Towards Argumentation with Symbolic Dempster-Shafer Evidence

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Abstract. This paper is concerned with the combination of argumentation with the Dempster-Shafer theory of evidence. In particular, we show how logical elements of evidence, associated with numerical degrees of belief, can be combined into arguments.

1. Introduction

Trust is a mechanism for managing the uncertainty about autonomous entities and the information they deal with. As a result, trust can play an important role in any decentralized system, and particularly in multiagent systems, where agents are often engage in competitive interactions. Following Castelfranchi and Falcone [1], we believe that trust is based on reasons. We interpret this to mean that there is an advantage in clearly identifying the sources of information and relating these to the conclusions drawn from them, the sources and their connections to the conclusion being the reasons. In prior work [15] we have described how to track these relationships using argumentation, and to summarize the resulting connections as a graph. These connections can then be presented to individuals who have to make decisions based on information that comes from acquaintances of varying trustworthiness.

The formal system in [15] combines work on propagating trust through a social network with argumentation, showing how the results of this propagation can be linked to Dung-style [4] argumentation — where the arguments are structured as in [5,12]. The result is a system in which one agent can reason using information from other agents that it knows through the social network, assigning belief to that information depending on how much the agents that are the source of the information are trusted (as in [10]). A key issue in this work is the propagation of numerical measures of trust, which in our work is input from the social network, through the resulting argumentation. The system in [15] uses an approach based on possibility theory [3]. In this paper we consider how we can combine our system of argumentation with the Dempster-Shafer theory of evidence [13], in a way that seamlessly connects with the use of Demspter-Shafer theory to model trust in social networks [7].

2. Basic notation

We start with a predicate language $\mathcal{L}$ based on a set $\mathcal{P}$ of symbols with standard connectives $\wedge$, $\vee$, $\rightarrow$, $\neg$ and standard semantics. We further constrain the domain of any term of a predicate in $\mathcal{P}$ to be finite and no functional symbols are allowed for any term of a predicate in $\mathcal{P}$. In this way, we will have a finite set of grounded predicates. For notational convenience, we also use $\mathcal{P}$ to denote the set of all grounded predicates.

The set of truth assignments to all ground predicates is denoted by $\Omega = 2^{\mathcal{P}}$ where Ω is taken as the *frame of discernment*. Following the standard semantics, every formula $\theta \in \mathcal{L}$ can be interpreted into a subset of truth assignments to $\mathcal{P}$, $\mathcal{I}(\theta) \subseteq \Omega$. Two special symbols for "false" and "true" are FALSE with $\mathcal{I}(\text{FALSE}) = \emptyset$, and TRUE with $\mathcal{I}(\text{TRUE}) = \Omega$.

Two formulae ϕ and φ, denoted by $\phi \equiv \varphi$, are equivalent iff $\mathcal{I}(\phi) = \mathcal{I}(\varphi)$, and an inference rule δ for $\mathcal{L}$ is of the form:

$$\delta = \frac{p_1, ..., p_m}{c} \tag{1}$$

where $p_1, .., p_m, c \in \mathcal{L}$. The p_i are the set of *premises* of the rule, and a specific p_i is denoted by $p_i(\delta)$. c is the *conclusion* of the rule, and is denoted by $c(\delta)$. The set of all valid rules is denoted by Δ.

From this language, we construct a knowledge base $\mathbf{K} = \langle \Sigma, \Delta \rangle$ consisting of a set of formulae and a set of rules for reasoning with the formulae. $\Sigma = \{\langle h, E \rangle\}$ is the set of formulae, where each formula h is associated with some supporting evidence E, and $\Delta = \{\langle \delta, E \rangle\}$ is a the set of rules, where each rule δ is also associated with some supporting evidence. Our key notion is that of the *evidence argument*:

Definition 1 *An* evidence argument *is a pair $\langle h, E \rangle$, where h is a formula in $\mathcal{L}$ and $E = \{e_1, \ldots, e_n\}$ is a set of formulae in $\mathcal{L}$.*

E is called the *supporting evidence* for h, denoted by $E(h)$. Every $e_i \in E(h)$ is a indivisible element of the evidence for h, therefore it is called a *focal element* of the evidence h[1]. It is possible that $\{\langle h, E_1 \rangle, \langle h, E_2 \rangle\} \subseteq \Sigma$ for the same formula $h \in \mathcal{L}$ with two different sets of evidence E_1 and E_2. If such distinguishable repetitions occur, we assume that we can identify different occurrences of h in Σ as different pieces of information with the associated evidence denoted by $E(h)$.

