Multiagent Preference Aggregation Using Fuzzy Quantifiers

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ABSTRACT: This paper concerns itself with the problem of aggregation of preference functions associated with a group of agents for the purpose of making a decision. We suggest a formulation for the overall preference function, the mediation rule, which makes use of concepts drawn from the fuzzy subset literature. In particular we use the idea of a linguistic quantifier to formulate the overall preference function. We also consider the the situation where there exists a priority over the agents with respect their power in getting their preferred solution

1. Introduction

An issue that is of considerable interest in E-commerce and other distributed systems is the selection of a solution acceptable to a collection agents when each agent has their own preferences among the set of alternative possible solutions [1-3]. The process of finding such a solution can be seen as a kind of group decision making involving these agents. This class of problems have a long history in the economic literature. We particularly note the work of Arrow [4], Nash [5] and the classic work by Luce and Raiffa [6]. In the fuzzy set literature one finds a considerable body of literature devoted to the closely related problem of multi-criteria aggregation starting with the classic work of Bellman and Zadeh [7]. Rosenchein and Zlotkin [8] have presented a comprehensive work discussing this issue from the distributed systems multi-agent point of view. A number of authors have looked at this from a philosophical point of view [9, 10].

Our pragmatic object here is to try to develop machinery to help in the electronic mediation between these multiple agents. We use the term mediation to distinguish it from negotiation. Whereas negotiation can be a dynamic process our concern here is with a more narrowly focused, although closely related, activity, that of combining agent preferences. At a formal level the goal here is to provide for the intelligent aggregation of the individual agent preference functions to obtain a group preference function to form the bases of selecting a solution from among a collection of possible solutions. We note that the aggregation technology presented here can be a part of a more complex negotiation process. The spirit of the technology presented here draws heavily upon the fuzzy multicriteria aggregation point of view initiated by Bellman and Zadeh [7].

2. Multi-Agent Preference Aggregation

Our point of departure here is a collection of n agents and a set of X alternatives from among which we must select one. We assume each agent has a preference function A_i over the set X such that for each alternative x the value $A_i(x) \in [0, 1]$ indicates the degree to which agent i is satisfied with this alternative. In the case of just a single agent the problem of finding a solution from among the possible alternatives is easily solved by just selecting the alternative that best satisfies the single agent's preference function. In the multiple agent environment the situation is more complex. Here each alternative can be viewed as an n tuple (or n vector) whose components are the satisfactions to the individual agents. Our task of choosing a best alternative then is to select among these vectors. This is generally a difficult task, for accept in the special case where one vector dominates all the others the choice is not clear. Here we shall take a different approach. We shall associated with each alternative a scalar value Val(x) and then compare these alternatives by simply comparing these scalar values.

Using this we can associate with each alternative $x \in X$ a value

$$Val(x) = F(A_1(x), A_2(x), \quad , A_n(x))$$

indicating the degree to which alternative x satisfies the group of agents. Once having calculated Val(x) for all the alternatives in X we then select, as our optimal alternative, the alternative x* which satisfies $Val(x^*) = Max_{x \in X}[Val(x)]$.

One feature of this type of approach where each alternative is separately scored and then these alternative scores compared is that it has the property of what Arrow [4] called indifference to irrelevant alternatives.

Definition: Let **DP** indicate some decision procedure. Let X be a set of alternative solutions being considered. Let **DP**(X) be the optimal choice under **DP** over the set X of alternatives. Let y be an additional alternative not in X. Let $X' = X \cup \{y\}$, the addition of y to X. The decision procedure **DP** is said to have the property of indifference to irrelevant alternatives if DP(X') is either DP(X) or y.

With this property the addition of y doesn't cause us to select some other element in X. The feature precludes the strategy of adding of an alternative to X with the purpose of helping some already available alternative. We note that it is the fact that Val(x) doesn't depend on the overall satisfaction of any alternative other than x that guarantees the property of indifference to irrelevant alternatives. We say F is pointwise.

