i : \mathcal{O} &\rightarrow I\!\!R \end{aligned}$$

- We capture preferences by **utility functions**:
- Agents have preferences over other agents.
- Assume $\mathcal{O} = \{w_1, w_2, \dots\}$ is the set of "outcomes" that agents over how the environment is.
- Agents are assumed to be **self-interested**: they **have preferences** over how the environment is.
- Assume we have just two agents: $Ag = \{i, j\}$.

2 Utilities and Preferences



1 What are Multiagent Systems?

- ... will be linked by other (organisational) relationships.
- ... have different "spheres of influence" (which may coincide).
- ... are able to act in an environment ...
- ... which interact through communication ...
- ... thus a multiagent system contains a number of agents ...

LECTURE 6: MULTIAGENT INTERACTIONS

http://www.csc.liv.ac.uk/~mjtw/pubs/mas/

Rational Action

- Suppose we have the case where **both** agents can influence the outcome, and they have utility functions as follows:
- With a bit of abuse of notation:
- Then agent i 's preferences are:
- "C" is the **rational choice** for i .
- (Because i prefers all outcomes that arise through C over all outcomes that arise through D .)

$u_i(D, D) = 1 \quad u_i(D, C) = 4 \quad u_i(C, D) = 1 \quad u_i(C, C) = 4$

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• An introduction to Multiagent Systems

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3 Multiagent Encounters

- We need a model of the environment in which these agents will act...
- agents simultaneously choose an action to perform, and as a result of the actions they select, an outcome in Ω will result;
- the **actual** outcome depends on the **combination** of actions;
- assume each agent has just two possible actions that it can perform C ("cooperate") and D ("defect").
- environment behaviour given by **state transformer function**:
- agent i 's action agent j 's action
$$\tau : A^G \times A^G \rightarrow \Omega$$

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What is Utility?

- Utility is **not** money (but it is a useful analogy).
- Typical relationship between utility & money:
- Here is a state transformer function:
- Here is another:
- Neither agent has any influence in this environment.
- And here is another:
- This environment is controlled by j .

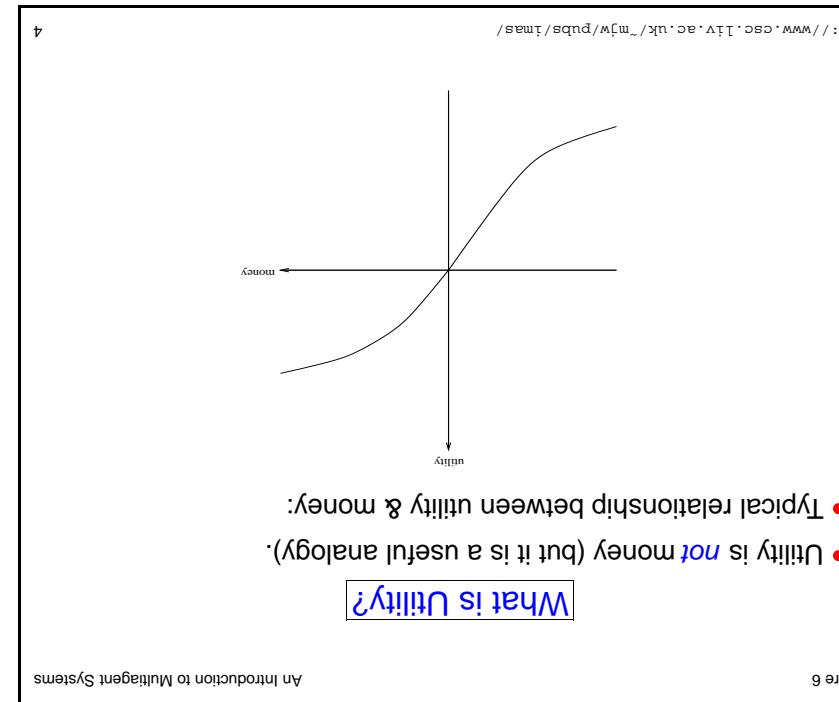
$\tau(D, D) = w_1 \quad \tau(D, C) = w_2 \quad \tau(C, D) = w_1 \quad \tau(C, C) = w_2$

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$\tau(D, D) = w_1 \quad \tau(D, C) = w_2 \quad \tau(C, D) = w_3 \quad \tau(C, C) = w_4$

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Competitive and Zero-Sum Interactions

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- Zero sum encounters in real life are very rare . . . but people tend to act in many scenarios as if they were zero sum.
- Zero sum implies strictly competitive.
- Zero sum implies strictly competitive.

$$u_i(w) + u_j(w) = 0 \quad \text{for all } w \in \mathcal{Q}.$$

- Zero-sum encounters are those where utilities sum to zero.
- Where preferences of agents are diametrically opposed we have **strictly competitive** scenarios.
- Zero-sum encounters are those where utilities sum to zero.

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Dominant Strategies

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- Given any particular strategy s (either C or D) agent i , there will be a number of possible outcomes.
- We say s_1 **dominates** s_2 if every outcome possible by i playing s_1 is preferred over every outcome possible by i playing s_2 .
- A rational agent will never play a dominated strategy.
- So in deciding what to do, we can **delete dominated strategies**.
- Unfortunately, there isn't always a unique undominated strategy.

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Nash Equilibrium

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- In general, we will say that two strategies s_1 and s_2 are in Nash equilibrium if:
- 1. under the assumption that agent i plays s_1 , agent j can do no better than play s_1 .
- 2. under the assumption that agent j plays s_2 , agent i can do no better than play s_2 ; and
- Neither agent has any incentive to deviate from a Nash equilibrium.