The key idea here is that the evidence associated with a formula $\theta \in \mathcal{L}$ or a rule $\delta \in \Delta$ summarises the data that shows the rule or formula holds. When we reason with the formulae, which we do by using the rules, we will then propagate the evidence, and so obtain the evidence that supports any conclusions. For every pair $\langle h, E \rangle$ it is then the case that:

1. $h = \theta \in \mathcal{L}$ or $h = \delta \in \Delta$; and
2. $E = \{e_1, ..., e_n\}$ is a set of evidence for h such that $e_1, ..., e_n \in \mathcal{L}$, $e_i \not\equiv e_j$ for any $i \neq j$.

[1] The term "focal element" is appropriated from Dempster-Shafer theory [13] since the e_i end up playing the same role as the focal elements do in that theory.

	p	q	
$\mathcal{I}(\neg p \wedge \neg q)$	0	0	
$\mathcal{I}(\neg p \wedge q)$	0	1	
$\mathcal{I}(p \wedge \neg q)$	1	0	$\}\mathcal{I}(h_1)$
$\mathcal{I}(p \wedge q)$	1	1	

(a)

	p	q	
$\mathcal{I}(\neg p \wedge \neg q)$	0	0	
$\mathcal{I}(\neg p \wedge q)$	0	1	$\}\mathcal{I}(h_2)$
$\mathcal{I}(p \wedge \neg q)$	1	0	
$\mathcal{I}(p \wedge q)$	1	1	$\}\mathcal{I}(h_2)$

(b)

Figure 1. Truth tables for Example 1. (a) Truth table for h_1. Obviously, $b(h_1) = m(E_1, p)$ because $\mathcal{I}(p) \subseteq \mathcal{I}(h_1)$, and $d(h_1) = m(E_1, \neg p \wedge q)$ because $\mathcal{I}(\neg p \wedge q) \cap \mathcal{I}(h_1) = \emptyset$. (b) Truth table for h_2. We can see that $b(h_2) = m(E_2, \neg p \wedge q)$ because $\mathcal{I}(\neg p \wedge q) \subseteq \mathcal{I}(h_2)$, $d(h_2) = m(E_2, \neg q)$ because $\mathcal{I}(\neg q) \cap \mathcal{I}(h_2) = \emptyset$, and $u(h_2) = m(E_2, p)$ because $\mathcal{I}(p) \cap \mathcal{I}(h_2) \neq \emptyset$.

In addition we assume the existence of a *probability mass function* $m(E) : E \mapsto [0, 1]$ defined on E which satisfies the constraint:

$$m(E, e_1) + ... + m(E, e_n) = 1$$

and for all $\phi \notin E$, we set $m(E, \phi) = 0$. In other words we associate some measure of belief $m(\cdot)$ wth every item of evidence, with the goal that from these we can calculate a measure for every h.

In Dempster-Shafer theory [13], it is this probability mass that is the focus, and it is the probability mass that constitutes the evidence. In the work we present here, the evidence is a combination of logical statements over which a probability mass can be defined. As in standard Dempster-Shafer theory, we use the probability mass to determine how much certain interesting hypotheses are believed. In our case, these hypotheses are the conclusions of arguments[2].

Definition 2 *Given an evidence argument* $A = \langle h, E \rangle$ *for a formula* $h \in \mathcal{L}$*, the* belief $b(h)$, disbelief $d(h)$*, and the* uncertainty $u(h)$ *of* h *are computed as follows:*

$$b(h) = \Sigma_{\mathcal{I}(e_i) \subseteq \mathcal{I}(h)} m(E, e_i) \qquad b(h) = \Sigma_{e_i \vdash h} m(E, e_i)$$

$$d(h) = \Sigma_{\mathcal{I}(e_i) \cap \mathcal{I}(h) = \emptyset} m(E, e_i) \qquad d(h) = \Sigma_{e_i \vdash \neg h} m(E, e_i)$$

$$u(h) = \Sigma_{\mathcal{I}(e_i) \cap \mathcal{I}(h) \neq \emptyset} m(E, e_i) \qquad u(h) = \Sigma_{e_i \nvdash h \text{ and } e_i \nvdash \neg h} m(E, e_i)$$

In other words (in the formulation on the left above), the belief in h is the sum of the mass of the all focal elements in E that are part of the evidence for h. Equivalently (in the formulation on the right above), the belief in h is the sum of the mass of all the formulae that imply h. Disbelief and uncertainty are similarly defined.