Another feature desired of any function F is what Arrow [4] calls positive association of group and individual values. This feature can be seen as essentially requiring that F is monotonic. That is if x and y are two alternatives and $A_j(x) \geq A_j(y)$ for all j then $F(A_1(x), \quad , A_n(x)) \geq F(A_1(y), \quad , A_n(y))$

Beyond being pointwise and monotonic the choice of the form for F models the mediation systems imperative for aggregating the preferences of the individual agents to obtain some group preference. Thus a fundamental problem that arises in this task is to provide an ability to use F to represent different types of mediation rules that be used to formulate a group preference function from individual preference functions.

Starting with the classic work of Bellman and Zadeh [7] fuzzy logic has been used as a tool to develop and model multicriteria and group decision problems. In this framework individual agents preference function can viewed as fuzzy subsets over the space of solution alternatives and fuzzy set operators are used to aggregate the individual agent preferences to form the overall group preference function. The choice of operators used to aggregate these individual agent preferences reflect the choice of a mediation rule. As originally suggested by Bellman and Zadeh the agents preferences can be combined by the use of an intersection operation which implicitly implies a requirement that **all** the agents **must** be satisfied by a solution to the problem. Here we are imposing the condition that any individual agent can doom a solution. Each agent has the power to totally reject an alternative. As suggested by Bellman and Zadeh if the relationship is that we desire **all** agents be satisfied then we can use $Val(x) = Min_j[A_j(x)]$. This is a reflection of the fact this situation is essentially modeling an "anding" between the preference functions. That is here the overall group satisfaction Val is modeling a desire to satisfy A_1 **and** A_2 **and** .... **and** A_n.

As noted by Yager [11] this condition of may not always be the appropriate relationship for combining the agents preferences. For as we noted this is a very strong imperative, any agent can unilaterally doom any alternative. Other mediation rules can be considered where all agents need not be satisfied and thereby relaxing this unilateral control. For example an acceptable solution may be obtained if ***most of*** *the agents are satisfied*. Furthermore other considerations can be included in the process. Do some agents have priority over others? Are some agents more important than others? In this work we look at an approach to the formulation of these softer aggregation rules which we call **quantifier guided aggregations**. This approach allows a natural language expression of the quantity of agents that need to agree on an acceptable solution. As we shall see the Ordered Weighted Averaging (OWA) operator [11] will provide a tool to implement these kinds of aggregations [12].

3. OWA Operators

In order to be able to formally model these quantifier guided aggregations we shall use the class of aggregation operators called **O**rdered **W**eighted **A**veraging (OWA) operators introduced by Yager [11].

Definition: An aggregation operator $F: I^n \rightarrow I$ is called an **O**rdered **W**eighted **A**veraging (OWA) operator of dimension n if it has associated with it a weighting vector $W = [w_1 w_2 \ . \ . \ w_n]$ such that

1. $w_j \in [0, 1]$

2. $\sum_{j=1}^{n} w_j = 1$

and where with b_j being the j^{th} largest of the a_i.we have

$$F(a_1, ..., a_n) = \sum_{j=1}^{n} w_j b_j$$

An essential feature of this aggregation is the reordering operation, a nonlinear operation, that is used in the process. In the OWA aggregation the weights are not associated directly with a particular argument but with the ordered position of the arguments. If id is an index function such that id(j) is the index of the jth largest of the a_i the we can express

$$F(a_1, ..., a_n) = \sum_{j=1}^{n} w_j a_{id(j)}.$$

The following example illustrates the usage of these operators.

Example: Assume F is an OWA operator of dimension four with weighting vector such that

$w_1 = 0.4, w_2 = 0.2, w_3 = 0.1$ and $w_4 = 0.3$.

If we desire to evaluate F(0.5, 1, 0.8, 0.9) then we assign

$b_1 = 1, b_2 = 0.9, b_3 = 0.8$ and $b_4 = 0.5$

and F(0.5, 1, 0.8, 0.9) = (1) (0.4) + (0.9) (0.2) + (0.8) (0.1) + (0.5) (0.3) = 0.81.

