SOME AUCTION THEORY

What is auction theory?

- Concerned with how you analyse an auction.
- Since auctions are multi-party events, you have to:
  - Reason about what others will do.
  - Make assumptions about how others will act.
- Thus, need to model participants.
- Usual assumption is that participants are rational in some sense.
  - Seek to maximise their own benefit.
  - Conditional on risk attitude and information.
- Also need some notion of what a good auction is.

Results from our English auctions

- We ran two English auctions.
- As for all these experiments, the distribution of private values was:
  25, 50, 75, 100, 125, 150, 175, 200, 225, 250.
- In the first auction, thanks to some confusion about the experimental setup, the sale price was 170 (with a profit of 5).
- In the second, the sale price was 235 (a profit of 15), and the two previous high bids were 225 and 230.
- This second outcome is in pretty good agreement with the theory.

Why does this agree?

- What we aim for are auctions which *maximise social value*.
- This is code for (in this single item, sell-side auction):

  The item is sold to the buyer who values it most.
- Given assumptions about the buyers, we can also predict sale price.
- In a setup in which one buyer has a higher valuation than any other buyer, we'd expect the sale price to be no more than that valuation, and no less than the second highest valuation.

Results from our Dutch auctions

- We ran two Dutch auctions with the same distribution of values as before.
- The first item sold for 250 (a profit of 0).
- The second item sold for 230 (a profit of 20).
- Again, this is pretty close to what theory suggests will happen.
- In fact both results basically agree with the theory under different assumptions.
  - In one case the item sold for exactly what the bidder with the highest valuation valued it at.
  - In the other case the item sold for the price of the second highest valuation plus the minimum increment (the price was dropping in 10 cent intervals).

Results from our FPSB auctions

- The results in terms of trade price agree with the theory.
- Sales were for 235 and 240 (a tie between two bidders).
- The first was at a profit of 15, and 10 cents above the next highest valuation.
- Again this agrees with the theory.
- One of the second auction winners (we had many items for sale so we rapidly turned this single unit auction into a two unit auction) was at a profit of 10, and again agrees with the theory.
- The second winner was at a loss of 15.
- The winning bidder was playing more strategically than the theory allows for, factoring in her profit from the previous auction.

Results from the SPSB auctions

- Here the results deviated from what simple theory would predict.
- The first auction was won with a bid of 700, though the price paid was only 255 (still a loss of 80).
- However a number of participants overbid, sometimes by quite a margin.
  - "overbid" means bid more that their value for the item
- In the second auction the winning bid was 400, and the price was 300.

Optimality of the auctions

- Overall the auctions, then, we can measure the ability of the mechanisms to ensure the good goes to the person who values it most.
- Does it maximise *social welfare*.
- This happened in 5 of the 8 cases.
- Another way to look at this:
  - it happened in all the cases other than the misunderstood first English auction and the overly-strategised Vickrey auctions.
  - not so bad.
- Can also be more precise and talk about *efficiency*.

- In this case we consider how close the actual revenue (sum of sale prices) is to what the revenue would be if the high valuation was paid every time.
- Think of this as how good it is for the seller
  - The seller wants to get as mch as possible.
  - That is the highest valuation in each auction.
- Over all auctions, we got 96% of the expected revenue.
- This was distorted by the windfall profits in the Vickrey auctions — without those we got 91% of the expected revenue.
- This includes the losses due to the first auction we ran when everyone was learning exactly how the experiment was running.
- Excluding this first auction, the numbers are 99.7% (with Vickrey) and 95% (without).

- If nothing else, this shows that auctions are a pretty robust form of mechanism.
- Without any prior training, you managed to extract more than 90% of the theoretical profits/revenue.
- And that was with valuations that were easily learnt, so that the high valuation holder could easily shade their bid, knowing that any bid above 225 should win.
  - Would win, unless someone made a loss.

