

A Robust Logic for Rule-based Reasoning under Uncertainty

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Abstract

Reasoning with uncertain information is a problem of key importance when dealing with real knowledge. The more information required by the procedure used to handle the knowledge, the higher the probability of failure of the reasoning system. The theory of rough sets is not information intensive and is thus a good basis for reasoning in domains where knowledge is sparse. We present a logic based on rough set theory that is suitable for reasoning under uncertainty, a rough inference rule, and demonstrate its effectiveness in rule-based reasoning.

1. Introduction

Any system designed to reason about the real world must be capable of dealing with uncertain information. This is a consequence of the complexity of the real world and the finite size of the knowledge base that such a system has at its disposal. A number of mathematical formalisms have been developed to cope with uncertainty in knowledge-based systems [4]. These formalisms all suffer from the fact that they are *information intensive*, requiring large amounts of precise information in order to deal with uncertainty. Thus truth values of relations between variables are required in the form of grades of membership and probability distributions. Such values are often unknown, or expensive to obtain, and methods that are not information intensive are often desirable.

2. Rough set theory

Consider a set of elementary objects $\mathbf{A} = \{A_1, \dots, A_n\}$. The members of this set are used to define a set of objects $\mathbf{E} = \{E_1, \dots, E_m\}$ each of the A_i corresponding to a possible attribute of the E_j . Since the specification of the E_j must be based on the A_i alone, it could well be the case that some of the specifications of the E_j are indistinguishable since the values that distinguish them are not identified by the A_i . Thus the use of a finite set \mathbf{A}

implies the existence of an equivalence relation and a consequent partition on \mathbf{E} ; $\mathbf{P} = \{P_1, \dots, P_r\}$ where $\cup P_i = \mathbf{E}$ and $P_i \cap P_j = \emptyset$ for $i \neq j = 1, \dots, r$ and each $P_s = (E_{s_1}, \dots, E_{s_z})$ is an equivalence class. Let $T \in \mathbf{S}$ be an object, whose attributes are T_A/\mathbf{A} , that we wish to describe in terms of the set of partitioned attributes E_i .

$$T^c(\mathbf{P}, \mathbf{E}) = \{e: e \in P_i, P_i \cap T \neq \emptyset\}$$

$$T^e(\mathbf{P}, \mathbf{E}) = \{e: e \in P_i, P_i \cap T = \emptyset\}$$

where $T^c(\mathbf{P}, \mathbf{E})$ is the core of T based on \mathbf{E} and \mathbf{P} , and $T^e(\mathbf{P}, \mathbf{E})$ is the envelope. The pair $[T^c(\mathbf{P}, \mathbf{E}), T^e(\mathbf{P}, \mathbf{E})]$ is a *rough set* [3].

We can determine the degree to which logical combinations of roughly defined objects may themselves be defined. Logical operations such as disjunction, implication and negation may be defined in terms of set operations on the core and envelope of the objects concerned:

$$\begin{aligned} (A \vee B)^c &\alpha A^c \cap B^c & (A \wedge B)^c &= A^c \cap B^c \\ (A \vee B)^e &= A^e \cap B^e & (A \wedge B)^e &/ A^e \cap B^e \\ (\neg A)^c &= \sim(A^e) & (A \rightarrow B)^c &\alpha \sim A^e \cap B^c \\ (\neg A)^e &= \sim(A^c) & (A \rightarrow B)^e &= \sim A^c \cap B^e \end{aligned}$$

where $\sim A$ is the complement of set A .

3. A logic of rough truth values

Consider a Boolean algebra of propositions (P, \vee, \wedge, \neg) . We define a rough measure R on P such that $p \in P$, $R(p) = [p^c, p^e]$. $R(p)$ may be related to the truth of the proposition p by careful choice of the set \mathbf{A} [2]. We can distinguish the following limiting values of $R(p)$, for $\emptyset \prec X \prec \mathbf{A}$, and $\emptyset \prec Y \prec \mathbf{A}$, which form a lattice, ordered by set inclusion:

$$\begin{array}{cccccc} [\emptyset, \emptyset] & / & [\emptyset, X] & / & [\emptyset, \mathbf{A}] & / & [Y, \mathbf{A}] & / & [\mathbf{A}, \mathbf{A}] \\ \text{false} & & \text{roughly false} & & \text{unknown} & & \text{roughly true} & & \text{true} \end{array}$$

We then introduce a rough truth measure RV which identifies which of these five values each $p \in P$ takes on.

