Argumentation and qualitative decision making

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Abstract. This paper presents a system of argumentation which captures the kind of reasoning possible in qualitative probabilistic networks, including reasoning about expected utilities of actions and the propagation of synergies between actions. In these latter regards it is an extension of our previous work on systems of argumentation which reason with qualitative probabilities.

1 Introduction

In the last few years there have been a number of attempts to build systems for reasoning under uncertainty that are of a qualitative nature—that is they use qualitative rather than numerical values, dealing with concepts such as increases in belief and the relative magnitude of values. Three main classes of system can be distinguished—systems of abstraction, infinitesimal systems, and systems of argumentation. In systems of abstraction, the focus is mainly on modelling how the probabilities of hypotheses change when evidence is obtained. Such systems provide an abstract version of probability theory, known as qualitative probabilistic networks (QPNs) [25], which is sufficient for planning [25], explanation [5] and prediction [18] tasks. Infinitesimal systems deal with beliefs that are very nearly 1 or 0, providing formalisms that handle order of magnitude probabilities. Such systems may be used for diagnosis [4] and have been extended with infinitesimal utilities to give complete decision theories [21, 26]. Systems of argumentation are based on the idea of constructing logical arguments for and against formulae. Such systems of have been applied to problems such as diagnosis, protocol management and risk assessment [11], as well as handling inconsistent information [1], and providing a framework for default reasoning [10, 16].

In a previous paper [17], we provided a hybridisation of the argumentation and abstraction approaches by introducing a system called the qualitative probabilistic reasoner ($\mathcal{QPR}$) which constructed arguments about how probabilities change. In this paper we extend the kind of reasoning possible using $\mathcal{QPR}$ to deal with information about changes in utilities, thus providing a qualitative utility reasoner $\mathcal{QUR}$ which provides an abstraction of classical decision making rather than just of probability theory and so captures the kind of reasoning possible in QPNs.

2 The logical language

This section introduces the language used by our system. We build on the language of $\mathcal{QPR}$ by introducing notions of utility, but to save space here we only deal with non-categorical changes in value, simplify the language by not dealing with logical conjunction, restrict ourselves to causally directed reasoning, and cut the discussion of those features drawn from $\mathcal{QPR}$. A fuller account is contained in [19].

2.1 Basic concepts

We start with a set of atomic propositions $\mathcal{L}$ which includes the symbol V. We also have a set of connectives $\{\neg, \rightarrow, \uplus, \stackrel{v}{\rightarrow}, \leadsto, \stackrel{v}{\leadsto}\}$, and the following set of rules for building the well-formed formulae (*wff* s) of the language.

1. If $l \in \mathcal{L}$ then l is a well-formed simple formula (*swff*).
2. If l is an *swff*, then $\neg l$ is an *swff*.
3. If l and m are *swff* s, then $l \rightarrow m$ is a well-formed implicational formula (*iwff*).
4. If l is an *swff*, then $l \stackrel{v}{\rightarrow} V$ is a well-formed value formula (*vwff*).
5. If l, m and n are *swff* s, then $l \uplus m \leadsto n$ and $l \uplus m \stackrel{v}{\leadsto} V$ are well-formed synergistic formulae (*ywff* s).

We denote the set of all *swff* s which can be defined using $\mathcal{L}$ by $\mathcal{S}_{\mathcal{L}}$, while $\mathcal{I}_{\mathcal{L}}$, $\mathcal{Y}^{+}_{\mathcal{L}}$ and $\mathcal{V}_{\mathcal{L}}$ denote the corresponding sets of *iwff* s, *ywff* s and *vwff* s respectively. The set of all *wff* s which can be defined using $\mathcal{L}$ is $\mathcal{W} = \mathcal{S}_{\mathcal{L}} \cup \mathcal{I}_{\mathcal{L}} \cup \mathcal{Y}^{+}_{\mathcal{L}} \cup \mathcal{V}_{\mathcal{L}}$. $\mathcal{W}$ may then be used to build up a database Δ where every item $d \in \Delta$ is a triple $(i : l : s)$ in which i is a token uniquely identifying the database item (for convenience we will use the letter 'i' as an anonymous identifier), $l \in \mathcal{W}$, and s gives information about the probability of l. In particular we take triples $(i : l : \uparrow)$ to denote the fact that $\Pr(l)$ increases (due to some piece of evidence), and similar triples $(i : l : \downarrow)$, to denote the fact that $\Pr(l)$ decreases. Triples $(i : l : \leftrightarrow)$ denote the fact that $\Pr(l)$ is known to neither increase nor decrease, and triples $(i : l : \updownarrow)$ denote we don't know whether $\Pr(l)$ increases or decreases. It should be noted that the triple $(i : l : \uparrow)$ indicates that $\Pr(l)$ either goes up, or does not change—this inclusive interpretation of the notion of "increase" is taken from QPNs—and of course a similar proviso applies to $(i : l : \downarrow)$.

2.2 Non-material implication

Now, "$\rightarrow$" does not represent material implication but a connection between the probabilities of antecedent and consequent. We take an *iwff*, which we will also call an "implication", to denote that the antecedent of the *iwff* has a probabilistic influence on the consequent. Thus we are not concerned with the probability *of* the *iwff*, but what the *wff* says *about* the probabilities of its antecedent and consequent. More precisely we take the triple $(i : a \rightarrow c : +)$ to denote the fact that:

$$\Pr(c|a, X) \geq \Pr(c|\neg a, X) \quad (1)$$

for all $X \in \{x, \neg x\}$ for which there is a triple $(i : X \rightarrow c : s)$ (where s is any sign) or $(i : c \rightarrow X : s)$. The effect of the X in this inequality is to ensure that the restriction holds whatever is known about formulae other than c and a—whatever the probabilities of a and c, the constraint on the conditional probabilities holds. It is possible to think of this as meaning that there is a constraint on the probability distribution over the formulae c and a such that an increase in the probability of a entails an increase in the probability of c. The triples $(i : a \rightarrow c : -)$ and $(i : a \rightarrow c : 0)$ denote that (1) holds with $\geq$ replaced by $\leq$ and $=$ respectively. We also have implications such as $(i : a \rightarrow c : ?)$ which denotes the fact that the relationship between $\Pr(c|a, X)$ and $\Pr(c|\neg a, X)$ is not known, so that if the probability of a increases it is not possible to say how the probability of c will change.

