Dempster-Shafer Argument Schemes

Yuqing Tang¹, Nir Oren², Simon Parsons³, and Katia Sycara¹

 ¹ Robotics Institute, Carnegie Mellon University 5000 Forbes Ave, Pittsburgh, PA 15213, USA. {yuqing.tang, katia}@cs.cmu.edu
 ² Department of Computing Science, University of Aberdeen Aberdeen, AB24 3UE, Scotland n.oren@abdn.ac.uk
 ³ Department of Computer & Information Science, Brooklyn College City University of New York, Brooklyn, NY 11210, USA. parsons@sci.brooklyn.cuny.edu

Abstract. Dempster-Shafer theory, which can be regarded as a generalisation of probability theory, is a widely used formalism for reasoning with uncertain information. The application of the theory hinges on the use of a rule for combining evidence from different sources. A number of different combination rules have been applied, each of which makes certain assumptions about the evidence. Here we describe several of these rules as argument schemes, using critical questions to capture the assumptions behind the rules. An example illustrates how we imagine these argument schemes might be used.

1 Introduction

There is a rich body of work investigating the use of uncertainty within argumentation theory. Such work typically concerns itself with reasoning about what conclusions can be drawn from uncertain information. Therefore, this body of work concerns itself with reasoning *with* uncertainty. Less attention has been given to the dual of this approach, namely reasoning *about* uncertainty. More specifically, while most existing frameworks determine how uncertain some conclusion is, given some uncertain premisses and rules for processing this uncertainty, little work addresses *which* rules should be applied in what context in order to obtain correct conclusions in the presence of uncertain information.

The ability to argue both with, and about uncertainty leads to a complete, extensible system for uncertain reasoning. Here, rules capable of dealing with uncertainty are applied to uncertain facts, resulting in new facts with an associated level of uncertainty. These facts in turn could be used to determine which rules should be applied, and with what confidence⁴. The process would repeat over these additional rules and inferences. Such a system identifies likelihoods of conclusions both due to uncertainty regarding facts in the world, and differences (again due to uncertainty) regarding which reasoning rules should be applied.

⁴ Potentially leading to multiple probabilistic extension distributions

In order to create such a system, this paper introduces a class of argument schemes for reasoning over uncertain information. These are based on Dempster-Shafer theory [19] (DST). Our aim is to identify under what conditions specific techniques for combining uncertain information should be used, and to encode these through argument schemes (Dempster-Shafer theory is a particularly good subject for this because a number of different combination rules have been applied, each with different assumptions and intended applications.) By encoding conditions as argument schemes, we are able to argue about which combination method should be applied, and, by using multiple extension semantics, can explicitly recognise the different outcomes that can be obtained through the use of different combination methods, all of which could be appropriate in some setting.

The rest of this paper is as follows. Section 2 lays out a framework that combines argumentation with an explicit representation of evidence. Section 3 looks at different approaches for combining evidence and shows how to cast the application of these as argument schemes. Section 4 applies these schemes to an example reasoning task, and Section 5 discusses related work on argument schemes. Section 6 then concludes.

2 Basic notation

In this section we describe a basic framework, previously described in [23] that combines argumentation with an explicit representation of evidence.

A predicate language \mathcal{L} based on a set \mathcal{P} of symbols with standard connectives $\land, \lor, \rightarrow, \neg$ and standard semantics is assumed in this work. We further constrain the domain of any term of a predicate in \mathcal{P} to be finite and no functional symbols are allowed for any term of a predicate in \mathcal{P} . In this way, we will have a finite set of grounded predicates. For notational convenience, we also use \mathcal{P} to denote the set of all grounded predicates.

The set of truth assignments to all ground predicates is denoted by $\Omega = 2^{\mathcal{P}}$ where Ω is taken as the *frame of discernment*. As is standard, every formula $\theta \in \mathcal{L}$ can be interpreted into a subset of truth assignments to $\mathcal{P}, \mathcal{I}(\theta) \subseteq \Omega$. Truth and falsity are denoted by the symbols \top and \bot , with $\mathcal{I}(\bot) = \emptyset$, and $\mathcal{I}(\top) = \Omega$. Two formulae ϕ and φ , denoted by $\phi \equiv \varphi$, are equivalent iff $\mathcal{I}(\phi) = \mathcal{I}(\varphi)$.

An inference rule δ for \mathcal{L} is of the form:

$$\delta = \frac{p_1, ..., p_m}{c}$$

where $p_1, ..., p_m, c \in \mathcal{L}$. The p_i are the set of *premises* of the rule, and a specific p_i is denoted by $p_i(\delta)$. c is the *conclusion* of the rule, and is denoted by $c(\delta)$.

In order to represent evidence, we start with a knowledge base $\mathbf{K} = \langle \Sigma, \Delta \rangle$ consisting of a set of formulae and a set of rules for reasoning with the formulae. $\Sigma = \{\langle h, E \rangle\}$ is the set of formulae, where each formula *h* is associated with some supporting evidence *E*, and $\Delta = \{\langle \delta, E \rangle\}$ is the set of rules, where each rule δ is also associated with some supporting evidence. Our key notion is that of an *evidence argument*:

Definition 1. An evidence argument is a pair $\langle h, E \rangle$, where h is a formula in \mathcal{L} and $E = \{e_1, \ldots, e_n\}$ is a set of formulae in \mathcal{L} .

E is called the *supporting evidence* for *h*, denoted by E(h). An element $e_i \in E(h)$ is called a *focal element* of the evidence for h^5 , and represents an indivisible chunk of information serving as evidence. It is possible that $\{\langle h, E_1 \rangle, \langle h, E_2 \rangle\} \subseteq \Sigma$, such that $E_1 \neq E_2$. In such cases, we assume that E(h) is able to identify E_1 and E_2 separately.

Informally, the evidence associated with a formula $\theta \in \mathcal{L}$ or a rule $\delta \in \Delta$ summarises the data that supports a rule or formula. When we reason with the formulae, which we do by using the rules, we propagate the evidence, and so obtain the evidence that supports any conclusions. For every pair $\langle h, E \rangle$ it is then the case that:

1.
$$h = \theta \in \mathcal{L}$$
 or $h = \delta \in \Delta$; and

2. $E = \{e_1, ..., e_n\}$ is a set of evidence for h such that $e_i \neq e_j$ for any $i \neq j$.