In [11] Yager shows that OWA aggregation has the following properties:

(1) Commutativity: The indexing of the arguments is irrelevant

(2) Monotonicity: If $a_i \geq \hat{a}_i$ for all i then

$F(a_1, ..., a_n) \geq F(\hat{a}_1, ..., \hat{a}_n)$.

(3) Idempotency: F(a,...a) = a.

(4) Boundedness

$Max_i[a_i] \geq F(a_1, ..., a_n) \geq Min_i[a_i]$

We note that these conditions imply that the OWA operator is a mean operator. The OWA operators can also be seen as providing a generation of alpha trimmed means [13].

The form of the aggregation is very strongly dependent upon the weighting vector used. In [14] Yager investigate various different families of OWA aggregation operators. A number of special cases of weighting vector are worth noting. The weighting vector W^* defined such that $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$ gives us the aggregation $F^*(a_1, ..., a_n) = Max_i[a_i]$. Thus W^* provides the largest possible aggregation. The weighting vector W_* defined such that $w_n = 1$ and $w_i = 0$ for $i \neq n$ gives us the aggregation $F_*(a_1,...a_n) = Min_i[a_i]$. This weighting provides the smallest aggregation of the arguments.

The weighting vector W_A defined such that $w_i = \frac{1}{n}$ for all i gives us the simple average

$$F_A(a_1,...a_n) = \frac{1}{n}\sum_{i=1}^{n} a_i.$$

The weighting vector W^k defined such that $w_k = 1$ and $w_i = 0$ for $i \neq k$ gives us $F(a_1,...a_n) = b_k$ where b_k is the k^{th} largest of the a_i.

Another class of OWA aggregation is called the Olympic operator, in this case $w_1 = w_n = 0$ and $w_i = \frac{1}{n-2}$. Here we are eliminating the extreme scores.

In [11] Yager associated with any OWA aggregation a measure which he called its attitudinal character. In particular, if we have a weighting vector W of dimension n then this attitudinal character is defined as

$$A\text{-}C(W) = \frac{1}{n-1}\sum_{i=1}^{n} (n - i)w_i.$$

It is easy to show that this measure lies in the unit interval. Furthermore it was shown in [11] that $A\text{–}C(W^*) = 1$, $A\text{–}C(W_{ave}) = 0.5$ and $A\text{–}C(W_*) = 0$. In the framework of multi-agent preference this can be seen as being inversely related to an individual agent's power in rejecting an alternative. Thus A-C(W) = 0 indicates that any individual agent can unilaterally doom an alternative. As A–C(W) moves from zero to one the individual agents power of rejection decreases, more agreement is needed to doom an alternative. In this framework we shall characterize a mediation rule with weighting vector W by

$$\text{U-D}(W) = 1 - \text{A-C}(W) = \frac{1}{n-1}\sum_{i=1}^{n}(i - i)w_i$$

where U-D can be seen as an acronym for **U**nilateral **D**ooming.

4. Quantifier Guided Aggregation

Assume we are faced with a decision problem in which we have a group of n agents whose preferences are to be considered in the decision. We denote these agents as A_1, ..., A_n. For any possible solution x we assume we have available the degree to which it is satisfactory to any of the agents. We shall denote these scores as $A_i(x)$ and assume they lie in the unit interval, [0, 1]. In this framework A_i can be viewed as a fuzzy subset over the set of alternatives. In order to determine the appropriateness of a particular alternative x as the solution to our problem we must aggregate its satisfaction to the individual agents to find some overall single value to associate with the alternative, the group satisfaction. This aggregation process constitutes a process of mediation between the agents. In order to obtain this overall evaluation some information must be provided about the form of a mediation rule used to implement the aggregation of the agents preference functions.

