DOUBLE AUCTIONS

Basic terminology

- We deal with:
  - interactions between agents, *traders*; and
  - commodities that they own.
- We deal with two commodities:
  - *good*;
  - *money*.
- Goods are indivisible, while money is divisible.
- The interactions between the agents lead to *exchange*.

- In an exchange, traders *freely* alter the allocation of commodities.
  - They may choose to swap a good for some quantity of money
  - (Or some quantity of money for a good).
  - The amount they choose is related to the value they place on the good.
- This value is called the:
  - *private value*;
  - *limit price*.
- If a buyer pays more than its private value, it is trading at a loss.
- If a seller sells for less than its private value it is trading at a loss.

- The *market institution* defines how this exchange takes place.
- The institution lays down rules about what the traders can do:
  - What messages can be exchanged
- The institution lays down rules for determining the final allocation given the actions of traders:
  - How the messages turn into trades.
- The change of allocation is the market *clearing*

- The clearing changes the allocation of goods.
- The difference between the initial allocation and the final allocation is the *net trade*.
- The net trade states for each trader which shows the change in money and good.
- The minimum net trade is when two traders have non-zero components.
- Any trader with a non-zero component has an associated:
  - *trade price*
  - *transaction price*
- This price is the absolute value of the money component divided by the good component.

- Traders with a positive good component:
  - Buyers.
- Traders with a negative good component:
  - Sellers.
- Typically we will deal with *one way traders* which are wither buyers or sellers (not both).
  - Though there are some situations in which it is interesting to consider traders who can buy or sell as they see fit.

- An *auction* as a market institution in which messages from traders include some price information.
- An offer to buy at some price:
  - Bid
- An offer to sell at some price:
  - Ask
- An auction also gives priority to higher bids and lower asks.
- A *quasi-auction* has most of the features of an auction, but not all.
  - posted offer institution.

Double auctions

- A *two-sided* auction has both buyers and sellers making offers.
  - *double* auction.
- A double auction can be *one-shot* or it can be *repeated*.
- A repeated auction has two or more *trading periods*.
- Under "conditions of normal supply and demand", traders receive a new allocation of commodities at the end of every period.
  - Simulates ongoing markets.

- In a *periodic* or *discrete time* double auction, clearing happens at a certain time.
- Clearing might happen several times in each trading period, or may happen just once at the end of the trading period.
- The classic example of a periodic double auction is the *call market* or *clearing house*
- This was used to determine the opening prices on the NYSE.
- Traders place bids and asks until the clearing.
- The supply and demand curves can then be established, and from these the clearing price is determined to balance supply and demand.

- In a *continuous* double auction, clearing can happen at any time.
  - As soon as there is a bid that exceeds an ask, the corresponding traders can make a deal.
  - The precise rules of the institution determine whether traders *must* make a deal, or whether they just have the option.
- The transaction price is local to those two traders.
  - *discriminatory pricing* rather than *uniform pricing*.
- This is the model for markets like the NYSE during daily trading.