The advantages of such a measure are its extreme simplicity and robustness. The logical behaviour of the measure RV is established from set operations on the measure R for relevant propositions. For example:

$$RV(p \& q) = \max(RV(p), RV(q))$$

We can adapt reasoning patterns of classical logic such as modus ponens for this rough valued logic:

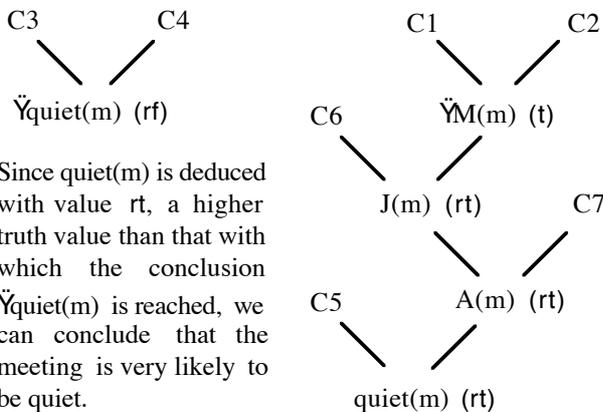
$$\begin{aligned} RV(p \rightarrow q) &= \alpha \\ RV(p) &= \beta \end{aligned}$$

$$\alpha \geq RV(q) \geq \min(\alpha, \beta)$$

4. An example

Consider the following simple example, based on the 'meeting problem' [1], from which we can deduce two conflicting results:

- C1. If Robert comes to a meeting, then Mary does not come. $R(x) \rightarrow \neg M(x)$ (t).
- C2. Robert is coming to the meeting tomorrow. $R(m)$ (t).
- C3. If Beatrix comes to the meeting tomorrow, it is unlikely to be quiet. $B(m) \rightarrow \neg \text{quiet}(m)$ (rt).
- C4. Beatrix may come to the meeting tomorrow. $B(m)$ (rf).
- C5. If Albert comes to a meeting, then it is almost certain that the meeting will be quiet. $A(x) \rightarrow \text{quiet}(x)$ (rt).
- C6. If MARY does not come to a meeting, then John almost definitely comes. $\neg M(x) \rightarrow J(x)$ (rt).
- C7. If John comes to the meeting tomorrow, it is rather likely that Albert will come. $J(m) \rightarrow A(m)$ (rt).



5. Rule-based reasoning

In many domains detailed numerical estimates of the certainty of rules and facts may be impossible to obtain, and the inference mechanism adopted must be capable of dealing with vague estimates.

The knowledge base of a typical rule-based system consists of a series of rules of the form 'if p then q' with a certainty value attached to each. In forward chaining inference starts with one or more facts, also with an associated certainty, which match the antecedents of particular rules. These rules are fired to obtain their consequents, with the certainty of the consequent being determined by a combination of the certainties of rule and

antecedent, and the consequents used to fire more rules.

If we assume that rules of the form 'if p then q' are translated by use of material implication into logical statements of the form $p \rightarrow q$, then the mechanism of forward chaining is the modus ponens. We can establish when consequent values of unknown certainty will be generated, taking the lower bound on the value of q:

t	

p	
	t

No combinations of values of antecedent and rule other than those shown, have either antecedent, consequent, or rule valued as unknown. The truth tables show that the value of the consequent can be determined if the value of the rule and the antecedent is not unknown, or, if one has an unknown value the other has the value false (f) or roughly false (rf). Thus the logic has the ability to absorb vague data.

6. Conclusion

We have presented a symbolically quantified logic for reasoning under uncertainty that is based upon the concept of rough sets. This mathematical model provides a simple yet sound basis for a robust reasoning system. We have supplied a rule of inference analogous to modus ponens, and shown how it might be used by a reasoning system to determine the most likely outcome under conditions of uncertain knowledge. An analysis of the robustness of the logic in rule-based reasoning has also been presented.

References

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