COMPUTATIONAL SOCIAL CHOICE

This presentation

- A very brief introduction to some of the topics in computational social choice theory.
- Leans heavily on:

  Y. Chaevaleyre, U. Endriss, J. Lang and N. Maudet, “A short introduction to computational social choice”

- For more details and a lengthy bibliography, see that paper.

What is it?

- Intersection of computer science and *social choice theory*.
- Social choice theory deals with the design and analysis of methods for collective decision making.
  - A classic example is the design of voting systems.
- Social choice theory has typically dealt with the existence of procedures with certain properties.
- *Computational* social choice is more concerned with computing those properties.

For example

- Consider a voting protocol.
- We would like the protocol to be such that no voter can manipulate the result of the vote.
- Social choice theory would rule out a mechanism if there is any way that the result can be manipulated.
- However, if the procedure for rigging the vote is computationally intractable, then we might decide that we will risk the manipulation.

Another example

- Consider a procedure for dividing a resource between agents.
- If the procedure is fair, then social choice theory would say it is a good procedure.
- If it is intractable, then we might not choose to use it.

Preference aggregation

- We want to combine a set of preferences:

$$P = \langle P_1, \ldots, P_n \rangle$$

  into a collective preference relation P^*.
- Requires circumventing Arrow's impossibility theorem (see on).

Voting theory

- Voting as a means of reaching a decision.
- Many rules proposed, but the computational properties are not typically studied.
- A *positional scoring rule* computes a score based on the individual profiles and selects the candidate with the maximum score.
- An example is the *plurality rule*.
  - score of 1 to each most preferred candidate, 0 otherwise.
- Another is the *Borda rule*.
  - score of m to the most preferred candidate $m-1$ to the next most preferred, down to 1 to the least preferred.

- A candidate preferred to any other by a strict majority of votes is a *Condorcet winner*.
- a Condorcet winner is unique, but not guaranteed to exist.
- A voting rule that will elect any existing Condorcet winner is called *Condorcet-consistent*.

Resource allocation/Fair division

- Assignment of resources from a finite set R to a set of agents N.
- By a central authority to whom agents give their preferences.
- In a decentralized way.
  - Through an exchange.
- Rate allocations by Pareto efficiency.
- Perhaps better is to rate by *envy-freeness*
  - An allocation is envy-free if no agent prefers the bundle held by another agent.

- Also consider the following.
- *Utilitarian social welfare*
  - The sum of the utilities experienced by the members of society.
  - Close to the economic notion of efficiency.
- *Egalitarian social welfare*
  - The utility of the agent with the lowest utility.
  - Maximising this can be considered to be a way of increasing the fairness of the allocation.
- These measures are often not compatible — there may well be no allocation that is Pareto efficient and envy-free.

Coalition formation

- A coalition is a set of agents that together can obtain higher rewards by working together than working alone.
- Two key problems.
- What coalitions will form for a particular problem, and how will the agents get together into coalitions?
- How should the benefits of the coalition be divided between the members?
- A key notion is that of *stability*.
- A coalition is stable if no agent has an incentive to leave the coalition.

Judgement aggregation

- How do you merge a set of opinions?
- In this case opinions are a set of related propositions.
- Models the case of:
  - Committees
  - Panels of experts.

Complexity of voting rules

- Many voting rules are tractable
  - Linear in number of voters
  - Linear or quadratic in number of candidates.
- Some interesting ones are not:
  - Kemeny, build the collective profile which is closest to the n profiles it is constructed from.
  - Dodgson, pick the candidate that is closest to being a Condorcet winner.
- These computations are NP-hard, as are the computations of other interesting rules.

Combinatorial domains

- If the set of alternatives (candidates) is small, it is easy to represent preferences between them
  - They can be listed explicitly.
- However, it is harder to express preferences over sets of alternatives with different attribute values.
  - The set over which preferences have to be expressed grow exponentially.
- This means that even simple voting rules become computationally hard to apply.
  - A function that is linear in the number of items still becomes intractable when applied to an exponential number of items.
- An important question is how to represent preferences compactly rather than explictly listing them all.

Strategy-proofness

- Is it possible to manipulate voting mechanisms?
- The Gibbard-Sattertwaite theorem says that with at least three candidates, if the voting mechanism is non-dictatorial, then the mechanism is manipulable for some sets of preferences.
- Given a set of preference profiles

$$\langle P_1, \ldots, P_n \rangle$$

which elect candidate c, we say j can manipulate the process if there is is a profile P'_j such that:

$$\langle P_1, \ldots, P_{j-1}, P'_j, P_{j+1}, \ldots P_n \rangle$$

elects candidate c', and j ranks c' higher than c.

- Three candidates c_1, c_2, c_3.
- Five voters, two with profile:

$$c_1 \succ c_2 \succ c_3$$

two with profile:

$$c_2 \succ c_1 \succ c_3$$

and one with

$$c_3 \succ c_1 \succ c_2$$

- If we use the plurality rule, the last voter's preferred candidate cannot win.
- So he has an incentive to vote insincerely and ensure that c_1 wins.

- Here, theory says that what we want is impossible, so settle for making it hard to manipulate.
- For example, Single Transferable Vote is NP-hard for single agents to manipulate.
- Some transformations to the voting process make manipulation harder
  - Adding a pre-round to eliminate half the candidates.
- However, typical complexity results only tell us that manipulation is hard in the *worst case*.
- Recent work suggests that there are no voting rules that are usually hard to manipulate.
  - Computationally tractable algorithm for computing an insincere profile succeeds in manipulating a large fraction of the time.

Communication requirements

- What communications are required by social choice mechanisms?
- Even centralized mechanisms may require lot of information to be exchanged.
  - social choice with *incomplete information*.
- Preference elicitation
  - Combinatorial auction with R items requires $2^{|R|} - 1$ bundles be valued.
  - For each bidder.
  - Are there protocols that will reduce this number?

Logics to analyse social procedures

- Logics, typically modal logics, used to analyse
  - specify
  - verify

  social mechanisms.
- Social software (Parikh)
  - Epistemic logic to capture knowledge of groups of agents.
- Coalition logic (Pauly)
  - Modality to capture effectiveness of coalitions.

Summary

- Gave a quick summary of some of the main areas studied in computational social choice.
- Problem areas such as
  - voting mechanisms
  - resource allocation.
- Related computational problems, like
  - the complexity of computing preference orderings
  - The number of messages required to reach agreement
- This is an active area of research, but one that has more questions than answers.