In addition we assume the existence of a *probability mass function*, defined for every set of evidence E, and mapping its constituent members to a measure of belief. Formally, $m(E, \cdot) : E \mapsto [0, 1]$ is defined on E, and satisfies the constraint:

$$m(E, e_1) + \dots + m(E, e_n) = 1$$

and for all $\phi \notin E$, we set $m(E, \phi) = 0$. In other words we associate some measure of belief $m(E, \cdot)$ with every item of evidence in E, with the goal that from these we can calculate a measure for every h. For notational convenience, we also denote E and $m(E, e_i) = m_i$ together as: $E = \{e_1 : m_1, ..., e_n : m_n\}$.

The focus of Dempster-Shafer theory revolves around this probability mass, which constitutes evidence. In the current work, the evidence is a combination of logical statements over which a probability mass can be defined. As in standard Dempster-Shafer theory, we use the probability mass to determine how much certain interesting hypotheses are believed. In our case, these hypotheses are the conclusions of arguments

Definition 2. Given an evidence argument $A = \langle h, E \rangle$ for a formula $h \in \mathcal{L}$, the belief b(h), disbelief d(h), and the uncertainty u(h) of h are computed as follows:

$$b(h) = \Sigma_{\mathcal{I}(e_i) \subseteq \mathcal{I}(h)} m(E, e_i) = \Sigma_{e_i \vdash h} m(E, e_i)$$

$$d(h) = \Sigma_{\mathcal{I}(e_i) \cap \mathcal{I}(h) = \emptyset} m(E, e_i) = \Sigma_{e_i \vdash \neg h} m(E, e_i)$$

$$u(h) = \Sigma_{\mathcal{I}(e_i) \cap \mathcal{I}(h) \neq \emptyset} m(E, e_i) = \Sigma_{e_i \lor h} and e_i \lor \neg h} m(E, e_i)$$

In other words, the belief in h is the sum of the mass of the all focal elements in E that are part of the evidence for h; the disbelief in h is the sum of all the mass for all the focal elements that are evidence for $\neg h$; and the uncertainty is the sum of all the mass that for evidence that is assigned to neither h nor $\neg h$. Equivalently, the belief in h is the sum of the mass of all the formulae that imply h; the disbelief in h is the sum of the mass of the formulae that imply $\neg h$; and the uncertainty is the sum of the mass of the formulae that imply $\neg h$; and the uncertainty is the sum of the mass of the formulae that imply $\neg h$.

Example 1. Let $\langle h_1, E_1 \rangle = \langle p, \{p, \neg p \land q\} \rangle$, where $m(E_1, p) = 0.4$ and $m(E_1, \neg p \land q) = 0.6$. Then, as explained in Table 1(a):

$$b(h_1) = m(E_1, p) = 0.4, d(h_1) = m(E_1, \neg p \land q) = 0.6, \text{ and } u(h_1) = 0$$

⁵ This term originates from DST, as e_i plays the same role as focal elements in that theory.

p q	$p \; q$
$\overline{\mathcal{I}(\neg p \land \neg q) \ 0 \ 0}$	$\mathcal{I}(\neg p \land \neg q) \ 0 \ 0$
$\mathcal{I}(\neg p \land q) 0 \ 1$	$\mathcal{I}(\neg p \land q) = 0 \ 1 \ \mathcal{I}(h_2)$
$\mathcal{I}(p \wedge \neg q) = 1 0 \Big]_{\mathcal{T}(h_{-})}$	$\mathcal{I}(p \wedge \neg q) = 1 \ 0$
$\mathcal{I}(p \wedge q) = 1 \ 1 \ \int^{\mathcal{L}(n_1)}$	$\mathcal{I}(p \wedge q) = 1 \ 1 \ \mathcal{I}(h_2)$
(a)	(b)

Fig. 1. Truth tables for Example 1. (a) Truth table for h_1 . $b(h_1) = m(E_1, p)$ as $\mathcal{I}(p) \subseteq \mathcal{I}(h_1)$, and $d(h_1) = m(E_1, \neg p \land q)$ since $\mathcal{I}(\neg p \land q) \cap \mathcal{I}(h_1) = \emptyset$. (b) Truth table for h_2 . We can see that $b(h_2) = m(E_2, \neg p \land q)$ because $\mathcal{I}(\neg p \land q) \subseteq \mathcal{I}(h_2)$, $d(h_2) = m(E_2, \neg q)$ because $\mathcal{I}(\neg q) \cap \mathcal{I}(h_2) = \emptyset$, and $u(h_2) = m(E_2, p)$ because $\mathcal{I}(p) \cap \mathcal{I}(h_2) \neq \emptyset$.



Fig. 2. Four types of evidence

Let $\langle h_2, E_2 \rangle = \langle q, \{\neg p \land q, \neg q, p\} \rangle$, where $m(E_2, \neg p \land q) = 0.5$, $m(E_2, \neg q) = 0.3$, and $m(E_2, p) = 0.2$. Then, as explained in Table 1(b):

 $b(h_2) = m(E_2, \neg p \land q) = 0.5, d(h_2) = m(E_2, \neg q) = 0.3$, and $u(h_2) = m(E_2, p) = 0.2$.

3 Combining evidence

Having introduced a framework in which evidence and arguments can be associated, we turn to the question of how to combine arguments for the same conclusion.

The problem of combining evidence can be stated in the context of sensor fusion. Multiple sensors, each of which demonstrate different properties of uncertainty about the world, generate different characterizations of the world as observed through the evidence they can obtain. In the framework introduced above, the evidence obtained by each sensor can be characterized by the basic probability assignments to the frame of discernment of world as the above. The critical issue here is then how to combine this evidence. [21] suggests four different kinds of evidence, consonant evidence, consistent evidence, disjoint evidence, and arbitrary evidence. These different types of evidence are pictured in Figure 2 as sets of elements of the frame of discernment for where there are non-zero basic probability assignments. In the next section, we consider each of these types of evidence in more detail.

