Using Hypergames to Select Plans in Adversarial Environments

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Abstract. Hypergames are an emerging tool for combining decision theoretic and game theoretic information. This paper explains their use in choosing a course of action when the decision-maker has a belief-context about an opponent's probable play, by explaining the Model Opponent strategy. Several findings about expected value calculations in hypergames are given. A refined definition of expected value for hypergame strategies is proposed that considers the effects of strategic outguessing. A contemporary military situation is provided as an illustrative example of the technique.

1 Introduction

Fundamentally, this work employs a decision theoretic framework to combine a limited number of reasoning contexts. Each reasoning context is constructed as a subgame of a larger more complete game with an attached belief-context. These subgames represent belief-context models of opponent behavior that are aggregated into a new representation, the hypergame [1]. Game theory has no mechanism for combining games that are different, because this violates its axioms [2,3,4]. Decision theory places no constraints on what an opponent might know; therefore a decision theoretic mechanism was proposed to combine such models (subgames).

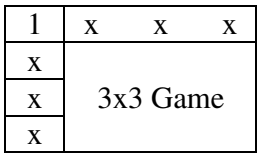
Simply, the decision-maker (the Row Player) forms a decision theoretic model about an opponent (the Column Player) who sees the contest as one of several probable subgames. Row calculates Column's behavior in each subgame and weights Column's choices by Row's belief-context that Column is playing that subgame. This belief-weighted vector is called the Column Mixed Strategy explicit, CMS explicit.

This paper explores some of the insights gained by experimental and mathematical manipulation of these concepts [5,6,7]. It provides a clear example of the new representation. The reader will find that there are many other variations on this theme. This approach performs better than or equal to game theory in all cases. Game players who have commented on this work have mentioned that this clearly

explains why superior players may play differently against novices than against other well-known and equally talented players. Lastly, as Row knows less and less about Column, then the solution tends to the Nash Equilibrium Mixed Strategy for the full game.

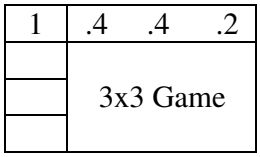
2 Representation of the Problem

There are two regions in any hypergame representation: the scenario belief-context and the m x n game. The scenario belief-context is shown in the upper left-hand corner. The m x n game is shown in the lower right-hand corner. Additionally, there are two mutually exclusive aspects: Column's expected behavior or a subgame context. Column's expected behavior is shown in the rows which have numbers above the game. Subgame contexts are shown by x's which indicate the rows and columns active in the subgame. The following three hypergames (figures 1-3) represent reasoning scenarios that often occur and an associated hypergame for each one.



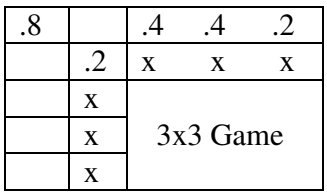
Hypergame with Subgame Context

Fig. 1. The full game is the first reasoning scenario, which corresponds to standard game theory. The Row player believes that the situation is completely described by the 3 x 3 game, hence the "1" in the upper left-hand corner. It has a subgame context that is 3 x 3.



Hypergame with Column Behavior

Fig. 2. In the second scenario, Row believes that he knows Column's behavior, as signified by the "1" in the upper left-hand corner, which is analogous to decision theory. It has Column's expected behavior as (.4 .4 .2).



Hypergame with uncertainty

Fig. 3. In the third scenario, Row is less sure. It shows that Row believes that it is .8 likely that he knows Column's expected behavior. The remaining .2 belief is assigned to the full 3 x 3 game.

Let p_i be the value of belief-context$_i$, expressed as a probability, of an opponent playing subgame *i* of the full game. Note that the scenario belief context sums to one.

$$1 = \sum_{i=1}^{n} p_i \tag{1}$$

In other words, *n* accounts for all of the possible games that the opponent might play. Let s_{ij} be the Column player's NEMS vector for the each such subgame. Any subgame strategy determined by using standard game theory assumptions results in a subgame rational strategy. Unfortunately for Column, this is a local optimization of only one game. To determine S_j, the jth component of CMS explicit, determine the belief-context weighted sum of Column's subgame strategies.

$$S_j = \sum_{i=1}^{n} p_i s_{ij} \tag{2}$$

where p_i is the belief-context weight of subgame *i*, s_{ij} is the probability weight for column*j* for each subgame *i*, and *n* is the number of scenario-contexts (subgames) considered.