1. Not every interaction scenario has a Nash equilibrium.

2. Some interaction scenarios have more than one Nash equilibrium.

Unfortunatelly:

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Payoff Matrices

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		Agent i	
		defect	coop
Agent j	defect	1	4
	coop	4	1

We can characterise the previous scenario in a **payoff matrix**

• Agent i is the **column player**.

• Agent j is the **row player**.

- Surely they should both cooperate and each get payoff of 3!
- But *intuition* says this is *not* the best outcome:
- So defection is the best response to all possible strategies: both agents defect, and get payoff = 2.
- This guarantees a payoff of at most 1.
- The *individual rational* action is *defect*.

- They are told that:
- if one confesses and the other does not, the confessor will be freed, and the other will be jailed for three years;
 - if both confess, then each will be jailed for two years.
 - Both prisoners know that if neither confesses, then they will each be jailed for one year.

4 The Prisoner's Dilemma

- Payoff matrix for prisoner's dilemma:
- | | defect | coop |
|--------|--------|------|
| defect | 1, 1 | 4, 4 |
| coop | 4, 4 | 3, 3 |

- This apparent paradox is *the fundamental problem of multi-agent interactions*.
- It appears to imply that *cooperation will not occur in societies of self-interested agents*.
- Real world examples:
 - nuclear arms reduction ("why don't I keep mine...")
 - free rider systems — public transport
 - in the UK — television licences.
- Can we recover cooperation?
- The prisoners dilemma is *ubiquitous*.

the best strategy.

- Playing the prisoners' dilemma known number of rounds, defection is pre-determined, common knowledge.

This is the **backwards induction** problem.

Incentive to defect there, too.

But this makes round $n - 2$ the last "real", and so you have an

On round $n - 1$, you have an incentive to defect, to gain that extra

bit of payoff... .

But... suppose you both know that you will play the game

exactly n times.

4.2 Backwards Induction

- One answer: **Play the game more than once.**
- If you know you will be meeting your opponent again, then the incentive to defect appears to evaporate.
- Cooperation is the rational choice in the infinitely repeated prisoners' dilemma.
- (Hurray!)

4.1 The Iterated Prisoner's Dilemma

- Conclusions that some have drawn from this analysis:
- the game theory notion of rational action is wrong!
- somehow the dilemma is being formulated wrongly
- we are not all machiavellian
- the other prisoner is my twin!
- The shadow of the future... .

Arguments for Recovering Cooperation

- **ALLD:** Axelrod suggests the following rules for succeeding in his tournament:
 - **Don't be envious:** Don't play as if it were zero sum!
 - **Be nice:** Start by cooperating, and reciprocate cooperation.
 - **Retaliiate appropriately:** Always punish defection immediately, but use "measured" force — don't overdo it.
 - **Don't hold grudges:** Always reciprocate cooperation immediately.

Recipes for Success in Axelrod's Tournament

- Given the 4 possible outcomes of (symmetric) cooperation/defection games, there are 24 possible orderings on outcomes.
- 6 Other Symmetric 2 x 2 Games

	- CC \succ_i DC \succ_i DD \succ_i CD	- CC \succ_i CC \succ_i CD \succ_i CD	- DC \succ_i CC \succ_i CD \succ_i CD	- DC \succ_i CC \succ_i DD \succ_i CD
	- CC \succ_i CD \succ_i DC \succ_i DD	- DC \succ_i CD \succ_i CC \succ_i DD	- DC \succ_i DD \succ_i CC \succ_i CD	- CC \succ_i CD \succ_i DD \succ_i CD
	- Cooperition dominates.	- Deadlock. You will always do best by defecting.	- Prisoner's dilemma.	- Chicken.
	- Deadlock. You will always do best by defecting.	- Prisoner's dilemma.	- Chicken.	- Stag hunt.

Given the 4 possible outcomes of (symmetric) cooperation/defection games, there are 24 possible orderings on outcomes.

Given the 4 possible outcomes of (symmetric) cooperation/defection

6 Other Symmetric 2 x 2 Games

- **Joss:** On 1st round, defect. If the opponent retaliated, then play TIT-FOR-TAT. Otherwise intersperse cooperation & defection.
- **Tester:** On round $n = 0$, cooperate. Do what your opponent did on round $n - 1$.
- **TIT-FOR-TAT:** Axelrod suggests the following rules for succeeding in his tournament:
 - **Always defect** — the **hawk** strategy;
 - **Always cooperate**;
 - **Don't be envious**;
 - **Be nice**;
 - **Retaliiate appropriately**;
 - **Don't hold grudges**:
- **Consider another type of encounter — the game of chicken:**
 - **5 Game of Chicken**
- **Difference to prisoners dilemma:**
 - **coop, driving straight = defect**)
 - **Think of James Dean in Rebel without a Cause: swerving =**
- **Whereas sucker's payoff is most feared in prisoners dilemma.)**
- **Strategies (c,d) and (d,c) are in Nash equilibrium**

Multiplayer is most feared outcome. Whereas sucker's payoff is most feared in prisoners dilemma.) Strategies (c,d) and (d,c) are in Nash equilibrium	(Whereas sucker's payoff is most feared in prisoners dilemma.) Multiplayer is most feared outcome. Whereas sucker's payoff is most feared in prisoners dilemma)
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i	j	1	2	3
defect	defect	4	4	2
coop	coop	1	2	3
defect	coop	4	4	1

5 Game of Chicken

- Consider another type of encounter — the game of chicken:
 - **5 Game of Chicken**
- **Difference to prisoners dilemma:**
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