3.1 Types of evidence

In the case of *Consonant evidence*, the evidence can be framed as a nested structure of subsets in a frame of discernment Ω . (The importance of consonant sets has long been recognised in Dempster-Shafer theory [19].) For example, consider four sentences -A, B, C, D — in the language \mathcal{L} such that:

$$A \vdash B \vdash C \vdash D$$

In other words, the models of these sentences form a nested structure with respect to the subset relation over the set of models of the discernment frame such that:

$$\mathcal{I}(A) \supseteq \mathcal{I}(B) \supseteq \mathcal{I}(C) \supseteq \mathcal{I}(D)$$

This is the situation depicted in Figure 2(a). An example that fits this mould is the following:

- Sensor 1 observes A in the evidence it obtains: $E_1 = \{A : 1\}$.
- Sensor 2 observes A and B in the evidence it obtains: E₂ = {A : 0.6, B : 0.4}.
 Sensor 3 observes A, B, and C in the evidence it obtains: E₃ = {A : 0.4, B : 0.4, C: 0.2.
- Sensor 4 observes A, B, C and D in the evidence it obtains: $E_4 = \{A : 0.3, B :$ 0.3, C: 0.35, D: 0.1.

Now consider the case of Consistent evidence, as depicted in Figure 2(b). Here, there is at least one element is common to all subsets. For example, if $A, B, C, D \in \mathcal{L}$, then a case in which the evidence is consistent is the case in which:

$$A \equiv A \land B \land C \land D$$

Namely, A is the common subset of each of the subsets of the frame of discernment. Equally, we might have:

$$A \land B \land C \land D \vdash A$$

where A is a subset of the intersection of all the other sets. An example of this type of situation is the following:

- Sensor 1 observes A in the evidence it obtains: $E_1 = \{A : 1\}$.
- Sensor 2 observes A and B in the evidence it obtains: $E_2 = \{A : 0.6, B : 0.4\}$.
- Sensor 3 observes A and C in the evidence it obtains: $E_3 = \{A : .0.45, C : 0.55\}$.
- Sensor 4 observes A and D in the evidence it obtains: $E_4 = \{A : 0.3, D : 0.7\}$.

In the case of *Disjoint evidence*, as illustrated in Figure 2(c), the observed subsets in the evidence are disjoint and so do not intersect. As an example, consider the four observations A, B, C and D, which are pair-wise disjoint, and so

$$A \wedge B = \bot$$
$$B \wedge D = \bot$$
$$\vdots$$

A concrete example of this situation is as follows.

- Sensor 1 observes A in the evidence it obtains: $E_1 = \{A : 1\}$.
- Sensor 2 observes B in the evidence it obtains: $E_2 = \{B : 1\}.$
- Sensor 3 observes C in the evidence it obtains: $E_3 = \{C : 1\}.$
- Sensor 4 observes D in the evidence it obtains: $E_4 = \{D : 1\}$.

In the terminology of [21], Arbitrary evidence is the case where none of the above structures appear in the evidence. For example, consider the case in which the sensors observe the same evidence with $A, B, C, D \in \mathcal{L}$ as in the case of consistent evidence above, but, as in Figure 2(d), we have:

$$C \vdash A$$
$$A \land B = \bot$$

In [21], these different situations are related to different rules for combining evidence. We consider these rules in the next section.

3.2 Argument schemes for conflicting and uninformative evidence

A

In general, a scenario that contains multiple arguments, supported by different evidence, will represent some form of conflict. That is, those different pieces of evidence will point to somewhat different conclusions⁶. The way that this conflict is handled should reflect the types of evidence being combined with respect to the various assumptions made by those introducing their arguments, and their preferences. Argument schemes provide a convenient way of encoding these factors.

In this section we discuss several ways of combining evidence that have been suggested in the context of Dempster-Shafer theory, considering them all to fit into the general pattern of:

– A rule pattern in Δ :

$$\delta = \frac{p_1, \dots, p_n}{c}$$

- A Dempster-Shafer argument scheme specifying the
 - the pattern of the evidence of the premises: $\langle h_1, E_1 \rangle, ..., \langle h_n, E_n \rangle$
 - optional evidence for rule applicability E_{δ}
 - an associated conclusion evidence derivation process: $E(c) = f(E_1, ..., E_n)$ is the derivation process to compute E(c) from the evidence of the premises $E(p_1), ..., E(p_n)$; optionally, the conclusion evidence derivation process might explicitly involve with the rule applicability evidence: $E(c) = f(E_{\delta}, E_1, ..., E_n)$
- Premise instances with evidence: $\langle h_1, E_1 \rangle, ..., \langle h_n, E_n \rangle$
- A conclusion instance with derived evidence $\langle c, E\rangle$ following the derivation process.
- Critical questions: These are the questions whose answers will lead to either the acceptance of the application of such a scheme, or the rejection of the application⁷

⁶ If this is not the case, the arguments and their evidence will be identical and one will arguably not have multiple arguments so much as multiple copies of the same argument.

⁷ A third possibility exists, namely that critical questions can *reduce* the likelihood of a scheme's conclusion. We assume that a critical question must exceed some burden of proof to be applied, and if so, the scheme's application is rejected outright. Otherwise, the critical question does not affect the scheme. We intend to investigate this third possibility in future work.

Each argument scheme has, associated with it, a set of critical questions which control the application of the scheme. For notational simplicity, in this paper we will focus on schemes regarding how the conclusion evidence can be derived: Given $E_1, ..., E_n$ for the premises, how E(c) can be derived for the conclusion.

Dempster's rule of combination: This is the basic rule used in Dempster-Shafer theory [19], and as [21] reminds us, is a generalization of Bayes' rule which concentrates on the consensus between pieces of evidence:

$$m(E_1 \otimes^D E_2, A) = \frac{\sum_{A=B \wedge C} m(E_1, B) m(E_2, C)}{1 - \sum_{B \wedge C = \bot} m(E_1, B) m(E_2, C)}$$

The effect of this rule is to keep only the consensus between two pieces of evidence, and to reject all the conflicts. This is achieved by the normalization — having all the mass that would have been assigned to \emptyset within the frame of discernment be reassigned to the those parts of the frame of discernment that are supported by both pieces of evidence.