Example 1 *Let* $\langle h_1, E_1 \rangle = \langle p, \{p, \neg p \wedge q\} \rangle$*, where* $m(E_1, p) = 0.4$ *and* $m(E_1, \neg p \wedge q) = 0.6$*. Then, as explained in Table 1(a):*

[2]Though, as we discuss in Section 6, others have considered the connection between Dempster-Shafer theory and logic, none before us have done as we will here focussed so much on the logical structure of the evidence — this is is the reason that we distinguish our work as being about *symbolic* Dempster Shafer theory.

$$b(h_1) = 0.4, d(h_1) = 0.6, \text{ and so } u(h_1) = 0.$$

Let $\langle h_2, E_2 \rangle = \langle q, \{\neg p \wedge q, \neg q, p\} \rangle$*, where* $m(E_2, \neg p \wedge q) = 0.5$*,* $m(E_2, \neg q) = 0.3$*, and* $m(E_2, p) = 0.2$*. Then, as explained in Table 1(b):*

$$b(h_2) = 0.5, d(h_2) = 0.3, \text{ and so } u(h_2) = 0.2.$$

3. Combining evidence

In reasoning about complex situations, we often find that we have to deal with evidence from multiple sources. Dempster-Shafer theory provides several ways to do this. In this paper we explore one, and will expand on others in a longer version of this paper.

Conjunctive combination is the approach used to combine evidence from sources in a way that has the characteristics of a logical and. For example, it is appropriate to use it to fuse data from two sensors which both have to indicate the presence of some object in order for us to infer that the object is there. Suppose one sensor collects evidence of intrusion by detecting movement, a second collects evidence based on temperature, and we need to establish the movement of a warm body to detect intrusion. In this case, conjunctive combination should be adopted to combine evidence from the two sensors.

Given a list of formula $\{p_1, ..., p_k\}$ with a set of independent pieces of evidence $\{E_1, , ..., E_k\}$ respectively, we can combine the evidence into a set of evidence for the whole list $\{p_1, ..., p_k\}$ in a conjunctive way:

$$E(p_1 \wedge p_2 \wedge ... \wedge p_k) = E_1 \otimes E_2 \otimes \ldots \otimes E_k = \{e'\}$$

such that $e' = e_{1,j_1} \wedge e_{2,j_2} \wedge ... \wedge e_{k,j_k}$ where $e_{i,j_i} \in E_i$, and

$$m(E_1 \otimes ... \otimes E_k, e') = \frac{\Sigma_{e' \equiv \bigwedge_{i=1,...,k} e_{i,j_i}} \Pi_{i=1,...,k} m(p_i, e_{i,j_i})}{\Sigma_{\langle e_{1,j_1}, ..., e_{k,j_k} \rangle \in \Pi_{i=1,...,k} E_i} \Pi_{i=1,...,k} m(p_i, e_{i,j_i})},$$

Example 2 (Conjunctive combination) *Let* $E_1 = \{p, \neg p \wedge q\}$ *and* $E_2 = \{\neg p \wedge q, \neg q, p\}$ *with* $m(E_1, p) = 0.4$*,* $m(E_1, \neg p \wedge q) = 0.6$*,* $m(E_2, \neg p \wedge q) = 0.5$*,* $m(E_2, \neg q) = 0.3$ *and* $m(E_2, p) = 0.2$*. Then:*

$$\begin{aligned} & E_1 \otimes E_2 \\ & = \{p \wedge (\neg p \wedge q), p \wedge \neg q, p \wedge p, \neg p \wedge q \wedge \neg p \wedge q, \neg p \wedge q \wedge \neg q, \neg p \wedge q \wedge p\} \\ & = \{\bot, p \wedge \neg q, p, \neg p \wedge q\} \end{aligned}$$

where:

$$\begin{aligned} m(E_1 \otimes E_2, \bot) &= 0.4 \times 0.5 + 0.6 \times 0.3 + 0.6 \times 0.2 = 0.5 \\ m(E_1 \otimes E_2, p \wedge \neg q) &= 0.4 \times 0.3 = 0.12 \\ m(E_1 \otimes E_2, p) &= 0.4 \times 0.2 = 0.08 \\ m(E_1 \otimes E_2, \neg p \wedge q) &= 0.6 \times 0.5 = 0.3 \end{aligned}$$

4. Deductive reasoning with evidence

Given that we want to combine the use of evidence with logical reasoning using rules of the form in (1), the combination rules introduced above do not help us directly. Rather we need to build on them to create combination rules for logical combinations.