In a their classic work Bellman and Zadeh [7] suggested an approach to this type of individual preference aggregation problem which uses

$$\mathbf{Agg}(A_1(x), A_2(x), ...A_n(x)) = \text{Min}_i[A_i(x)].$$

Essentially, this approach is assuming a form of mediation in which we require that **all** the agents be satisfied by an acceptable solution. One then selects, as the best solution, the alternative with the highest aggregated value.

In the above in formulating our overall aggregation function we have essentially implemented the following linguistic agenda for our mediation rule

__All__ agents must be satisfied by an acceptable solution.

As we noted in many situations the requirement that ***all*** agents be satisfied is to strong. Examples of alternative and perhaps more reasonable mediation rules would be

__Most__ agents must be satisfied by an acceptable solution.

__At least about half__ the agents must be satisfied by an acceptable solution.

The above statements are examples of what we call **quantifier guided aggregations.** In these statements the underlined terms are examples of what Zadeh [15] called relative linguistic quantifiers.

In natural language we find many examples of relative linguistic quantifiers. These objects are exemplified by terms such as *all, most, many, at least half, some* and *few*. In [15] Zadeh suggested a formal representation of these linguistic quantifiers using fuzzy sets. He suggested that any relative linguistic quantifier can be expressed as a fuzzy subset Q of the unit interval, I. In this representation for any proportion $y \in I$, Q(y) indicates the degree to which y satisfies the concept conveyed by the term Q. In applications to quantifier guided aggregation we mainly use a special class of these linguistic quantifiers called **R**egular **I**ncreasing **M**onotone (RIM) quantifiers. These types of quantifiers have the property that as more agents are satisfied our overall satisfaction can't decrease. Formally these quantifiers are characterized in the following way: **1.** $Q(0) = 0$, **2.** $Q(1) = 1$ and **3.** $Q(x) \geq Q(y)$ if $x > y$. Examples of this kind of quantifier are *all, most, many, at least* α.

The quantifier **for all** is represented by the fuzzy subset Q_* where $Q_*(1) = 1$ and $Q_*(x) = 0$ for all $x \neq 1$. The quantifier **any** is defined as $Q^*(0) = 0$ and $Q^*(x) = 1$ for all $x \neq 0$. Both of these are examples of RIM quantifiers.

Having introduced the OWA aggregation operator we are now in a position to describe the process of quantifier guided aggregation. Again consider that we have a collection of A_1 ....., A_n of agents. These agents have their preferences represented as a fuzzy subset over the set of alternatives X. In the process of quantifier guided aggregation a group mediation protocol is expressed in terms of a linguistic quantifier Q indicating the proportion of agents whose agreement is necessary for a solution to be acceptable. This structure constitutes the rule for automated mediation. The form of mediation implicit in this approach is

Q agents must be satisfied by an acceptable solution,

where Q is a quantifier.

The formal procedure used to evaluate this decision function is expressed in the following. The quantifier Q is used to generate an OWA weighting vector W of dimension n. This weighting vector is then used in an OWA aggregation to determine the overall evaluation for each alternative. For each alternative the argument of this OWA aggregation is the degree of satisfaction of that alternative to each of the agents, $A_i(x)$, i = 1....n. Thus the process used in quantifier guided aggregation is as follows:

(1) Use Q to generate a set of OWA weights, $w_1, \ldots w_n$.

(2) For each alternative x in X calculate the overall evaluation

$$D(x) = F(A_1(x), A_2(x), \ldots A_n(x))$$

The procedure [12] used for generating the weights from the quantifier is

$$w_i = Q(\frac{i}{n}) - Q(\frac{i-1}{n})$$

for i = 1....n

Because of the nondecreasing nature of Q it follows that $w_i \geq 0$. Furthermore from the regularity of Q, Q(1) = 1 and Q(0) = 0, it follows that $\Sigma_i w_i = 1$. Thus we see that the weights generated are an acceptable class of OWA weights.

The use of a RIM quantifier to guide the aggregation essentially implies that the more agents satisfied the better the solution. This condition seems to be one that is naturally desired in multi–agent mediation. Thus most quantifier guided aggregation would seem to be based upon the use of these types of quantifiers.