We can cast the use of Dempster's rule as being the application of the following argument scheme:

- Premises: evidence $E_1, ..., E_n$ Conclusion: new evidence $E_1 \otimes^D ... \otimes^D E_n$ where \otimes^D is defined as above.

Following the discussion of the applicability of Dempster's rule (such as those mentioned in [21]), the critical questions for this scheme can be as follows. As with all the schemes we assume that the critical questions have to be answered "yes" for the rule to be applicable:

- 1. Is the evidence for consonant, consistent or arbitrary focal subsets of the frame of discernment?
 - Sub-question: In the case of consonant and consistent evidence, is the consonant or consistent piece of evidence effectively providing evidence (both positive and negative) for the conclusion of interest?
- 2. Is each piece of evidence equally reliable?
- 3. Is each piece of evidence independent?
- 4. Should conflict between evidence be ignored in the mass assignments that result from combination?
- 5. Is there a restricted stochastic process behind the evidence which can be exploited to obtain a more accurate combination?
- 6. Is the evidence all informative (i.e. no E_i in the premises contains the totally uncertain focal element Ω with $m(E_i, \Omega) > 0$)?

In the example depicted in Figure 2(a), let us assume that knowledge base Σ and rule base Δ are as follows:

$$\begin{split} \boldsymbol{\Sigma} &= \{ \langle A, \{A\} \rangle, \langle A, \{A, B\} \rangle, \langle A, \{A, B, C\} \rangle, \langle A, \{A, B, C, D\} \rangle \} \\ \boldsymbol{\Delta} &= \{ \frac{\langle p, E_1 \rangle, ..., \langle p, E_n \rangle}{\langle p, E_1 \otimes^D \ldots \otimes^D E_n \rangle} \} \end{split}$$

Following Dempster's rule of combination, as every conjunction constructed from a representative element from each evidence E_1, E_2, E_3 and E_4 is A, we will have

$$E = E_1 \otimes^D \dots \otimes^D E_n = \{A\}$$

m(E, A) = 1 × (0.6 + 0.4) × (0.4 + 0.4 + 0.2) × (0.3 + 0.35 + 0.25 + 0.1) = 1.

For the example depicted in Figure 2(b), as every conjunction constructed from a representative element from each evidence E_1, E_2, E_3 and E_4 is A, we will again have:

$$E = \{A\}$$
$$m(E, A) = 1.$$

However, for the examples depicted in Figure 2(c) and Figure 2(d), as as every conjunction constructed from a representative element from each evidence E_1, E_2, E_3 and E_4 is \perp , we will have

$$E = E_1 \otimes^D \dots \otimes^D E_n = \{\}$$

m(E, \cdot) = UNDEFINED.

We now turn our attention to the scheme's critical questions. We will detail the discussion of critical question 1 with examples as follows. For the examples in Figure 2(a), Figure 2(b) and for the examples in Figure 2(d), the answer to critical question 1 is "yes" and this scheme applies; for the example in Figure 2(c), the answer to critical question 1 is "no", this scheme doesn't apply unless further justification is available. For the examples in Figure 2(a), Figure 2(a), Figure 2(b), the answers to the sub-question of 1 are "yes" so this scheme applies: $E = \{A\}$ can effectively support the conclusion A resulting in belief b(E, A) = 1, disbelief d(E, A) = 0 and uncertainty u(E, A) = 0. However, for the example in Figure 2(d), evidence $\{\}$ can not effectively support A, and so this scheme does not apply.

If instead of having the general rule on a sentence $p \ \frac{\langle p, E_1 \rangle, \dots, \langle p, E_n \rangle}{\langle p, E_1 \rangle \otimes D \dots \otimes D E_n \rangle}$, we have a more concrete syntactic rule $\frac{\langle A, E_1 \rangle, \dots, \langle A, E_n \rangle}{\langle \neg A, E_1 \otimes D \dots \otimes D E_n \rangle}$, then the answer to the sub-question of critical question 1 will still be "yes" since it provides negative evidence to the conclusion, leading to the rejection of the conclusion with no uncertainty: $b(E, \neg A) = 0$, $d(E, \neg A) = 1$ and $u(E, \neg A) = 0$. On the other hand, if we have a rule $\frac{\langle A, E_1 \rangle, \dots, \langle A, E_n \rangle}{\langle F, E_1 \otimes D \dots \otimes D E_n \rangle}$ where neither $A \vdash F$ nor $A \not\vdash F$, then the derived evidence will provide totally uncertain information regarding the conclusion F: b(E, F) = 0, d(E, F) = 0 and u(E, F) = 1. This would prevent the application of the scheme.

Critical question 2 tells us that if the evidence is not all equally reliable then Dempster's rule might not be applicable (we might, for example, have to discount some evidence). Critical question 3 is about the independence assumption behind Dempster's rule. If the evidence is not independent, this rule might double-count the evidence. Critical question 4 is about conflict handling. In many cases, instead of ignoring conflicts when doing the combination, the conflicting evidence might be useful to help to judge the conclusion by some means, e.g. by identifying how much weight these conflicts should play and count them as a form of uncertainty with discounts. Critical question 5 is a refinement question. Even if the scheme is determined as applicable given its inputs, it might be case that we can do better. For example, knowing that the underlying distribution of the basic mass function of the consonant evidence conforms with the Gaussian distribution, we might be able to obtain a more accurate estimation of the conclusion by estimating the parameters of Gaussian distribution from the evidence, and use this estimate to obtain a probabilistic mass function for the evidence of the conclusion.

Finally, note that as is normal within argumentation, if this — or any other — scheme is rejected due to a negative answer to one of the critical questions regarding the applicability, the scheme can be reinstated through better justifications.