3 An Approach to Solving the Problem

To choose the best row based on the belief-context, compute the highest utility row using the original full game and the S_j components of CMS explicit.

$$MO = \max_k (S_j \cdot u_{jk}) \quad \text{for j = 1 to n.} \tag{3}$$

where u_{jk} is the utility value for the outcome column *j* and row *k*.

The Modeling Opponent strategy, MO, maximizes the expected value of the situation-context (hypergame) by selecting the 'best' row, k, by weighting the utilities for any row, u_{jk}, by values of CMS explicit, $\boldsymbol{S}_j$. The result is the expected value of the Modeling Opponent strategy. Since the decisions of the Row and Column players are now decoupled, *only a pure strategy is recommended*, which is a valuable and important result. This means that the planner can make a definite recommendation, rather than espousing a mixture of rows. Modeling Opponent seems to be a tractable, theoretical plan selection extension of game theory.

This result uses the summation of the subgames to determine the strategy-vector for Column, but then solves the problem as though it was the result of a decision theoretic elicitation (as per figure 4). The upper left-hand corner represents the scenario-belief context and the lower right contains the m x n game (u_{ij}).

					Column	Col 1	Col 2	Col 3	...	Col n
	P_k					c_{k1}	c_{k2}		...	
		...				...	...	...	...	...
			P_1				c_{12}	c_{13}		
				1-ΣP		c_{01}	c_{02}	c_{03}	...	c_{0n}
MO						Col 1	Col 2	Col 3	...	Col n
EV 1	r_{k1}	...	r_{11}	r_{01}	Row 1	u_{11}	u_{12}	u_{13}	...	u_{1n}
EV 2	r_{k2}	...	r_{12}	r_{02}	Row 2	u_{21}	u_{22}	u_{23}	...	u_{2n}
...	...	...	...	...	...	...	...	...	...	...
EV m	r_{km}	...	r_{1m}	r_{0m}	Row m	u_{m1}	u_{m2}	u_{m3}	...	u_{mn}

Generalized Hypergame

Fig. 4. The normal form with MO solution is a hypergame with the expected value of each row attached on the left. It includes all of the information of the new representation, as well as the actual strategy-vectors and expected values for the game.

There is a value computed for every column that an entry occurs in the new problem form. If the strategy evaluates to 0, a '0' placeholder is still placed in the entry, so that one can still reconstruct the thought process of the elicitation. The CMS explicit's terms are determined and the expected values for each row are calculated. Obviously the maximum value is the pure strategy to choose. This form will be used in the explanations that follow, so that a step by step construction of the solutions can be presented.

Finding #1 – The new representation allows the decision-maker to record a consistent belief-context about an opponent's choices and the explicit situation (see figure 4).

Finding #2 – If a decision-maker can not allocate all belief in a belief-context (belief-context sums to less than one), the remaining belief should be assigned to the full game Nash Equilibrium Mixed Strategy (NEMS).

Proof for Finding #2:

1. Suppose Row's belief(remaining) > 0, then Row has no belief about what Column is considering for Row's unallocated belief. This is the same as saying that Row can not provide the scenario-context for some of Row's belief.
2. Let CMS explicit be any Column Mixed Strategy (CMS) that does not equal CMS m x n (Column's part of the NEMS).
3. If CMS explicit ≠ CMS m x n, then EV (RMS m x n, CMS explicit) > EV (RMS m x n, CMS m x n). This means that if Row uses the Row Mixed Strategy associated with the NEMS solution against any CMS explicit, an advantage is obtained.
4. Row should not assign a strategy to Column that can be exploited, when no belief exists for that strategy.
5. Therefore the unknown CMS should be set to CMS m x n.