5. Importance Weighted Quantifier Guided Aggregation

In this section we turn to the problem of multi-agent mediation based on quantifier guided aggregation in environments in which the agents involved in the mediation have differing importances associated with them. In this environment we shall again assume we have a set of n agents whose preference are again expressed as fuzzy subsets A_i over the space of alternative solutions X. In introducing quantifier guided aggregation we considered our mediation rule to be the statement *Q agents are satisfied by x*. We now additionally assume that we can associate with each agent a value V_i indicating the importance of that agent in this mediation process. We shall consider the V_i's to lie in the unit interval $V_i \in [0, 1]$. We make no restrictions on the total value of importances, that is they need not sum to one.

In this case, with importances, we modify the mediation rule to be

Q <u>important</u> agents must be satisfied by an acceptable solution.

In the following we describe the procedure to evaluate the overall satisfaction of alternative x. First we note for a given alternative x we have a collection of n pairs $(V_i, A_i(x))$. The first step in this process is to order the $A_i(x)$'s in descending order. Thus we let b_j be the j^{th} largest of $A_i(x)$, $b_j = a_{id(j)}$. Furthermore, we let u_j denote the importance associated with the agent that has the j^{th} largest satisfaction to x, $u_j = V_{id(j)}$. Thus if $A_5(x)$ is the largest of the $A_i(x)$ then $b_1 = A_5(x)$ and $u_1 = V_5$. At this point we can consider our information regarding the alternative x to be a collection of n pairs (u_j, b_j) where the b_j's are in descending ordering.

Our next step is to obtain the OWA weights associated with this aggregation. We obtain these weights as follows

$$w_j = Q(\frac{S_j}{T}) - Q(\frac{S_{j-1}}{T})$$

where $S_j = \sum_{k=1}^{j} u_k$ and $T = \sum_{K=1}^{n} u_k$, the total sum of importances. Having obtained the weights we can now calculate the evaluation D(x) associated with x, $D(x) = \sum_{j=1}^{n} b_j w_j(x)$. We emphasize that the weights used in this aggregation will generally be different for each x. This is due to the fact that the ordering of the A_i's will be different and in turn lead to different u_j's.

It can be shown in the special case where Q(r) = r, the unitor quantifier, this approach leads to the ordinary weighted average D(x) = $\frac{1}{T}\sum_{i=1}^{n} A_i(x) V_i$.

The following example illustrates the application of the above method

Example: Assume we have two alternatives x and y. We shall assume four agents A_1, A_2, A_3, A_4. The importances associated with these agents are $V_1 = 1$, $V_2 = 0.6$, $V_3 = 0.5$, $V_4 = 0.9$. We assume the satisfaction to each of the agent by alternative x is given by the following

$A_1(x) = 0.7$ $A_2(x) = 1$ $A_3(x) = 0.5$ $A_4(x) = 0.6$

$A_1(y) = 0.6$ $A_2(y) = 0.3$ $A_3(x) = 0.9$ $A_4(x) = 1$

We shall assume the quantifier guiding this aggregation to be **most** which is defined by $Q(r) = r^2$ We first consider the aggregation for x. In this case the ordering of the agent satisfaction give us

	b_j	u_j
A_2	1	0.6
A_1	0.7	.1
A_4	0.6	0.9
A_3	0.5	0.5

We note $T = \sum_{i=1}^{4} u_j = 3$. Calculating the weights associated with x, which we denoted $w_i(x)$, we get

$w_1(x) = Q(\frac{0.6}{3}) - Q(\frac{0}{3}) = (0.2)^2 - 0 = .04$,

$w_2(x) = Q(\frac{1.6}{3}) - Q(\frac{0.6}{3}) = .28 - .04 = .24$

$w_3(x) = Q(\frac{2.5}{3}) - Q(\frac{1.6}{3}) = .69 - .28 = .41$,

$w_4(x) = (\frac{3}{3}) - Q(\frac{2.5}{3}) = 1 - .69 = .31$

In this case $D(x) = \sum_{i=1}^{4} w_i(x)\, b_i = (.04)(1) + (.24)(.7) + (.41)(.6) + (.31)(.5) = .609$.