Discount and combine rule: Here each piece of evidence E_i is characterized by a reliability factor α_i and the combination of evidences occurs through an operation on the beliefs:

$$\overline{b}_i(A) = \alpha_i b_i(A)$$

The result of this is that information from less reliable sources is discounted (i.e. is associated with a lower belief value). This can be contrasted with Dempster's rule which assumes that all the sources of evidence are equally good. The combination of all the evidence is then:

$$\bar{b}(E_1 \otimes^{DC} E_2 \otimes^{DC} \dots \otimes^{DC} E_n, A) = \frac{1}{n}(\bar{b}_1 + \bar{b}_2 + \dots + \bar{b}_n(A)).$$

As the name implies, this approach averages out the evidence during the combination. Since this operation utilises beliefs rather than probability masses (as in the case of Dempster's rule), Definition 2 must be applied prior to applying this rule.

We can consider this approach to be the application of an argument scheme characterised as follows.

- Premises: $E_1, ..., E_n$
- Conclusion: new evidence $E_1 \otimes^{DC} \dots \otimes^{DC} E_n$ where \otimes^{DC} is as defined above.

The critical questions for this scheme are:

- 1. Does the evidence indicate that the frame of discernment is made up of consonant, consistent or arbitrary focal subsets?
- 2. Should conflict between evidence be ignored in the mass assignments that result from combination?
- 3. Is there a restricted stochastic process behind the evidence which can be exploited to obtain a more accurate combination?

It should be clear that this discount and combine rule applies in a more general context than Dempster's rule by allowing for sources of different reliability. The associated argument scheme can be used in cases where critical question 2 for Dempster's rule cannot be answered positively.

Yager's rule: This is a modified version of Dempster's rule [31, 32] which treats conflicting evidence as uncertainty. A new notion, the combined ground probability assignment, is defined as

$$q(E_1 \otimes^Y \dots \otimes^Y E_n, A) = \sum_{A = \bigwedge_{i=1}^n A_i} \prod_{i=1}^n m(E_i, A_i)$$

This allows $q(\perp) \ge 0$ and assigns probability mass to the full frame of discernment in the combined evidence as:

$$m(E_1 \otimes^Y \ldots \otimes^Y E_n, \Omega) = q(E_1 \otimes^Y \ldots \otimes^Y E_n, \Omega) + q(E_1 \otimes^Y \ldots \otimes^Y E_n, \bot).$$

This can be cast as an argument scheme in which:

- Premises: $E_1, ..., E_n$ Conclusion: new evidence $E_1 \otimes^Y ... \otimes^Y E_n$ where \otimes^Y is as defined above

The critical questions for this scheme are then:

- 1. Does the evidence indicate that the frame of discernment is made up of consonant, consistent or arbitrary focal subsets?
- 2. Should conflict between evidence be explicitly represented in the mass assignments that result from combination?
- 3. Is there a restricted stochastic process behind the evidence which can be exploited to obtain a more accurate combination?

This scheme can be used in cases where critical question 4 for Dempster's rule cannot be answered positively.

Inagaki's rule: This rule, described in [7], extends Yager's rule by distributing the probability assigned to conflicting evidence in a way that is similar to Dempster's rule⁸. The combination of evidence according to this rule is then:

$$\begin{split} & m(E_1 \otimes^I \dots \otimes^I E_n, C) \\ &= q(E_1 \otimes^Y \dots \otimes^Y E_n, C) + f(E_1 \otimes^I \dots \otimes^I E_n, C) q(E_1 \otimes^Y \dots \otimes^Y E_n, \bot) \end{split}$$

where $C \neq \perp$ and $\Sigma_{C \subseteq \Omega, C \neq \perp} f(E_1 \otimes^I ... \otimes^I E_n, C) = 1$ and the corresponding argument scheme is:

- Premises: $E_1, ..., E_n$
- Conclusion: new evidence $E_1 \otimes^I \ldots \otimes^I E_n$ where \otimes^I is as defined above.

The critical questions for this scheme are:

- 1. Does the evidence indicate that the frame of discernment is made up of consonant, consistent or arbitrary focal subsets?
- 2. Should conflict between evidence be ignored in the mass assignments that result from combination?
- 3. Is there a restricted stochastic process behind the evidence which can be exploited to obtain a more accurate combination?

This scheme can be used in cases where critical questions 4 and 5 for Dempster's rule cannot be answered positively.

⁸ The most general case of Inagaki's rule allows the redistribution to be controlled by a parameter, thus including both Dempster's rule and Yager's rule as special cases. This case is more general than the situation described here.

Rule of uninformative evidence: This is our modified Dempster's rule to handle uninformative evidence⁹. It preserves uninformative evidence and propagates it to the resulting combined evidence:

- For
$$A \neq \Omega$$
:
 $m(E_1 \otimes^{UI} \dots \otimes^{UI} E_n, A) = [\Pi_{i=1}^n (1 - m(E_i, \Omega))]m(E_1 \otimes^D \dots \otimes^D E_n, A)$
- For $A = \Omega$:
 $m(E_1 \otimes^{UI} \dots \otimes^{UI} E_n, A) = 1 - \Sigma_{B \neq \Omega} m(E_1 \otimes^{UI} \dots \otimes^{UI} E_n, B).$

It only takes into account the informative evidence in the combination and discounts the combined evidence with informative mass in both pieces of evidence. This can be cast as an argument scheme in which:

- Premises: $E_1, ..., E_n$
- Conclusion: new evidence $E_1 \otimes^{UI} \dots \otimes^{UI} E_n$ where \otimes^{UI} is as defined above

The critical questions for this scheme are then:

- 1. Does the evidence contain uninformative evidence on Ω ?
- 2. Should uninformative evidence be explicitly removed in the mass assignments that result from combination?
- 3. Should conflict between evidence be explicitly represented in the mass assignments that result from combination?
- 4. Is there a restricted stochastic process behind the evidence which can be exploited to obtain a more accurate combination?
- 5. Can any of the input evidence complement the uncertainty?