Note that the expected value of a Modeling Opponent solution is not constrained to lie in the MAXIMIN range for the full game.

Finding #3 - There exists at least one row, that is part of the full game, that will produce an expected value greater than the EV (NEMS m x n), when CMS explicit ≠ CMS m x n (see above for CMS explicit).

Proof for Finding #3:

1. Suppose the EV (for every row that is an element of the game's solution rows, CMS explicit) ≤ EV (NEMS m x n).
2. Then CMS explicit could guarantee Column a better result than the CMS m x n.
3. But that is contrary to the definition of the NEMS, so EV (some row 'a', CMS explicit) > EV (NEMS m x n).

Finding #4 - For all games, the EV(NEMS m x n-1) is always greater than or equal to EV(NEMS m x n) and EV(NEMS m-1 x n-1).

Proof for Finding #4:

1. EV (NEMS m x n) = EV (RMS m x n, CMS m x n)
2. EV (NEMS m x n-1) = EV (RMS m x n-1, CMS m x n-1)
3. EV (RMS m x n, CMS m x n) ≤ EV (RMS m x n-1, CMS m x n-1), because Column cannot do better by losing a column.
4. EV (NEMS m-1 x n-1) = EV (RMS m-1 x n-1, CMS m-1x n-1)
5. EV (RMS m-1 x n-1, CMS m-1 x n-1) ≤ EV (RMS m x n-1, CMS m x n-1), because Row cannot do better by losing a row.
6. Therefore, EV (NEMS m x n-1) ≥ EV (NEMS m x n) or EV (NEMS m-1 x n-1).

4 An Approach to Outguessing

An answer the problem of recording expert belief-contexts about an adversary has been proffered. It ignores an important source of uncertainty for such situations -- it has ignored strategic outguessing. What if Column is capable of reasoning about Row's Modeling Opponent solution? It is asserted that if Column was omniscient, then the best result that Row could achieve is limited to the minimum value, G, in the chosen row. Let's assert that the utility of any hypergame selection strategy should account for this, which is shown in figure 5.

Finding #5 - Definition: Utility of any hypergame selection strategy equals (1-g) • EV (selection strategy, CMS) + g • EV(selection strategy, G) for $0 \leq g \leq 1$. CMS is the explicit Column Mixed Strategy and G is the omniscient worst case column(see figure 5).

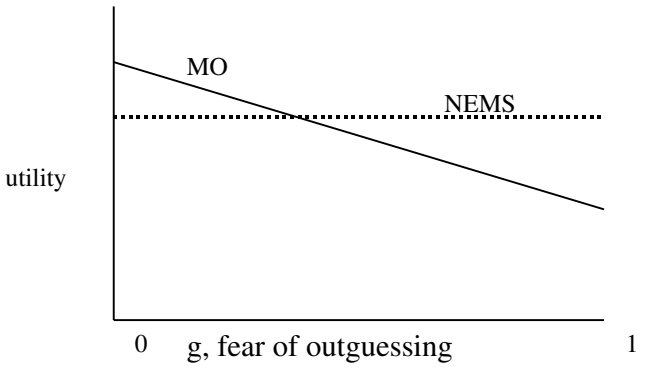


Fig. 5. This figure represents uncertainty in the range of 0-1 as g. Both the Model Opponent and the NEMS strategy are plotted. As Row becomes less certain of Column's capability to outguess (as g increases), the NEMS solution becomes more attractive.

For the lower values of g, the Modeling Opponent strategy is better than the NEMS in the notional hypergame.

5 An Example: Column's 3x3 in a 6x6

It's time for an example. The following scenario was borrowed as a notional decision from the Desert Storm War. In figure 6, the Iraqi High Command is the Column player. The Allied Command is the Row player. The values in the game vary from

–5 (decisive defeat) to 5 (decisive victory). A six-step process guides the computation.