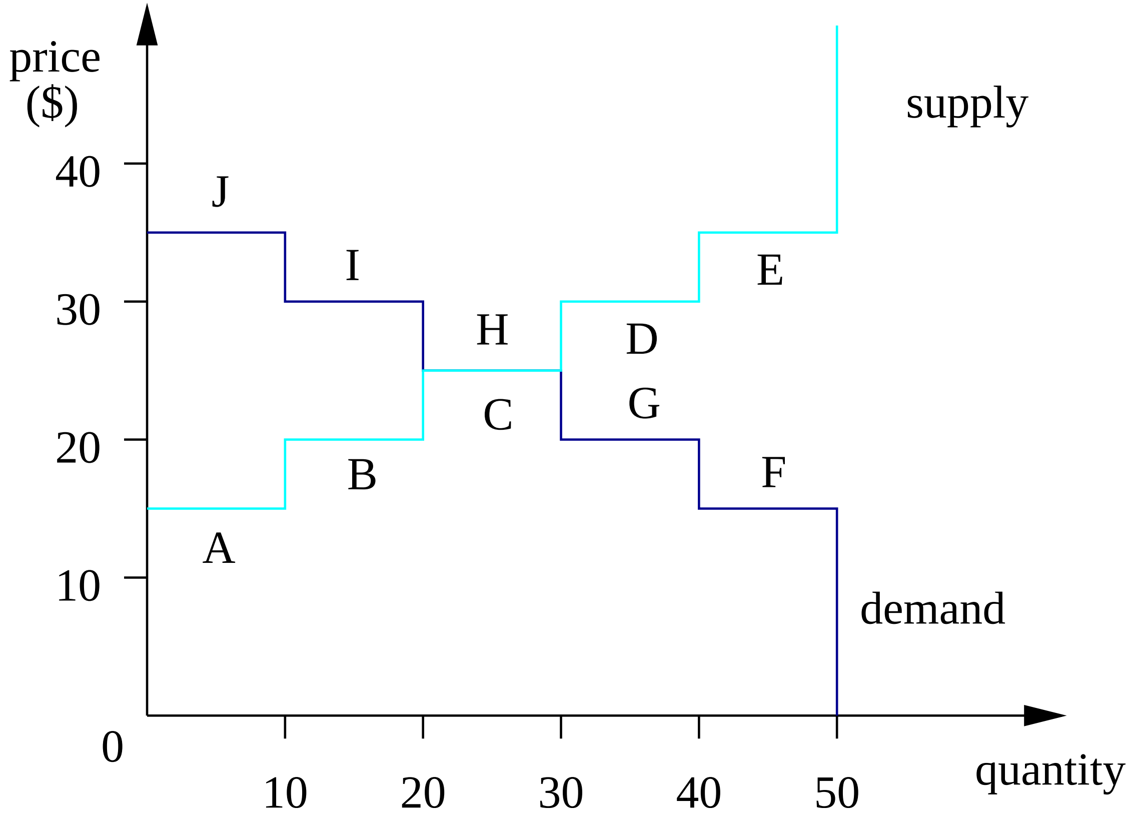
* As it was:

- As it is:

- Where the real action is:

Some theory

- If we know the private values of the traders, we can construct the supply and demand curves for the market.