Property 3 Not $\neg$. *Given an evidence argument* $\langle h, E\rangle$*, we can derive an evidence argument* $\langle \neg h, E\rangle$ *for* $\neg h$ *such that:*

$$b(\neg h) = d(h) = \Sigma_{\mathcal{I}(e_i)\cap\mathcal{I}(h)=\emptyset} m(E, e_i)$$
$$d(\neg h) = b(h) = \Sigma_{\mathcal{I}(e_i)\subseteq\mathcal{I}(h)} m(E, e_i)$$
$$u(\neg h) = u(h) = \Sigma_{\mathcal{I}(e_i)\cap\mathcal{I}(h)\neq\emptyset} m(E, e_i)$$

In this case h and $\neg h$ share the same evidence, but the belief and disbelief will be computed differently given that evidence. Note that h and $\neg h$ share the same uncertainty.

The proofs of all the properties in this section follow quickly from the definitions introduced above, and are omitted in the interests of space.

Property 4 And $\wedge$. *Given two evidence arguments* $\langle h_1, E_1\rangle$ *and* $\langle h_2, E_2\rangle$ *with independent evidence, we can derive evidence argument* $\langle h_1 \wedge h_2, E\rangle$ *for* $h_1 \wedge h_2$ *where* $E = E_1 \otimes E_2$.

Example 3 *Following Example 1, let* $\langle h_1, E_1\rangle = \langle p, \{p, \neg p \wedge q\}\rangle$*, where* $m(h_1, p) = 0.4$ *and* $m(h_1, \neg p \wedge q) = 0.6$*, and* $\langle h_2, E_2\rangle = \langle q, \{\neg p \wedge q, \neg q, p\}\rangle$*, where* $m(h_2, \neg p \wedge q) = 0.5$*,* $m(h_2, \neg q) = 0.3$*, and* $m(h_2, p) = 0.2$*. Then,*

$$\langle h_1 \wedge h_2, E_1 \otimes E_2\rangle$$
$$= \langle p \wedge q, \{p \wedge \neg p \wedge q, p \wedge \neg q, p, \neg p \wedge q, \neg p \wedge q \wedge \neg q, p \wedge \neg p \wedge q\}\rangle$$
$$= \langle p \wedge q, \{\bot, p \wedge \neg q, p, \neg p \wedge q\}\rangle$$

where $m(E_1 \otimes E_2, \bot) = 0.5$*,* $m(E_1 \otimes E_2, p \wedge \neg q) = 0.12$*,* $m(E_1 \otimes E_2, p) = 0.8$*, and* $m(E_1 \otimes E_2, \neg p \wedge q) = 0.3$*. Then,*

$$b(h_1 \wedge h_2) = 0$$
$$d(h_1 \wedge h_2) = m(E_1 \otimes E_2, \bot) + m(E_1 \otimes E_2, p \wedge \neg q) + m(E_1 \otimes E_2, \neg p \wedge q) = 0.92$$
$$u(h_1 \wedge h_2) = m(E_1 \otimes E_2, p) = 0.08.$$

Property 5 GMP. *Given* m *evidence arguments* $\langle h_1, E_1\rangle$*,* $\langle h_2, E_2\rangle$*,. . . ,* $\langle h_m, E_m\rangle$*, and an evidence argument for a rule:*

$$\langle \delta = \frac{h_1, ..., h_m}{h}, E_\delta\rangle$$

with independent evidence, we can derive an evidence argument $\langle h, E\rangle$ *for* h *where* $E = E_1 \otimes E_2 \otimes \ldots \otimes E_m \otimes E_\delta)$.

Thus both rules combine evidence using the conjunctive combination rule. Note that while GMP has the form of the generalized modus ponens rule, it differs in that our rules δ are defeasible rules, not material implications.

5. Propagating evidence within an argumentation framework

Having shown how to combine evidence during inference, we can go on to consider how evidence is propagated as arguments are constructed. The propagation of evidence allows us to identify conclusions along with the evidence that supports them. This not only allows us to compute the belief in conclusions, but—unlike other approaches which only manipulate the numerical values during reasoning—allows the symbolic evidence itself to be used in reaching further conclusions. Using the framework from [15], we consider an argument to be a graph constructed by chaining elements of Δ:

Definition 6 *A* rule network $\mathcal{R}$ *is a directed hypergraph* $\langle V^r, E^r \rangle$ *where (1) the set of vertices* V^r *are elements of* $\mathcal{L}$*; (2) the set of edges* E^r *are inference rules* δ*; (3) the initial vertices of an edge* $\delta \in E^r$ *are the premises of the rule; and (4) the terminal vertex of that edge is the corresponding conclusion* c*.*

