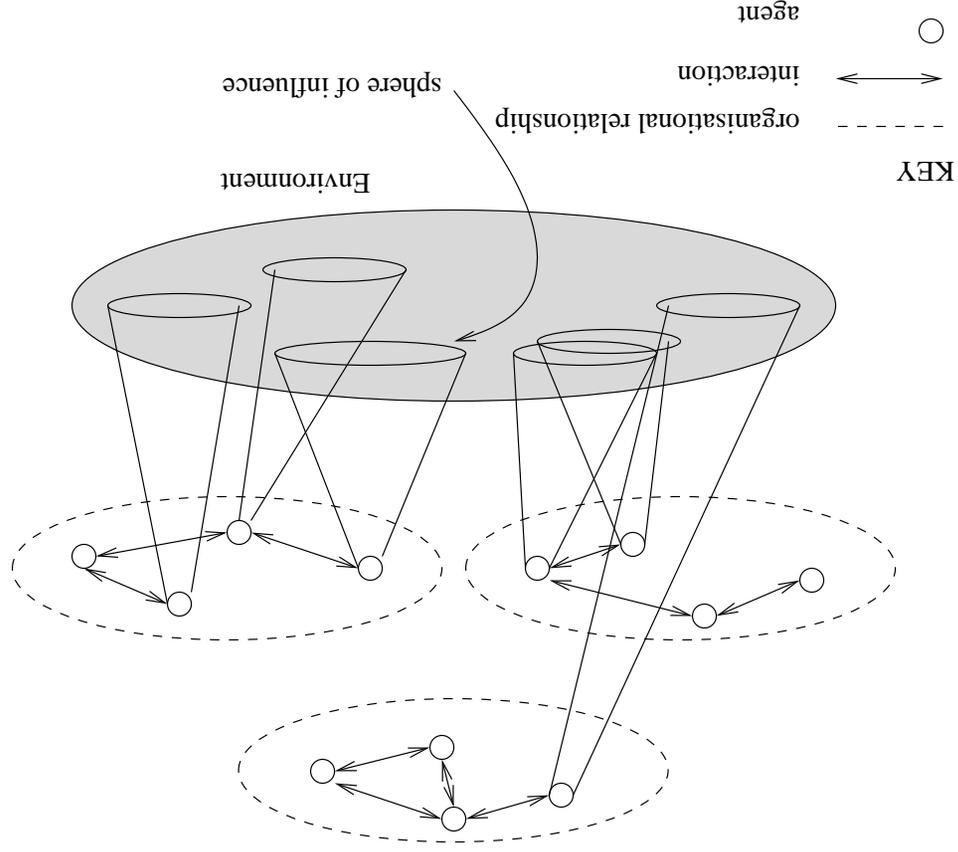


# LECTURE 6: MULTIAGENT INTERACTIONS

An Introduction to Multiagent Systems

<http://www.csc.liv.ac.uk/~mjw/pubs/imas/>

# 1 What are Multiagent Systems?



Thus a multiagent system contains a number of agents . . .

- . . . which interact through communication . . .
- . . . are able to act in an environment . . .
- . . . have different “spheres of influence” (which may coincide) . . .
- . . . will be linked by other (organisational) relationships.

## 2 Utilities and Preferences

- Assume we have just two agents:  $Ag = \{i, j\}$ .

- Agents are assumed to be *self-interested*: they *have preferences over how the environment is*.

- Assume  $\Omega = \{\omega_1, \omega_2, \dots\}$  is the set of “outcomes” that agents have preferences over.

- We capture preferences by *utility functions*:

$$u_i : \Omega \rightarrow \mathbb{R}$$

$$u_j : \Omega \rightarrow \mathbb{R}$$

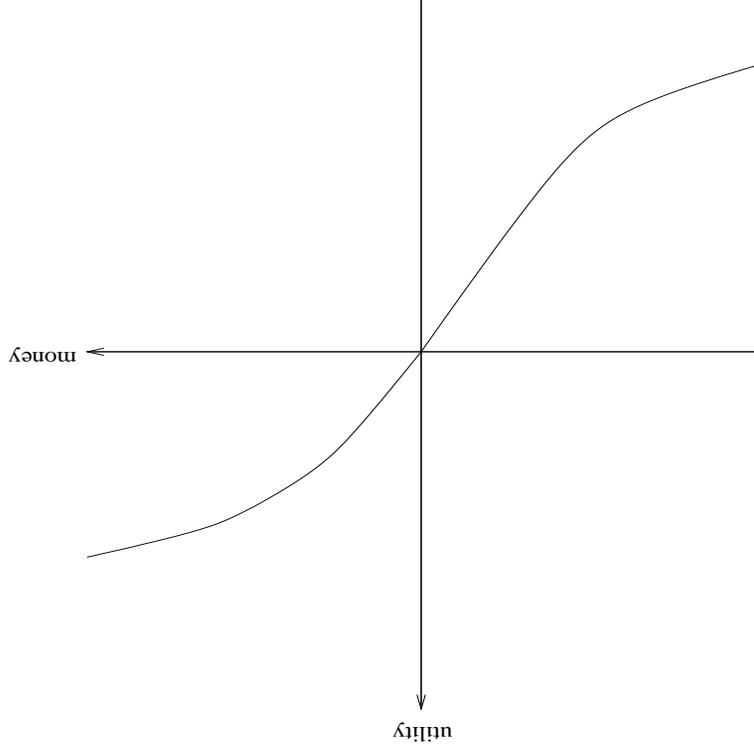
- Utility functions lead to *preference orderings* over outcomes:

$$\omega \succsim_i \omega' \text{ means } u_i(\omega) \geq u_i(\omega')$$

$$\omega \succ_i \omega' \text{ means } u_i(\omega) > u_i(\omega')$$

## What is Utility?

- Utility is *not* money (but it is a useful analogy).
- Typical relationship between utility & money:



### 3 Multiagent Encounters

- We need a model of the environment in which these agents will act. . .

- agents simultaneously choose an action to perform, and as a result of the actions they select, an outcome in  $\Omega$  will result;
- the *actual* outcome depends on the *combination* of actions;
- assume each agent has just two possible actions that it can perform  $C$  (“cooperate”) and “ $D$ ” (“defect”).

- Environment behaviour given by *state transformer function*:

$$T : \widehat{A_C} \times \widehat{A_C} \text{ agent } i\text{'s action} \times \text{agent } j\text{'s action} \rightarrow \Omega$$

- Here is a state transformer function:

$$\tau(D, D) = \omega_1 \quad \tau(D, C) = \omega_2 \quad \tau(C, D) = \omega_3 \quad \tau(C, C) = \omega_4$$

(This environment is sensitive to actions of both agents.)

- Here is another:

$$\tau(D, D) = \omega_1 \quad \tau(D, C) = \omega_1 \quad \tau(C, D) = \omega_1 \quad \tau(C, C) = \omega_1$$

(Neither agent has any influence in this environment.)

- And here is another:

$$\tau(D, D) = \omega_1 \quad \tau(D, C) = \omega_2 \quad \tau(C, D) = \omega_1 \quad \tau(C, C) = \omega_2$$

(This environment is controlled by  $j$ .)

## Rational Action

- Suppose we have the case where *both* agents can influence the outcome, and they have utility functions as follows:

$$\begin{aligned}
 u_i(\omega_1) = 1 & \quad u_i(\omega_2) = 1 & \quad u_i(\omega_3) = 4 & \quad u_i(\omega_4) = 4 \\
 u_j(\omega_1) = 1 & \quad u_j(\omega_2) = 4 & \quad u_j(\omega_3) = 1 & \quad u_j(\omega_4) = 4
 \end{aligned}$$

- With a bit of abuse of notation:

$$\begin{aligned}
 u_i(D, D) = 1 & \quad u_i(D, C) = 1 & \quad u_i(C, D) = 4 & \quad u_i(C, C) = 4 \\
 u_j(D, D) = 1 & \quad u_j(D, C) = 4 & \quad u_j(C, D) = 1 & \quad u_j(C, C) = 4
 \end{aligned}$$

- Then agent *i*'s preferences are:

$$C, C \succeq_i C, D \succ_i D, C \succeq_i D, D$$

- “*C*” is the *rational choice* for *i*.

(Because *i* prefers all outcomes that arise through *C* over all outcomes that arise through *D*.)

## Payoff Matrices

- We can characterise the previous scenario in a *payoff matrix*

	$i$	
defect	defect	1
defect	coop	4
	$j$	
defect	defect	1
defect	coop	4
	coop	4

- Agent  $i$  is the *column player*.

- Agent  $j$  is the *row player*.

## Dominant Strategies

- Given any particular strategy  $s$  (either  $C$  or  $D$ ) agent  $i$ , there will be a number of possible outcomes.
- We say  $s_1$  *dominates*  $s_2$  if every outcome possible by  $i$  playing  $s_1$  is preferred over every outcome possible by  $i$  playing  $s_2$ .
- A rational agent will never play a dominated strategy.
- So in deciding what to do, we can *delete dominated strategies*.
- Unfortunately, there isn't always a unique undominated strategy.

## Nash Equilibrium

- In general, we will say that two strategies  $s_1$  and  $s_2$  are in Nash equilibrium if:
  1. under the assumption that agent  $i$  plays  $s_1$ , agent  $j$  can do no better than play  $s_2$ ; and
  2. under the assumption that agent  $j$  plays  $s_2$ , agent  $i$  can do no better than play  $s_1$ .

- *Neither agent has any incentive to deviate from a Nash equilibrium.*

• Unfortunately:

1. *Not every interaction scenario has a Nash equilibrium.*
2. *Some interaction scenarios have more than one Nash equilibrium.*

## Competitive and Zero-Sum Interactions

- Where preferences of agents are diametrically opposed we have *strictly competitive* scenarios.
- Zero-sum encounters are those where utilities sum to zero:
 
$$u_i(\omega) + u_j(\omega) = 0 \quad \text{for all } \omega \in \Omega.$$
- Zero sum implies strictly competitive.
- Zero sum encounters in real life are very rare . . . but people tend to act in many scenarios as if they were zero sum.