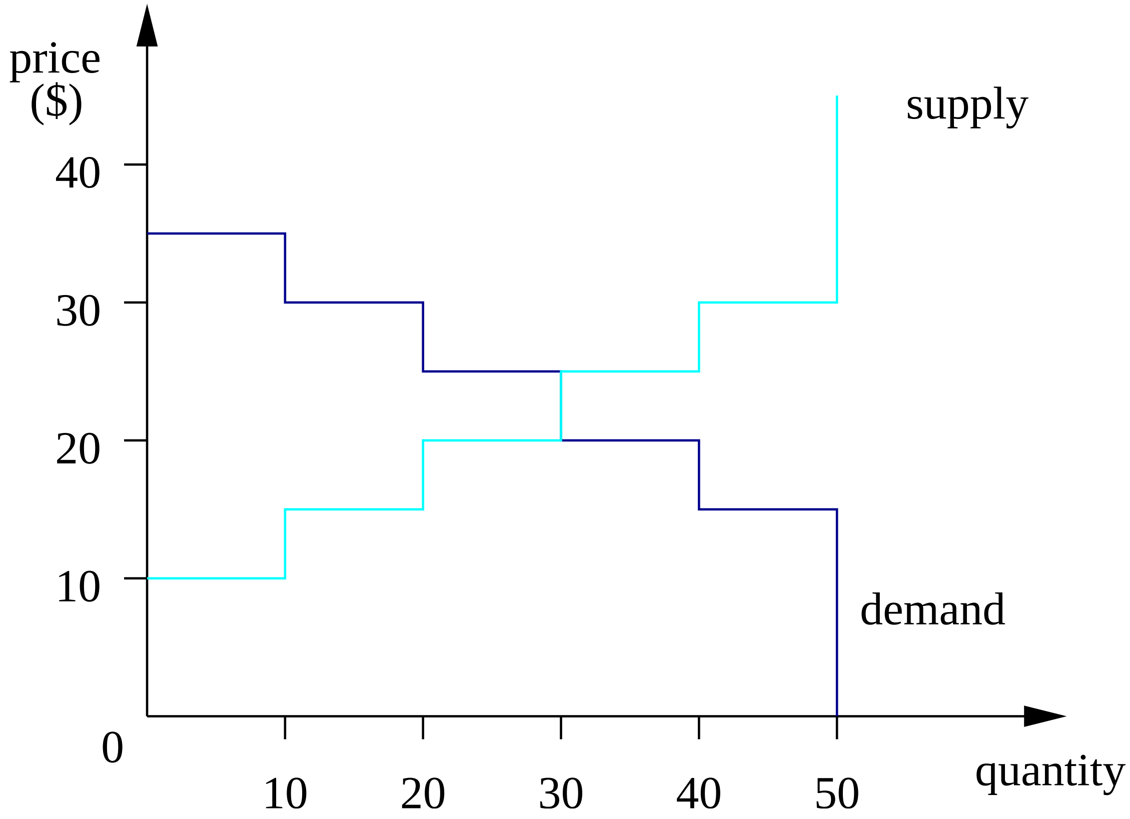


- These curves are based on:
  - traders A, B, C, D and E all looking to supply 10 goods with limit prices of \$15, \$20, \$25, \$30, and \$35.
  - F, G, H , I, and J looking to buy with limit prices of \$15, \$20, \$25, \$30, and \$35 respectively.
- If the trade price is set below \$15, no seller will be prepared to trade.
- If the price is between \$15 and \$20 then only A will trade.
- At a price above \$35 no buyer will trade, while at prices between \$30 and \$35 only J will trade.
- Though we won't, in general, know what these values are, the concept of the curves is a useful one.

- Once we have these curves, we can figure out the *equilibrium price*
  - p_0
- This is the price at which supply equals demand.
  - The curves cross.
- In our case this would be a price of $25.
- If one were to know the private values this is where one would set the transaction price for the traders.

The *k*-double auction

- The transaction price needn't be unique.



- Any price along the intersection is acceptable to all the agents.
- To set the trade-price between the lowest matched bid b and highest matched ask a:

$$kb + (1 - k)a$$

- Where $k \in [0, 1]$.
- If $k = 1$ we have the *buyer's bid* auction.
- $k = 0.5$ is commonly used.

Profit

- If the ith transaction matches buyer b_i with private value v_{b_i} and seller s_i with private value v_{s_i}, setting a price p_i, then:

$$pr_{b_i} = v_{b_i} - p_i$$

  is the profit that the buyer makes from the trade.

- Similarly,

$$pr_{s_i} = p_i - v_{s_i}$$

  is the profit that the seller makes.

- If a trader does not take part in a transaction, then its profit is 0.

Allocative efficiency

- The profit made by all the traders is

$$S_a = \sum_{i=1}^{n} v_{b_i} - p_i + \sum_{j=1}^{n} p_j - v_{s_j}$$

- We compare this against the profit that they would have made if all transactions were to have taken place at the equilibrium price

$$S_t = \sum_{i=1}^{n} v_{b_i} - p_0 + \sum_{j=1}^{n} p_0 - v_{s_j}$$

- Allocative efficiency is then:

$$e = \frac{S_a}{S_t}$$

expressed either as a fraction or as a percentage.

Market power

- How much have buyers or sellers managed to tip things in their favor?
- Buyer market power is calculated as:

$$mp_b = \frac{1}{n}\sum_{i=1}^{n}\frac{pr_{b_i} - epr_b}{epr_b}$$

  where i ranges across all n goods sold, pr_{b_i} is as before, and epr_b is the equilibrium profit of the buyers.
- The larger buyer market power, the further that buyers have managed to move the transaction price below the equilibrium price.

Coefficient of convergence

- "Smith's alpha"
- How much the transaction price deviates from the equilibrium price

$$\alpha = \frac{100\sigma_0}{p_0}$$

  where p_0 is the equilibrium price and σ_0 is the standard deviation of trade prices around the equilibrium.