Definition 7 *For a given knowledge base* $\mathbf{K} = \langle \Sigma, \Delta \rangle$*, a rule network* $\mathcal{R} = \langle V^r, E^r \rangle$ *is a* proof network *if and only if every premise of each* $\delta \in E^r$ *is either a member of* Σ *or the conclusion of some* $\delta' \in E^r$*.*

Definition 8 *A* tree argument A *from a knowledge base* $\mathbf{K}$ *and a rule base* Δ *is a pair* $\langle h, E \rangle$ *where* $E = \langle V^r, E^r \rangle$ *is a proof network for* h*, and* h *is the only leaf of* E*.*

Note that a tree argument $\langle h, \langle V^r, E^r \rangle \rangle$ is just an evidence argument, albeit one with some additional structure in terms of the associated graph. This structure places some restrictions on what arguments meet the conditions of Definition 8, and not all evidence arguments will be tree arguments. Given that the construction of a tree argument is equivalent to repeated applications of the GMP rule from Property 5, we can easily obtain a form of soundness result:

Proposition 9 *If an argument* $\langle h, \langle V^r, E^r \rangle \rangle$ *is constructed from a knowledge base* $\langle \Sigma, \Delta \rangle$*, using the Not, And and GMP rules, then the conclusion* h *follows from the application of a sequence of rules from* Δ*, and the premises are grounded on facts in* Σ*.*

We can compute the evidence associated with h from the evidence arguments for the elements of H:

Proposition 10 *If an argument* $\langle h, \langle V^r, E^r \rangle \rangle$ *is constructed from a knowledge base* $\mathbf{K}$*, then:*

$$E(h) = \bigotimes_{p \in P(h)} E(p) \otimes \bigotimes_{\delta \in E^r} E(\delta)$$

This follows immediately from Property 5 and the structure of $\langle h, \langle V^r, E^r \rangle\rangle$.

Finally, the system is complete in the sense that applying GMP will build all the tree arguments that can be built from a knowledge base [3].

6. Related work

This work starts with the association of basic probability mass with elements of a knowledge base and shows how beliefs measures may be derived for arguments constructed from that knowledge base, and then how those measures can be used to construct a preference-based argumentation system. In this way our work connects with existing approaches to preference-based argumentation, but allows the preference order to be established from what the agent knows, rather than assuming the existence of some pre-defined order. (And as discussed in [6], the measures we use here can be learned by the agent).

This work also connects to approaches that combining deductive reasoning and Dempster-Shafer, a connection that goes back to [2,14]. For example, [9] showed that it was possible to associate probability mass with formulae, reason with the formulae, and compute measures like belief in the conclusions of the reasoning. However, this approach has a limited notion of an argument — an argument is just a conjunction of literals — and the work is only concerned with the construction of arguments and the computation of belief. Our notion of an argument is closer to that in the argumentation literature. Further, as the longer version of this paper will show, our approach can be connected to Dungian semantics for argumentation systems, allowing our work to go beyond [9].

A more recent approach to combining argumentation and Dempster-Shafer theory is [11], which builds on subjective logic [8], a logic that incorporates measures from Dempster-Shafer theory. [11] established argumentation semantics solely based on the evidence and belief/disbelief/uncertainty, but its connection to Dung's argumentation semantics is not clear [11, Section 5], and, like [9], the focus is more on establishing the strength of individual arguments. We believe our approach has a stronger connection with the standard argumentation semantics, as well as fitting into our graphical approach to argumentation [15].

7. Summary

This paper has provided the foundations for a system of argumentation that combines logical reasoning and the Dempster-Shafer theory. The system allows arguments to be constructed from formulae in a predicate logic, each of which has a numerical measure associated with it, measures expressed using the Dempster-Shafer theory. These measures are appropriately combined as the formulae are constructed into arguments. The work is set in the context of a wider effort to use argumentation to reason in an environment when sources of information are

[3] The existence of such a completeness result can be seen from the analogy between our rules and Horn clauses, and the completeness of GMP for inference from sets of definite Horn clauses. A proof of this result will be given in the long version of this paper.

of varying trustworthiness. Well-founded approaches to reasoning about the trust in individuals have already been established and several of us have developed an approach that quantifies trust using the Dempster-Shafer theory [7]. Other among us have described how such reasoning about trust can be used as input for argumentation-based reasoning [15]. The work decribed here will allow for a seamless integration by providing a means to propagate the values established by the trust reasoning system through the resulting argumentation.

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