## 4 The Prisoner's Dilemma

Two men are collectively charged with a crime and held in separate cells, with no way of meeting or communicating. They are told that:

- if one confesses and the other does not, the confessor will be freed, and the other will be jailed for three years;
- if both confess, then each will be jailed for two years.

Both prisoners know that if neither confesses, then they will each be jailed for one year.

- Payoff matrix for prisoner's dilemma:

	$j$	
	defect	coop
$i$	defect	2, 2
	coop	4, 4

- Top left: If both defect, then both get punishment for mutual defection.

- Top right: If  $i$  cooperates and  $j$  defects,  $i$  gets sucker's payoff of 1, while  $j$  gets 4.

- Bottom left: If  $j$  cooperates and  $i$  defects,  $j$  gets sucker's payoff of 1, while  $i$  gets 4.

- Bottom right: Reward for mutual cooperation.

- The *individual rational* action is *defect*.  
This guarantees a payoff of no worse than 2, whereas cooperating guarantees a payoff of at most 1.
- So defection is the best response to all possible strategies: both agents defect, and get payoff = 2.
- But *intuition* says this is *not* the best outcome:  
Surely they should both cooperate and each get payoff of 3!

- This apparent paradox is *the fundamental problem of multi-agent interactions*.
- It appears to imply that *cooperation will not occur in societies of self-interested agents*.
- Real world examples:
  - nuclear arms reduction (“why don’t I keep mine...”)
  - free rider systems — public transport;
  - in the UK — television licenses.
- The prisoner’s dilemma is *ubiquitous*.
- Can we recover cooperation?

## Arguments for Recovering Cooperation

- Conclusions that some have drawn from this analysis:
  - the game theory notion of rational action is wrong!
  - somehow the dilemma is being formulated wrongly
- Arguments to recover cooperation:
  - We are not all machiavelli!
  - The other prisoner is my twin!
  - The shadow of the future. . .

## 4.1 The Iterated Prisoner's Dilemma

- One answer: *play the game more than once.*  
If you know you will be meeting your opponent again, then the incentive to defect appears to evaporate.
- *Cooperation is the rational choice in the infinitely repeated prisoner's dilemma.*  
(Hurrah!)

## 4.2 Backwards Induction

- But... suppose you both know that you will play the game exactly  $n$  times.
  - On round  $n - 1$ , you have an incentive to defect, to gain that extra bit of payoff...
  - But this makes round  $n - 2$  the last "real", and so you have an incentive to defect there, too.
- This is the *backwards induction* problem.
  - Playing the prisoner's dilemma with a fixed, finite, pre-determined, commonly known number of rounds, defection is the best strategy.

### 4.3 Axelrod's Tournament

- Suppose you play iterated prisoner's dilemma against a *range* of opponents . . .  
What strategy should you choose, so as to maximise your overall payoff?
- Axelrod (1984) investigated this problem, with a computer tournament for programs playing the prisoner's dilemma.

## Strategies in Axelrod's Tournament

- ALLD:
  - “Always defect” — the *hawk* strategy;
- TIT-FOR-TAT:
  1. On round  $n = 0$ , cooperate.
  2. On round  $n > 0$ , do what your opponent did on round  $n - 1$ .
- TESTER:
 

On 1st round, defect. If the opponent retaliated, then play TIT-FOR-TAT. Otherwise intersperse cooperation & defection.
- JOSS:
 

As TIT-FOR-TAT, except periodically defect.

## Recipes for Success in Axelrod's Tournament

Axelrod suggests the following rules for succeeding in his tournament:

- *Don't be envious:*  
Don't play as if it were zero sum!
- *Be nice:*  
Start by cooperating, and reciprocate cooperation.
- *Retaliate appropriately:*  
Always punish defection immediately, but use "measured" force — don't overdo it.
- *Don't hold grudges:*  
Always reciprocate cooperation immediately.

## 5 Game of Chicken

- Consider another type of encounter — the *game of chicken*:

	$i$	
	defect	coop
$j$	defect	coop
	1	4
	1	3
	2	3

- (Think of James Dean in *Rebel without a Cause*: swerving = coop, driving straight = defect.)
- Difference to prisoner's dilemma:

*Mutual defection is most feared outcome.*

- Whereas sucker's payoff is most feared in prisoner's dilemma.)
- Strategies (c,d) and (d,c) are in Nash equilibrium

## 6 Other Symmetric 2 x 2 Games

- Given the 4 possible outcomes of (symmetric) cooperate/defect games, there are 24 possible orderings on outcomes.

–  $CC \succ CD \succ DC \succ DD$

*Cooperation dominates.*

–  $DC \succ DD \succ CC \succ CD$

*Deadlock.* You will always do best by defecting.

–  $DC \succ CC \succ DD \succ CD$

*Prisoner's dilemma.*

–  $DC \succ CC \succ CD \succ DD$

*Chicken.*

–  $CC \succ DC \succ DD \succ CD$

*Stag hunt.*