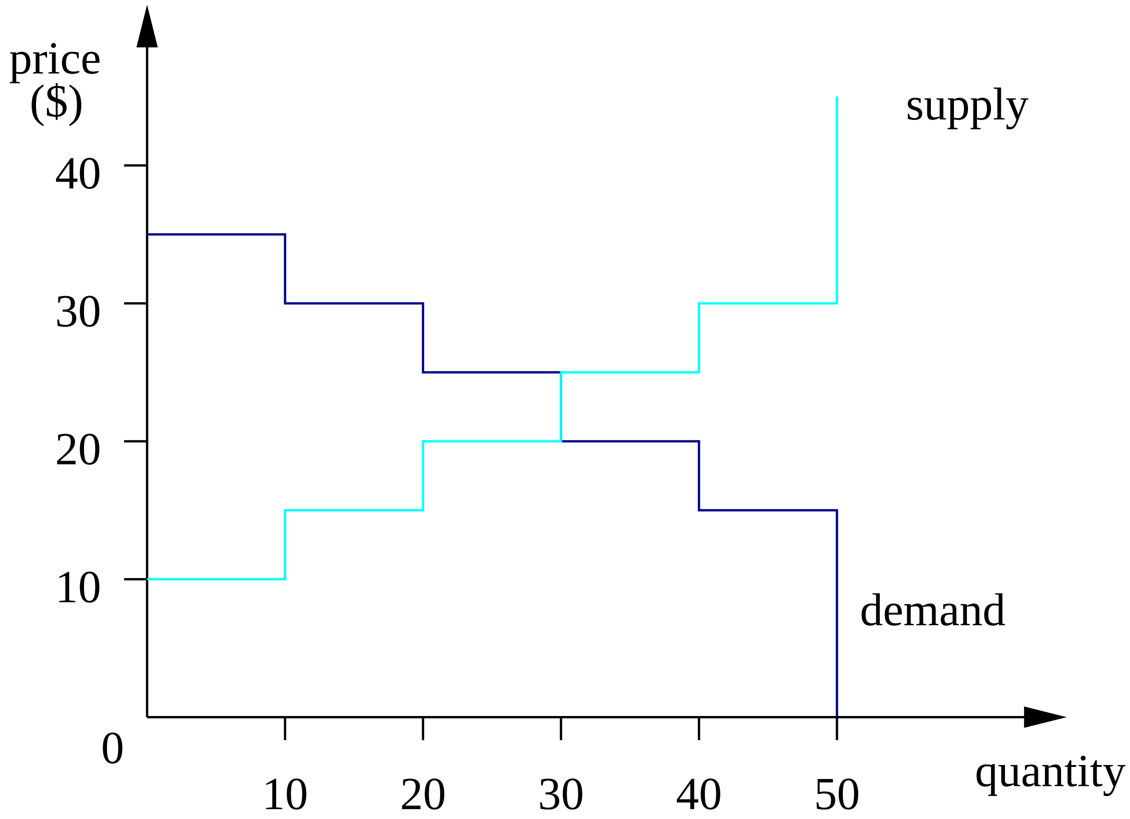
- Alternatively this is written:

$$\alpha = \frac{100}{p_0}\sqrt{\frac{1}{n}\sum_{i=1}^{n}(p_i - p_0)^2}$$

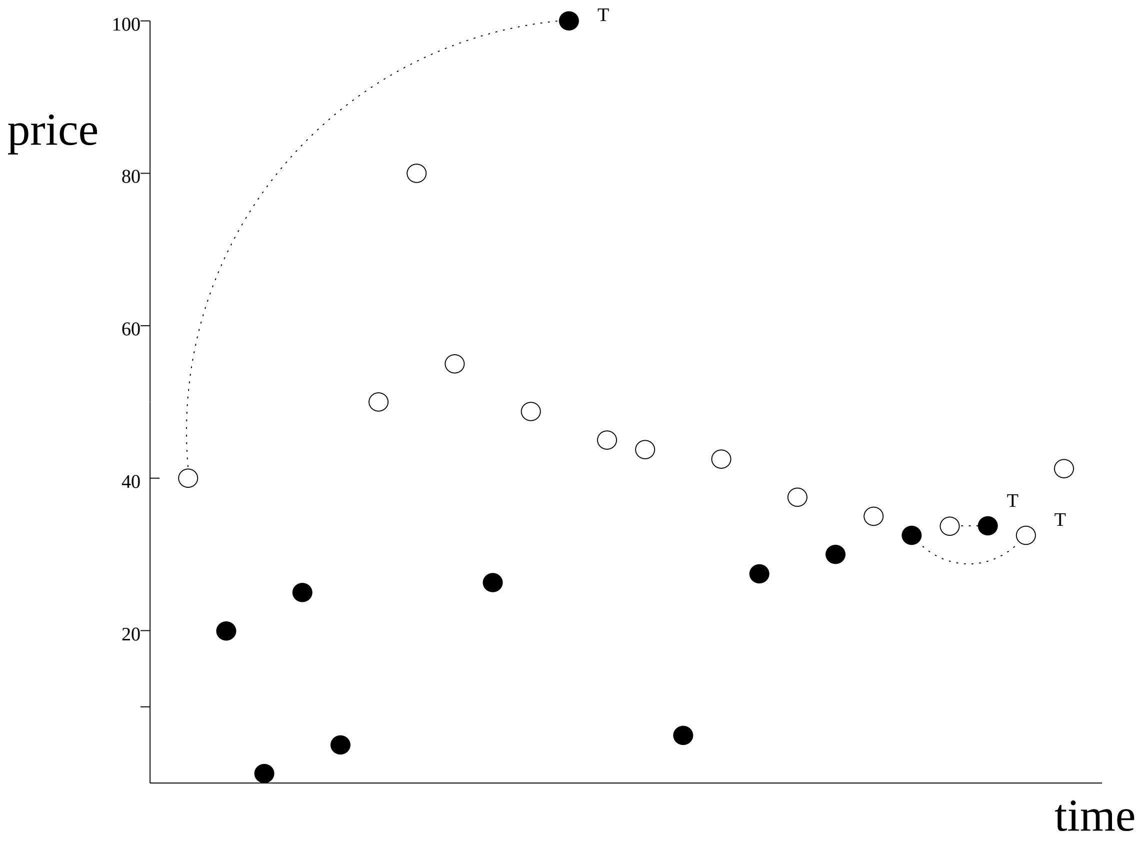
  where p_i is the trade price of transaction i.

Less than perfect equilibrium

- In a clearing house, deviation from equilibrium arises because of *strategic bidding*.
- In other words, traders falsely report their values to make a profit.
- (This despite the fact that the expected profits rapidly decrease as the number of traders increases).
- In a continuous double auction, additional distortions arise.
  - Order bids are made.
  - Extra-marginal traders.


price
($)
40
30
20
10
0
10
20
30
40
50
quantity
supply
demand

- A sample set of bids from a continuous double auction.



Program traders/trading agents

- With more and more electronic trading, there is a lot of interest in *program traders*
- In computer science, these tend to be called *trading agents*.
- Two basic areas of study:
  - Efficient trading/price formation
  - Effects of speculation
- We, like most of the computer scientists who do this work, will focus on the former.

Zero-intelligence traders

- Gode and Sunder proposed "no intelligence necessary" for efficient trade.
- Simple program traders which pick prices at random.
  - ZI-U
  - ZI-C
- For "sensible" supply and demand, surprisingly efficient (96%).
  - α tends to be high.
- ZI-U can be terribly inefficient given arbitrary private values.
- ZI-C makes a good benchmark.

Zero-intelligence plus

- ZI-C may be a decent benchmark, but nobody pretends it is a very sensible strategy.
  - Doesn't systematically look for a profit.
- Cliff proposed a "minimum intelligence" extension, ZIP.
- For a range of sensible scenarios ZIP has high efficiency and converges well to the equilibrium price.
  - $e \approx 1$
  - $\alpha < 5$

- For buyers.

```
if the last bid or ask was accepted at price P
then
    if V_B ≥ P then increase profit margin
    if the last offer was an ask
    then
        if V_B ≤ P then decrease profit margin
else
    if the last offer was a bid
    then
        if V_B ≤ P then decrease profit margin
```

- Sellers behave symmetrically.