To calculate the evaluation for y we proceed as follows. In this case the ordering of the agent satisfaction is

	b_j	u_j
A_4	1	0.9
A_3	0.9	0.5
A_1	0.6	1
A_2	0.3	0.6

The weights associated with the aggregation are: $w_1(y) = 0.09$, $w_2(y) = 0.13$, $w_3(y) = 42$ and $w_4(y) = 0.36$. We calculate $D(y) = \sum_{i=1}^{4} w_i(y)\, b_i = (.09)(1) + (.13)(.9) + (.42)(.6) + (.36)(.3) = .567$

Hence in this example x is the preferred alternative.

6. Prioritized Agent Aggregation

In this section we consider the situation in which the information about the importances of the individual agents is captured by a prioritization of the agents with respect to their importance in the mediation process. We note that in [16] we have looked at this problem in considerable detail. Simply speaking by saying agent A_1 has a higher priority then agent A_2 we are mean to indicate that we are not willing to tradeoff satisfaction to agent A_1 for a gain in satisfaction to agent A_2. National security interests generally take a priority over economic concerns. In many decision problems the preferences of the agent responsible for safety and security often are of the highest priority. Another example is in the domain of air travel, decisions that tradeoff savings in gasoline cost at the expense of passenger safety are not acceptable. In many organizations a natural priority exists based upon the position of the individual in the organization. This prioritization can naturally transfer over to the agents representing the individual. This concept of prioritization can be seen as a reflection of the fact the higher priority agents have more power to doom an alternative they are not happy with.

As we shall see in this work, when calculating the group satisfaction to an alternative one method [16] of including priority type information in the mediation process is by associating with lower priority agents importance weights that are related to the alternatives satisfaction to higher priority agents. This of course means that these induced importance weights are alternative dependent. Let us formally look at this model.

Our point of departure here is a group of n agents and a basic mediation rule *Q agents must satisfied by an acceptable solution.* However here we have an additional constraint of a prioritization of the agents. Here rather then having agent importances captured by some prescribed weights associated with each of the agents we have a prioritization over the set of agents indicating their priority with respect to considering their preferences.

Formally we assume we have a priority ordering in which p(j) is the index of the agent with the j^{th} highest priority. Thus $A_{p(1)} > A_{p(2)} > \quad > A_{p(n)}$. Using this notation we let $G_j = \{A_{p(i)}(x) | i = 1 \text{ to } j\}$, it is the scores under alternative x of the agents with j highest priorities. We shall also find it convenient to denote $Sat_x(G_j) = \underset{i = 1 \text{ to } j}{Min}[A_{p(i)}(x)]$, the degree of satisfaction to all the j highest priority agents.

In [16] it was suggested that an appropriate form for the multi-agent aggregation function in this environment is

$$D(x) = \sum_{j=1}^{n} w_j \, Sat_x(G_j)$$

$$D(x) == \sum_{j=1}^{n} w_j \underset{i = 1 \text{ to } j}{Min}[A_{p(i)}(x)]$$

where $w_j = Q(\frac{j}{n}) - Q(\frac{j-1}{n})$.

Let us investigate the properties of this formulation with respect its modeling the prioritization of agents in the framework of using the OWA aggregation method.