Zhang's center combination rule: We adapt the rule of [33] based on two frames of discernment S and T from two disjoint sub-language \mathcal{L}_S (based on predicate symbols $\mathcal{P}_S \subset \mathcal{P}$) and \mathcal{L}_T (based on predicate symbols $\mathcal{P}_T \subset \mathcal{P}$) of \mathcal{L} . It assumes that we are concerned with the truth of sentences in \mathcal{L}_T but we only have evidence expressed in \mathcal{L}_S and in $\mathcal{L}_S \cup \mathcal{L}_T$. For $A_T \in \mathcal{L}_T$, we are given two pieces of evidence E_1 in \mathcal{L}_S and E_2 in $\mathcal{L}_S \cup \mathcal{L}_T$. We combine these using:

$$b(E_1 \otimes^Z E_2, A_T) = \sum_{B_S \in E_1 \text{ such that } B_S \wedge A_T \in E_2} k \cdot m(E_1, B_S) \cdot m(E_2, B_S \wedge A_T)$$

where k is a re-normalization factor independent of $m(E_1, \cdot)$ and $m(E_2, \cdot)$, which is used to re-normalize $m(E_1 \otimes^Z E_2, \cdot)$ into a valid mass assignment. Here the evidence in E_2 acts as the logical constraint of the two frames in the original Zhang's rule. The corresponding argument scheme is:

- Premises: E_1 and E_2
- Conclusion: new evidence $E_1 \otimes^Z E_2$ where \otimes^Z is as defined above.

 $^{^9}$ Uninformative evidence is probability mass assigned to $\varOmega,$ the most general element in the frame of discernment.

The critical questions are:

- 1. Does the assumed structure on $\mathcal{L}_S, \mathcal{L}_T$ and $\mathcal{L}_S \cup \mathcal{L}_T$ exist in E_1 and E_2
- 2. Are we sure that E_2 is not disjoint from E_1 thus giving no further information regarding the focal sets in \mathcal{L}_S from the evidence in E_T
- 3. Is there a restricted stochastic process behind the evidence which can be exploited to obtain a more accurate combination.

This scheme can be used in cases where critical question 1 for Dempster's rule cannot be answered positively. It has a more detailed structural accounting of the focal elements. Zhang's rule is especially useful in transforming the evidence from a source domain \mathcal{L}_S into a targeting domain \mathcal{L}_T with the connection evidence in their super domain $\mathcal{L}_S \cup \mathcal{L}_T$.

Dubois and Prade's disjunctive consensus rule: This rule, as the name suggests, performs a combination that determines the consensus from disjunctive evidence:

$$m(E_1 \otimes^{DP} E_2, C) = \Sigma_{C=A \vee B} m(E_1, A) m(E_2, B).$$

The corresponding argument scheme is:

- Premises: E_1 and E_2 Conclusion: new evidence $E_1 \otimes^{DP} E_2$ where \otimes^{DP} is as defined above.

The critical questions are then:

- 1. Is it the case that double counting of the focal sets of the evidence occurs?
- 2. Is there a restricted stochastic process behind the evidence which can be exploited to obtain a more accurate combination?

This scheme can be used in cases where critical question 4 for Dempster's rule cannot be answered positively.

Mixing-or-average rule: When the Dempster-Shafer structure underlying the evidence of the premises is equivalent to the probability structure that can be obtained by averaging the probability structures corresponding to the evidence of individual premises, the combination is:

$$m(E_1 \otimes^M \dots \otimes^M E_n, A) = \frac{1}{n} \sum_{i=1}^n w_i \cdot m(E_i, A).$$

where $A \in E_1 \cup E_2 \cup ... \cup E_n$ and w_i is the weight assigned to reliability of the corresponding piece of evidence. The corresponding argument scheme is:

- Premises: E_1, \ldots, E_n
- Conclusion: new evidence $E_1 \otimes^M \dots \otimes^M E_n$ with the combination rule \otimes^M as defined above

The critical questions are:

- 1. Do the assigned weights not reflect the nature of the input evidence?
- 2. Is there a restricted stochastic process behind the evidence which can be exploited to obtain a more accurate combination?

This scheme can also be used in cases where critical question 4 for Dempster's rule cannot be answered positively.



Fig. 3. The example scenario

4 Example

To demonstrate the use of the argument schemes identified above, we present the following example which is loosely based on Operation Anaconda [11] and depicted in Figure 3. In this example, a decision is being made about whether to carry out an operation in which a combat team will move into a mountainous region to try to apprehend a high value target (HVT) believed to be in a village in the mountains.

We have the following information. If there are enemy fighters in the area, then an HVT is likely to be in the area. If there is a HVT in the area, and the mission will be safe, then the mission should go ahead. If the number of enemy fighters in the area is too large, the mission will not be safe. UAVs that have flown over the area have provided images that appear to show the presence of a significant number of camp fires, indicating the presence of enemy fighters. The quality of the images from the UAV is not very good, so they are not highly trusted. A reconnaissance team that infiltrated the area saw a large number of vehicles in the village that the HVT is thought to be inhabiting. Since enemy fighters invariably use vehicles to move around, this is evidence for the presence of many enemy fighters. Informants near the combat team base claim that they have been to the area in question and that a large number of fighters are present. In addition we have the default assumption that missions will be safe, because in the absence of information to the contrary we believe that the combat team will be safe.

Thus there is evidence from UAV imaging that sufficient enemy are in the right location to suggest the presence of an HVT. There is also some evidence from informants that there are too many enemy fighters in the area for the mission to be safe. Since informants are paid, their incentive is often to make up what they think will be interesting information and so they are not greatly trusted. However, this conclusion is supported by the findings of the reconnaissance team who are highly trusted.

To capture the scenario, \mathcal{P} includes the predicates described in Table 1. We also assume the following constants. VLM denotes the specific village in the mountain; HVT denotes the specific high value target.

From the scenario, we can extract the following facts in Σ

Predicate	Meaning
apprehend(TargetX, Loc Y)	Apprehend target $Target X$ at location $Loc Y$.
at(ObjectX, LocY)	ObjectX is at location $LocY$.
campFire(LocX)	A significant number of camp fires at location <i>LocX</i> .
enemyFighters(LocX)	A group of enemy fighters appear at location $LocX$.
largeNumEnemyFighters(LocX)	A large number of enemy fighters appear at location $LocX$.
largeNumVehicles(LocX)	A large number of vehicles appear at location $LocX$.
safe(LocX)	LocX is safe.