Truth telling

- Another useful benchmark.
- Just report your private value.
- Homeogeneous TT traders are 100% efficient in a clearing house.
- Much less efficient in a CDA
  - Depends on the private values.
  - $e \approx 0.8$.
- Nice measure of the possible gains from strategic play.

Kaplan

- This is a *sniping* strategy.
  - Invented for the Sante Fe Double Auction Tournament
- Waits until there is an ask that provides it with a nice profit.
  - Then snaps it up.
- If no juicy ask comes along and the auction is due to end, then bid at a slight profit.

Roth-Erev

- Comes from an attempt to model the way that people play games.
  - Modified to work in an auction setting.
- Reinforcement learning.
- Remember the result of bids/ask at different values.
  - Compute probability of a given bid winning.
- Estimate expected utility of bids.

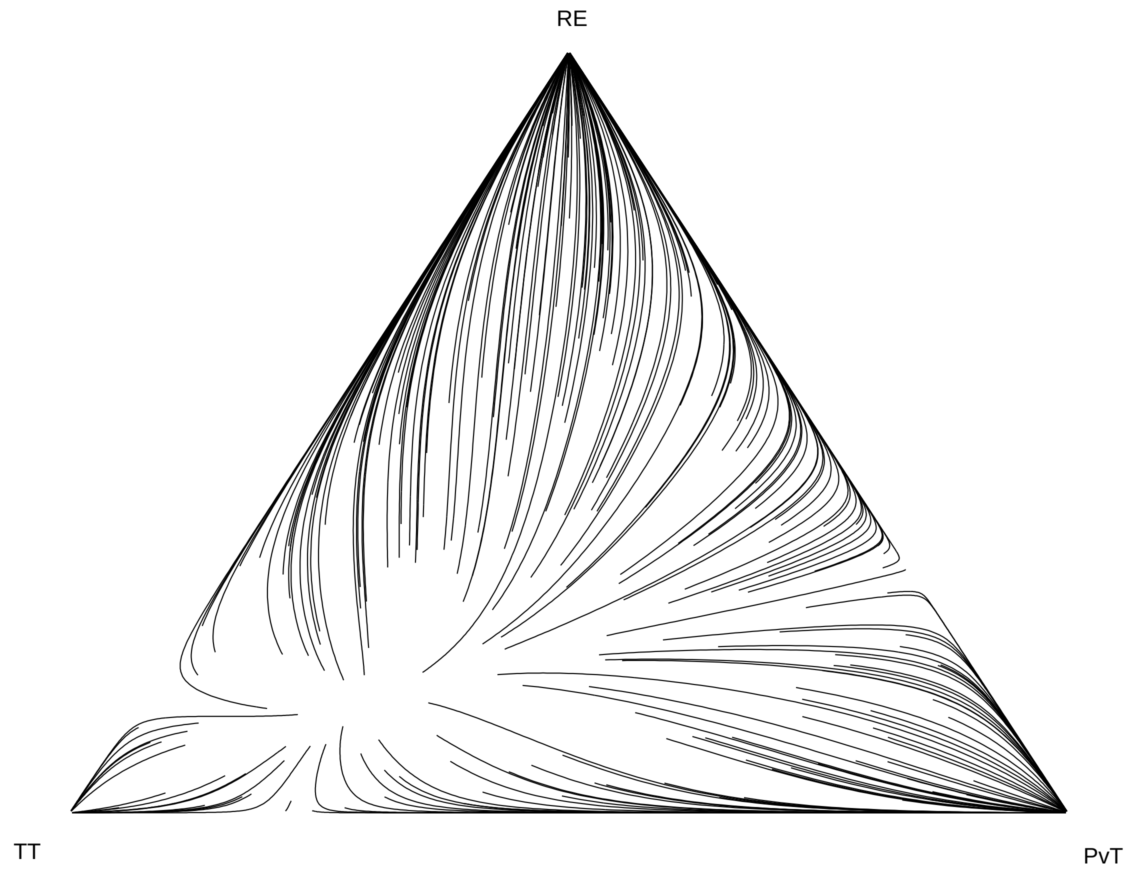
Gjerstad-Dickhaut

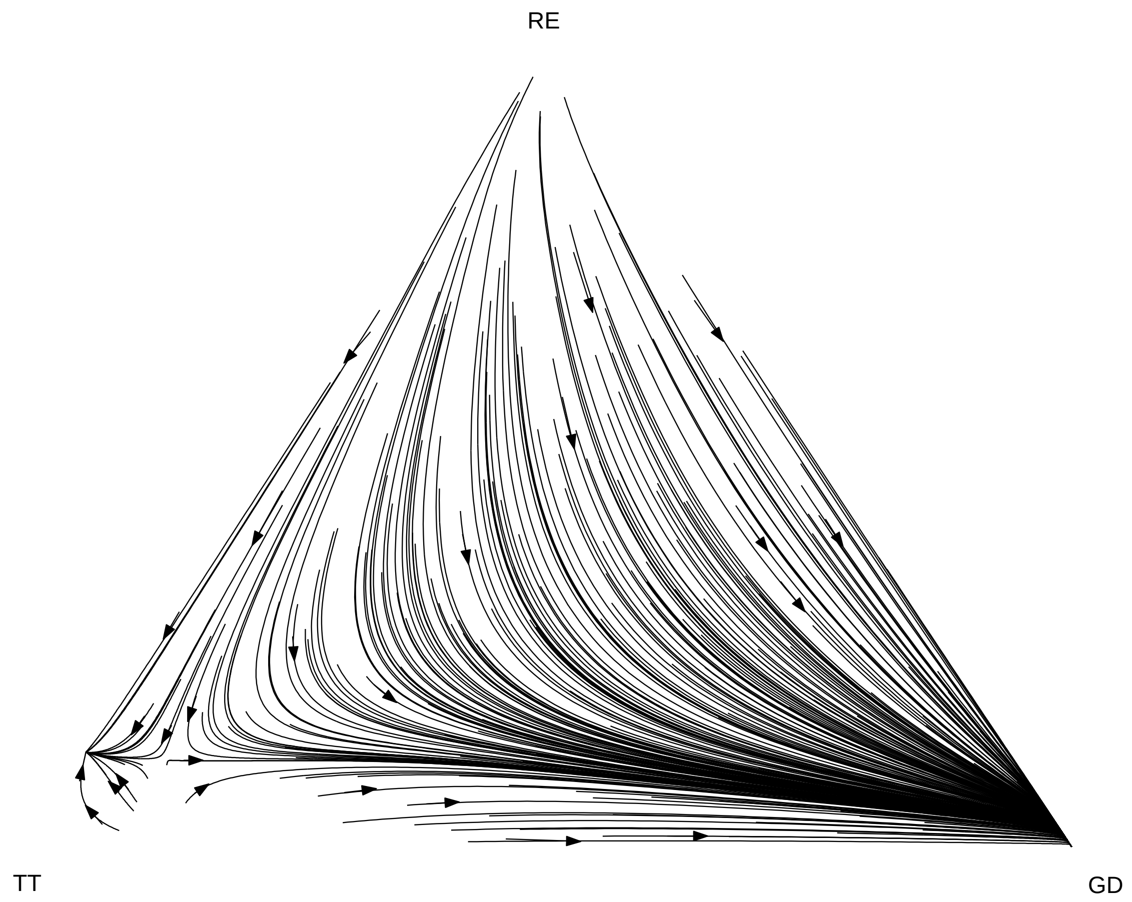
- Joel will talk about this at a later date.
- Tries to estimate the probability of a bid being accepted based on what *everyone* has bid before.
- Very high efficiency.

To note

- Different bidding strategies have rather different informational requirements.
  - Market quotes
  - Bids and asks
  - What bids/asks were accepted
- No uniform scenario for evaluating trading strategies.

Profitability of strategies

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Summary

- Introduced the basic structure of the double auction.
- Discussed commonly used measures of auction performance.
- Described some basic trading strategies.