Okay, some auction theory

- To make predictions (rather than analyze results) we need a model of bidder behavior.
- As mentioned before, we assume that bidders are *rational*, that is they want to maximise their profit.
  - This is the game theory notion of rationality.
- We also assume that bidders are *risk neutral*.
  - By that we mean that given a value for the good, the bidders will pay exactly that value.
  - They won't risk over-paying (which doesn't really make sense in this framework).
  - They won't risk underpaying and not getting the good (risk seeking).

- Just to be clear, we are only considering:
  - single item,
  - single unit,
  - sell-side

  auctions, just like the four standard ones.
- Of course, auction theory is broader than this, but an awfuloot of work has just considered the four standard auctions.
- We will consider what the outcome of the mechanisms are in terms of:
  - Maximising social welfare; and
  - Maximising revenue/profit.

Independent private values model

- Another part of the model is what we assume about how the bidders value goods.
- The simplest is the independent private values model.
- In the context of a single unit sell-side auction it says that:
  - All bidders have a private values for the good.
  - Private values are drawn independently from a distribution that is common knowledge.
  - Each bidder knows what their private value is.
- One can dispute the validity of these assumptions.

- What happens in a Dutch auction with IPV bidders?
  - Each will bid when the price drops to their value.
  - Good will sell for the highest valuation.
- This will *maximise social welfare* and be *efficient*.
- It is also the right strategy for IPV bidders to use — bid less than their value, and they might lose the good.
- What happens in an FPSB auction with IPV bidders?
  - Each will offer their PV.
  - Good will sell for the highest valuation.
- Results are *equivalent*.

- What happens in an English auction?
  - Each bidder will drop out when the price exceeds their private value.
  - Good will sell for just more than the second highest value.
- What happens in the Vickrey auction?
  - Each bidder will bid their value.
  - The highest value will win, and pay the second highest value.
- Thus the English and Vickrey auctions are more or less equivalent.

- In both cases, the bidder with the highest valuation wins.
  - This will maximise social welfare.
- However, since the bidder with the highest valuation only pays the second highest valuation, neither auction form will be completely efficient.
  - By whatever margin the second highest value is below the highest value.
- Note that when there is not much between the highest and second highest values, all these auction forms give almost exactly the same profit for the seller.

Common values model

- In the common values model, we have the same first pair of assumptions as in the IPV model
  - Bidders have private values.
  - These values are independently assigned from a known distribution.
- However, we relax the assumption that these are known to the bidders.
- Why would bidders be ignorant about their value for a good?

- Consider a typical "common values" scenario.
- You are bidding for the right to supply coffee to the graduate center for the next year.
- Clearly there is an actual value to this contract:
  - How much you can persuade students and faculty at the GC to pay you over that period.
- Similarly, you can determine how much it will cost you, per cup, to deliver the coffee.
  - Calculate from cost of coffee, wages, equipment and so on.
- Everybody can estimate these, but there is uncertainty in both calculations.
- Everbody just has an estimate of the common value.

- Some bidders will underestimate the real value, some will overestimate the real value.
- Consider a Dutch auction.
- Since bidders are risk neutral, each will be prepared to call out its value.
- The highest value wins.
- If we think of the estimation process generating a Gaussian error around the actual value, the winner will likely be a sample from the upper tail.
- The *winner's curse*.

- Actually, the "error" distribution doesn't have to be Gaussian.
- Whatever the distribution that generates private values, it is likely that the winner will be an overestimate.
- If you know the winner's curse exists, perhaps it is rational to factor that in.
  - Reduce your estimate of the value.
  - Don't bid up to your value.
  - *Shade* your bid.

- Think back to our Dutch auction example above.
  - If the high value holder shades her bid, then the auction may make less money.
  - If everyone shades their bid, then the auction *will* make less money.
- Might consider bid shading as not being risk neutral.
- It certainly opens the door to strategic thinking.

Correlated values

- In many scenarios, the IPV assumption that values are drawn independently from a common distribution is not valid.
- It is arguable whether it is valid in the coffee example.
  - Say the GC posts information about how many cups of coffee were sold last year.
  - Now any estimates that take account of this are not statistically independent.
- The *correlated values* model allows for bidders valuations to be positively correlated.
- In practice, this means that as bidders reveal increasing values, the values of all bidders are dragged upwards.