Table 1. The predicates used in our example, together with their meaning.

- Fact 1 from an UAV:

$$\langle campFire(VLM), E_{UAV} \rangle$$

 $E_{UAV} = \{ campFile(VLM) : 0.2, campFile(VLM) \lor \neg campFile(VLM) : 0.8 \}.$

- Fact 2 from a reconnaissance team:

 $\langle largeNumVehicles(VLM), E_{RECON} \rangle$

$$E_{RECON} = \{ largeNumVehicles(VLM) : 1 \}$$

- Fact 3 from an informant:

 $\langle largeNumEnemyFighters(VLM), E_{INFORM} \rangle$

$$\begin{split} E_{INFORM} &= \{largeNumEnemyFighters(VLM): 0.2, \\ & largeNumEnemyFighters(VLM) \\ & \lor \neg largeNumEnemyFighters(VLM): 0.8 \} \end{split}$$

- Assumption fact 4 encodes the default assumption by indicating total uncertainty about the safety of the mission.

$$\langle safe(VLM), E_{ASSUM} \rangle$$

 $E_{ASSUM} = \{ safe(VLM) \lor \neg safe(VLM) : 1 \}$

We also have the following rules in Δ :

– Rule δ_1 :

$$\frac{campFire(LocX)}{enemyFighters(LocX)}$$

$$E_{\delta_1} = \{ campFire(LocX) \land enemyFighters(LocX) : 0.80, \\ campFire(LocX) \land \neg enemyFighters(LocX) : 0.2 \}$$

– Rule δ_2 :

$$\label{eq:energy} \begin{split} \frac{enemyFighters(LocX)}{at(HVT,LocX)}\\ E_{\delta_2} = \{enemyFighters(LocX) \wedge at(HVT,LocX):1\} \end{split}$$

– Rule δ_3 :

$$\frac{at(HVT, LocX), safe(LocX)}{apprehend(HVT, LocX)}$$
$$E_{\delta_3} = \{safe(LocX) \land apprehend(HVT, LocX) : 1\}$$

– Rule δ_4 :

$$\frac{largeNumEnemyFighters(LocX)}{\neg safe(LocX)}$$

$$E_{\delta_4} = \{ largeNumEnemyFighters(LocX) \land \neg safe(LocX) : 1 \}$$

– Rule δ_5 :

$$\frac{largeNumEnemyFighters(LocX)}{enemyFighters(LocX)}$$

 $E_{\delta_5} = \{largeNumEnemyFighters(LocX) \land enemyFighters(LocX) : 1\}$

– Rule δ_6 :

$$\dfrac{largeNumVehicles(LocX)}{largeNumEnemyFighters(LocX)}$$

$$E_{\delta_6} = \{ largeNumVehicles(LocX) \land largeNumEnemyFighters(LocX) : 0.7, \\ largeNumVehicles(LocX) \land \neg largeNumEnemyFighters(LocX) : 0.3 \}$$

From the UAV evidence campFire(VLM) and the rule $\frac{campFire(LocX)}{enemyFighters(LocX)}$ associated with the specific evidence regarding the UAV's accuracy, we can derive the evidence for enemyFighters(VLM) using Dempster's rule:

 $E_1^D = \{ campFire(VLM) \land enemyFighters(VLM) : 0.8, \neg enemyFighters(VLM) : 0.2 \}.$

which results in a belief, disbelief and uncertainty of $b(E^D, EnemyFighters(VLM)) = 0.8$, $d(E^D, EnemyFighters(VLM)) = 0.2$, and $u(E^D, EnemyFighters(VLM)) = 0$ respectively. Alternatively, we can apply Zhang's rule to obtain the same result, ignoring the concerns with regards to campFire(VLM):

 $E_1^Z = \{enemyFighters(\textit{VLM}): 0.8, \neg\textit{enemyFighters}(\textit{VLM}): 0.2\}.$

However, doing so runs counter to intuition. Since there is uninformative evidence in the evidence from the UAV, the answer to critical question 6 is "no". This leads us to the use of the argument scheme built on the rule of uninformative information, resulting in the following refinement of the combined evidence:

$$\begin{split} E_1^{UI} &= \{ campFire(VLM) \land enemyFighters(VLM) : 0.16, \\ campFire(VLM) \land \neg enemyFighters(VLM) : 0.02, \\ enemyFigthers(VLM) \lor \neg enemyFighters(VLM) : 0.82 \end{split}$$

This results in a belief, disbelief and uncertainty of $b(E^{UI}, enemyFighters(VLM)) = 0.16, d(E^{UI}, enemyFighters(VLM)) = 0.02, and <math>u(E^{UI}, enemyFighters(VLM)) = 0.82.$

Let us assume that the rule of uninformative evidence is accepted at this point. By rule δ_2 , again using the rule of uninformative information, we will further combine evidence for at(HVT, VLM):

$$\begin{split} E_2^{UI} &= \{ campFire(VLM) \land enemyFighters(VLM) \land at(HVT, VLM) : 0.16, \\ & at(HVT, VLM) \lor \neg at(HVT, VLM) : 0.84 \} \end{split}$$

If we assume that this is deemed acceptable, then by including the safety assumption, we obtain the combined evidence for predicate apprehend(HVT, VLM) through rule δ_3 and Dempster's rule:

$$\begin{split} E_3^D &= \{ campFire(VLM) \land enemyFighters(VLM) \land \\ at(HVT, VLM) \land apprehend(HVT, VLM) : 0.16, \\ apprehend(HVT, VLM) \lor \neg apprehend(HVT, VLM) : 0.84 \} \end{split}$$

Note that as the assumption of safety assigns all the mass to the universe Ω , and so by Dempster's rule the conclusion of previous step is preserved. Assume, again, that this is deemed acceptable. We now can conclude that

$$\begin{split} b(E_3^D, apprehend(HVT, VLM)) &= 0.16\\ d(E_3^D, apprehend(HVT, VLM)) &= 0\\ u(E_3^D, apprehend(HVT, VLM)) &= 0.84. \end{split}$$

This gives us a belief of 0.16 regarding the success of carrying out the apprehend mission at the site, but with a relatively very high level of uncertainty 0.84.