			.80		3x3	1	0	0	-	-	-
				.20	6x6	0	.125	.25	0	.625	0
					Comb.	.8	.025	.05	0	.125	0
PS*	WS*	MO*	NEM S 3x3	NEM S 6x6		Defend	Defend Left	Defend Right	w/Re-serve	Screen	Counter Attack
0	.025	0	1	.125	Attack	-1	-1	-1	1	1	4
0	.1	0	0	.5	Attack Right	-1	-3	0	1	1	1
0	0	0	0	0	Attack Left	-1	0	-3	1	1	1
.556	.530	1	-	.375	Envelop Left	3	5	1	2	-1	-2
.444	.355	0	-	0	Invade Beach	1	-3	2	-1	-1	-2
0	0	0	-	0	Envelop Vertically	2	0	0	-3	-2	-1
1.64	**1.42**	**2.45**	**-1**	**.25**	**<--EV**						
-2	**-1.57**	**-2**	**-1**	**.25**	**<--G**						

Desert Storm Hypergame

Fig. 6. This hypergame shows a 3x3 subgame in a 6x6 full game. Row's belief about the scenario is shown in the upper left-hand corner. Three different hyperstrategies (Pick Subgame, Weighted Subgame, and Model Opponent) are shown in the leftmost three columns. The full game is included in the lower right-hand corner. Labels of strategies are provided to enrich the semantic content

1. The Row Player sees the possible attacks and defenses per figure 6.
2. The Column Player is deemed to be reasoning (.8 belief-context) that entrenched defenses are the only thing that will work against Row (given his air superiority). These plans are Defend, Defend Weighted Left, Defend Weighted Right. Column is also deemed to conclude that the Allies' fixation on Kuwaiti sovereignty will keep them within the borders of Kuwait. Under these circumstances only three frontal attacks are possible. These are Attack, Attack Right, Attack Left.
3. Each result is calculated and placed in the top three rows above the game.
4. Calculate each hyperstrategy. Note Weighted Strategy (WS) and Pick Subgame (PS) use a 6x3 subgame which is chosen because of finding #4. Sometimes these two strategies are useful, particularly when there is a higher likelihood that the opposing player (Column) understands Row's reasoning process.

5. Determine the Worst Case for each hyperstrategy. Plot as in figure 7. The Model Opponent strategy is clearly superior until it intersects with the NEMS solution. The Weighted Subgame and Pick Subgame are inferior to the combination of the Model Opponent strategy and the NEMS solution as shown in ensuing graph.
6. Determine Row's fear of being outguessed. If the Row player is willing to believe that Column will "know" his strategy at .2 belief, then g = .2. Calculating the expected value of the Model Opponent strategy to be about 1.5, it is best strategy for the Row Player to choose. The Row player chooses Envelop Left in this scenario with an .64 (.8 • .8) likelihood of obtaining '3'.

Utility with outguessing: 3x3 in a 6x6.

Fig. 7. The expected value of each hyperstrategy is based on the game in figure 6 and g.

In this scenario the Model Opponent strategy dominates, until g is very close to .5, primarily because of the .8 belief of Row that Column is reasoning on the 3x3 subgame and the subgame dominant single column strategy.

6 Applications

The implications of this approach are that four pieces need to be in place to have a respectable two-player operational planner:

1. a pattern recognition capability about opponent play to derive beliefs for the model (a manageable probability of false negatives or false positives),

2. a scale appropriate updating function that revises the battlefield assessment based on the size and latency of the organization for which the planning occurs (the larger, the less often to prevent thrashing),
3. an elegant high level estimate of how good or bad different courses of action are likely to be (such as the military often make or a computer game designer often has),
4. and the above selection technique.

People already practice techniques (1), (2), and (3) for many domains including the stock market, the military, and product competition. In the interactive entertainment field, the authors are convinced that a new generation of computer strategy games with very creditable computer opponents will result. These agents will not need to be propped up by favorably changing the combat results for the computer player.

7 Extensions to Multi-agent Scenarios

What are the implications when a number of opposing agents exist and a number of collaborative agents exist; such as in a sports event or a battlefield? Each of the two-player considerations above are still operative, but communication of the decision-maker's intent to other friendly agents would probably enhance success. Such communications can be thought of as added "rows" for the Row player. They can include instructions to "feint" or to exhort the friendly agents to "commit" to a course of action.

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