- What are the results of using the model?
- Dutch and FPSB auctions are still equivalent.
  - There is no information revelation to push prices up.
- English and Vickrey auctions are no longer equivalent.
  - English auction now has higher prices.
- Well, they are equivalent when there are only two bidders, but when there are more, the additional bidders in the English auction help to push the price up.
- Also, Vickrey auctions generate more revenue than FPSB auctions.

- Not clear how much this model makes good predictions.
- Dutch auctions lead to lower prices than first-price sealed bid
  - Maybe because people do not analyse the auction and determine strategy.
- Also claims that the "affiliation effect" is not significant.

Almost common values

- The scenario is the same as for the common values model.
  - Bidding for a good that has some value that nobody knows.
  - Bidders have different estimates.
- Different estimates introduce asymmetries between bidders.
- Bidder with higher value bids more aggressively.
  - Tends to force out opposition.
  - Therefore a lower winner's curse.

- LA licence in one of the spectrum auctions.
- Worth a bit more to PacTel since they already had customer contacts, brand name recognition etc..
  - Already operating in the area.
- Won, and with a price that seemed low by national standards.

- Appearance of aggression can be a good thing.
- Glaxo bidding for Welcome, £9 billion:

  "[will] almost certainly top a rival bid"
- Apparently frightened off bids of £10 billion and £11 billion from other companies.

Revenue equivalence

Theorem Assume each of a given number of risk-neutral potential buyers has a privately known valuation independently drawn from a strictly increasing atom-less distribution, and that no buyer wants more than one of the k identical indivisible objects. Then any mechanism in which (i) the objects always go to the k buyers with the highest valuations and (ii) any bidder with the lowest feasible valuation expects zero surplus, yields the same expected revenue (and results in each bidder making the same expected payment as a function of her valuation).

- Key aspect here is that values are independent.
  - Either values are independent or the signals from which the values are computed.
- Generalises to the common values model.
- Won't generalise to the correlated values model.
- Of course, the result only holds for idealised bidders — there is no guarantee that, in practice, bidders will behave in such a way that revenue equivalence holds.
- We have already discussed some reasons why real bidders won't match up to the idealisation, and so revenue equivalence won't hold.

Other complicating factors

- Here are some other complications.
- Are bidders symmetric?
  - Are they all operating under the same circumstances or are some privileged (or disadvantaged).

  If not, auctions may allocate goods in a way that doesn't maximise social welfare.
- Are payments a function of bids alone?
  - Does money flow into, or out of, the auction?
  - Royalties

  Both these cases will mean that revenue equivalence might not hold.

Bidder strategy

- Revenue equivalence says that in some senses there is no difference between auctions:
  - Profit made
  - Who wins
- Doesn't mean they are the same.
- How should bidders behave in the different auctions to maximise profit?
  - Strategic thinking
- English: Bid up to private value.
- Vickrey: Bid private value.
- These are *dominant* strategies.
  - Hold no matter what the other bidders do

- No dominant strategy for Dutch and FPSB auctions
  - Nash equilibrium strategy only.
- Not easy to state (or indeed compute in the general case) what this strategy is.
  - Depends on what other bidders valuations are.
  - Depends on beliefs about what other bidders valuations are.
- This might affect the choice of auction on the part of the seller.
  - If bidders dont understand the mechanism they may deviate from equilibrium and the revenue may change.

Comparison between auction types

- Revenue earned gives us a mechanism to choose between auctions.
  - Whose perspective are we taking here?
- Social welfare and efficiency are others.
- Transparency (though hard to quantify)
- Perhaps it is worth giving up some properties (like efficiency) for less obvious benefits like *incentive compatibility*
  - Removing gaming is a bigger benefit.
- Mechanism design takes us in this direction.

Summary

- We have looked at some basic auction theory, considering the following models of bidder behavior
  - Independent private values model
  - Common values model
  - Correlated values model
  - Almost common values model
- We also looked at the revenue equivalence and discussed the winner's curse.
- Looking deeper into this requires a prepraratory look at game theory.