Following the same procedure as above, we can introduce the information provided by the reconnaissance team and the informant. From the reconnaissance team's input, we can obtain the following evidence for $\neg safe(VLM)$ using Dempster's rule (the probabilistic mass is intentionally set to 1 to simplify the example; other values could be obtained through reasoning over the applicability of the different schemes).

$$E_{4,RECON}^{D} = \{ largeNumVehicles(VLM) \\ \land largeNumEnemyFighters(VLM) \land \neg safe(VLM) : 1 \}.$$

We can therefore conclude that

$$b(E_4^D, \neg safe(VLM)) = 1$$

$$d(E_4^D, \neg safe(VLM)) = 0$$

$$u(E_4^D, \neg safe(VLM)) = 0.$$

We assume that no critical questions are applicable, and can now apply standard preference-based argumentation semantics [1] (see [24] for a more formal notion of preference-based argumentation semantics with preferences derived from Dempster-Shafer belief, disbelief and uncertainty) to identify valid conclusions. We define these preferences by comparing the belief of $\neg safe(VLM)$ from the reconnaissance team's conclusions with the belief of safe(VLM) from the assumption. As the assumption assigns the probability mass totally to the universe (the uncertainty), the belief in safe(VLM) is 0. The argument for $\neg safe(VLM)$ from the reconnaissance team clearly defeats the assumption, and this undercuts the argument for apprehend(HVT, VLM), defeating this conclusion.

Another line of argumentation towards the conclusion $\neg safe(VLM)$ is from the information provided by the informant. With the rule of uninformative information, we obtain the following evidence:

$$E_{4,INFORM}^{D} = \{ largeNumEnemyFighters(VLM) \land \neg safe(VLM) : 0.2, \\ safe(VLM) \lor \neg safe(VLM) : 0.8 \}$$

After investigating the appropriate critical questions, this is accepted. Even though the belief over $\neg safe(VLM)$ is just 0.2, it can still be used to defeat the safe(VLM) obtained from the assumption. This can therefore not reverse the decision that the action apprehend(HVT, VLM) should be rejected.

Note that the conclusions on largeNumEnemyFighters(VLM) can be propagated by rule δ_5 to also conclude on apprehend(HVT, VLM). However, as the rule of uninformative information will be more appropriate to combine with the safety assumption with the location of the HVT, it will lead to total uncertain about whether to take the action apprehend(HVT, VLM). This will still lead to the rejection of the action.

5 Related work

There are two areas of related work that are relevant to this paper, namely work on argument schemes and work on argumentation that makes use of Dempster-Shafer theory to represent uncertainty.

Argument schemes were born out of the literature on informal logic, and the most influential work in this area, at least as far as work on computational argument is concerned, is the work of Walton et al. [27]. This provides a solid introduction to the use of argument schemes, and catalogues a large number of them. We consider these schemes to be very general in that they are not fitted to a specific domain. There are also a number of works which, like this paper, identify argument schemes for specific domains. For example, [3] and [29] consider argument schemes for legal reasoning, [18] discusses arguments schemes for agent communication, [13] considers argument schemes for decision support, and [25] looks at argument schemes for deliberation in the sense of [28], that is the process by which several entities reach a combined plan for action. Following this line, [15] presents the case for using argument schemes as an alternative to using logic as a means of knowledge representation (again focussing on the legal domain). In addition, we [14] have presented several argument schemes for reasoning about trust.

More recently, researchers have become interested in transforming argument schemes into computational versions to enable them to be used in systems that perform automated reasoning. For example, [2] gives a scheme for practical reasoning that has been widely used to facilitate reasoning about what to do in a number of different problem scenarios. Schemes concerning witness testimony and expert opinion have also been used in computational argument (e.g. [6]), and there have been several attempts to capture argument schemes from legal reasoning in various forms of logic[4, 17, 26]. Most recently, [30] described a functional language for a computational analysis of schemes.

This work also connects to approaches that combine deductive reasoning and Dempster-Shafer, a connection that goes back to [5, 20]. For example, [9] showed that it was possible to associate probability mass with formulae, reason with the formulae, and compute measures like belief in the conclusions of the reasoning. However, this approach has a limited notion of an argument — an argument is just a conjunction of literals — and the work is only concerned with the construction of arguments and the computation of belief. Our notion of an argument is closer to that in the argumentation literature. A more recent approach to combining argumentation and Dempster-Shafer theory is [12], which builds on subjective logic [8], a logic that incorporates measures from Dempster-Shafer theory. [12] established argumentation semantics solely based on the evidence and belief/disbelief/uncertainty measures we also use.

6 Conclusions and Future Work

This paper presented several argument schemes for performing information fusion in the presence of uncertainty, with critical questions aimed at identifying appropriate situations for the application of specific fusion rules. All of our schemes build on top of Dempster-Shafer theory, whose basic *rule of combination*, is a generalisation of Bayes' rule. This rule is not appropriate in a variety of situations, and we therefore utilize additional rules which allow us to deal with varying beliefs in the reliability of information and different ways of handling uncertainty, based on the context of the situation. By representing these rules within argument schemes, we allow a meta-argument process to decide which of the schemes is most appropriate in some specific situation. In doing so, we aim towards constructing a framework for reasoning about, and with, uncertainty.

We intend to extend this work in several ways. First, we intend to describe additional argument schemes with regards to uncertainty, and to investigate links with other formalisms such as Subjective Logic. We also intend to formalise the meta-argument level, formalising the critical questions in such a way so as to enable us to identify the most appropriate scheme(s) to use when reasoning about information. Finally, we intend to incorporate this work with our previous work [10, 22] on revising beliefs in response to external events, observations and inter-agent communication during argumentation.

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