As suggested in [16] we can consider the term $\alpha_j = Sat_x[G_{j-1}]^1 = \underset{i = 1 \text{ to } j-1}{Min}[A_{p(i)}(x)]$ as a kind of importance weight associated with the jth highest priority agent $A_{p(j)}$. Using this we get

$$D(x) = \sum_{j=1}^{n} w_j \, Sat_x(G_j)$$

$$D(x) = \sum_{j=1}^{n} w_j \, (Sat_x(G_{j-1}) \wedge A_{p(j)}(x)]$$

$$D(x) = \sum_{j=1}^{n} w_j \, (\alpha_j \wedge A_{p(j)}(x))$$

Here we observe that the importance associated with the j^{th} highest priority agent is equal to the minimal degree of satisfaction of all the higher priority agents. This provides a useful insight into the nature of agent prioritization. In particular we see that this hierarchical prioritization of the agents acts so that an agent can't contribute to the overall aggregated score unless all the agents with higher priorities are satisfied. Thus this formulation works to enforce our desire that we don't want to allow tradeoffs in satisfactions between higher priority agent and lower priority lower until some degree of satisfactions to higher priority agents is obtained.

As we see in the case of two agents where

$$D(x) = w_1 A_{p(1)}(x) + w_2 (A_{p(1)}(x) \wedge A_{p(2)}(x))$$

the ability of the lower priority agent $A_{p(2)}$ is constrained by the satisfaction to the higher priority agent $A_{p(1)}$. To simply illustrate this we consider the neutral case where $w_1 = w_2 = \frac{1}{2}$, here

$$D(x) = \frac{1}{2} A_{p(1)}(x) + \frac{1}{2} (A_{p(1)}(x) \wedge A_{p(2)}(x)).$$

We see that if $A_{p(1)}(x)$ is a small value even if $A_{p(2)}(x)$ is a large value the second term in the summation can't contribute to improving the value of D(x). Here then lack of support for alternative x by the higher priority agent dooms it to low overall support regardless of what the second agent believes. We summarize the essential idea of this below

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1 Here we assume by convention that where j = 1 we have $G_{j-1} = \varnothing$ and $Sat_x[\varnothing] = 1$.

Agent 1 for x	Agent 2 for x	Overall for x
Small	Anything	Small
Large	Small	$\frac{1}{2}$ Large
Large	Large	Large

The important implication here is that the higher priority agent can block any solution, although not necessarily guarantee its success. Its power to guarantee success is is related to the OWA weights w_1 and w_2. For example if $w_1 = 1$ then the higher priority agent can both guarantee success and failure of an alternative.

Let us look at some features of this type of aggregation.

Observation: For any weighting vector W if the highest priority agent is the least satisfied, this is the overall aggregated value, $D(x) = A_{p(1)}(x)$.

Justification: If $A_{p(1)}(x) = \text{Min}_i[A_i(x)]$ then $Sat_x[G_j] = \underset{i=1 \text{ to } j}{\text{Min}}[A_{p(i)}(x)] = A_{p(1)}(x)$ for all j and hence $D(x) = \sum_{j=1}^{n} w_j \, Sat_x[G_j] = A_{p(1)}(x)$

More generally we also see that since $Sat_x[G_j] \leq \underset{i=1 \text{ to } j}{\text{Min}}[A_{p(i)}(x)] \leq A_{p(1)}(x))$ then $D(x) = \sum_{j=1}^{n} w_j \; Sat_x[G_j] \leq A_{p(1)}(x)$, the aggregated value is never larger than the satisfaction to the highest priority agent.

We observe that if the weights are such that $w_q = 1$, $w_j = 0$ for all $j \neq q$, then

$$D(x) = \sum_{j=1}^{n} w_j \, Sat_x[G_j] = Sat_x[G_q]$$

$$D(x) = \underset{i=1 \text{ to } q}{\text{Min}}[A_{p(i)}(x))].$$

It is the least degree of satisfaction to any of the q highest priority agents. When q = 1, our desire is to satisfy any agent, thus $D(x) = [A_{p(1)}(x)]$, the score of the highest priority agent. If q = n, then we are required to satisfy all the agent and we get $D(x) = \text{Min}_i[A_i(x)]$.

While our formal methodology for multi-agent mediation is based on the OWA operator we believe the fundamental ideas presented here for addressing the issue of prioritization of agents by introducing importances related to the satisfaction by higher priority criteria transcend this choice of methodology and is applicable to other approaches to modeling multi-agent and group decision making

7. References

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