With this interpretation, implications correspond to qualitative influences in QPNs, and, as is the case in all probabilistic networks, [20] are causally directed in the sense that the antecedent is a cause of the consequent. This restriction is necessary to ensure that $\mathcal{QUR}$ is sound, for the reasons discussed in [17].

2.3 Values

The proposition V denotes the same thing as the value node in an influence diagram [13]—that is the utility of the decision maker. It can be used, just like any other *swff* to form triples, and these denote a change in utility. Thus $(i : V : \uparrow)$ means that utility increases. $\mathcal{QUR}$ also makes use of triples based on *vwff*s, and a *vwff* $(i : a \stackrel{v}{\rightarrow} V : +)$ is taken to mean:

$$U(a, X) \geq U(\neg a, X) \quad (2)$$

where, as before, X ranges across all other propositions which affect V, in this case all other propositions which are antecedents of *vwff*s. The meaning of the triple, as given by (2), is that a positively influences utility. Similar triples with sign $-$ and 0 denote that (2) holds with $\geq$ replaced by $\leq$ and $=$ respectively, and we use the sign ? to denote situations in which the relationship is not known.

2.4 Synergy

Being able to handle synergy relations is an important part of any qualitative probabilistic system. A detailed discussion of synergy is beyond the scope of this paper[1], but, informally, there is synergy between two variables with respect to a third if a change in the value of one of the first two has an effect on the relationship between the second and the third. Thus, A and B have a synergistic relationship with respect to C, if an increase in the probability of A changes the

[1] See [5, 6, 25] for detail on the subject.

strength of the probabilistic influence between B and C. In our system synergies are represented by formulae such as $a \uplus b \rightsquigarrow c$ which represents the synergy which exists between a and b with respect to c. Such synergistic formulae form the basis of triples such as $(i : a \uplus b \rightsquigarrow c : +)$ in just the same way as simple and implicational formulae do, but with yet another denotation. In particular, $(i : a \uplus b \rightsquigarrow c : +)$ denotes the fact that:

$$\Pr(c|a, b, X) + \Pr(c|\neg a, \neg b, X) \geq \Pr(c|\neg a, b, X) + \Pr(c|a, \neg b, X) \tag{3}$$

where as ever, X ranges across all other formulae such that there are triples $(i : X \rightarrow c : s)$ or $(i : c \rightarrow X : s)$. Similarly, $(i : a \uplus b \rightsquigarrow c : -)$ and $(i : a \uplus b \rightsquigarrow c : 0)$ denote that (3) holds with $\geq$ replaced by $\leq$ and $=$ respectively. As with the case of implications, synergies have sign ? when the relationship is not known. These synergy expressions are [18, 25] precisely the conditions necessary and sufficient to capture the fact that a change in $\Pr(a)$ has an effect on the influence of $\Pr(b)$ on $\Pr(c)$. It is perfectly possible to have synergies with respect to the value node represented by triples such as $(i : a \uplus b \overset{v}{\rightsquigarrow} V : +)$. This latter denotes the fact that:

$$U(a, b, X) + U(\neg a, \neg b, X) \geq U(\neg a, b, X) + U(a, \neg b, X) \tag{4}$$

where X is as before. Similarly, $(i : a \uplus b \rightsquigarrow V : -)$ and $(i : a \uplus b \rightsquigarrow V : 0)$ denote that (4) holds with $\geq$ replaced by $\leq$ and $=$ respectively. Note that all synergies are symmetrical, and that the synergies we deal with here are known as *additive* synergies. In contrast, $\mathcal{QPR}$ [17] deals only with *product* synergies.

3 The proof theory

The previous section introduced a language for describing probabilistic influences between formulae. For this to be useful, we need to give a mechanism for taking sentences in that language and using them to derive new sentences.

3.1 Arguments

We derive new sentences using the consequence relation $\vdash_{QU}$ which is defined in Figure 1. The definition is in terms of Gentzen-style proof rules where the antecedents are written above the line and the consequence is written below. The consequence relation operates on a database consisting of the kind of triples introduced in the previous section and derives *arguments* about formulae from them. There are two types of argument[2]:

Definition 1. *An* influence argument *for a well-formed formula p from a database Δ is a triple $S(p, G, s)$ such that $\Delta \vdash_{QU} S(p, G, s)$*

The *sign* s of an influence argument denotes something about the change in the probability of p which can be inferred given the *grounds* G—the elements of the database used in the derivation of p.

[2] The use of S and Y to denote the different types is taken from [25].

S-rules

$$\text{Ax1}\frac{}{\Delta \vdash_{QU} S(St, \{i\}, Sg)}\ (i : St : Sg) \in \Delta,\ St \in \mathcal{S}_{\mathcal{L}} \cup \mathcal{I}_{\mathcal{L}} \cup \mathcal{V}_{\mathcal{L}}$$

$$\neg\text{-E}\frac{\Delta \vdash_{QU} S(\neg St, G, Sg)}{\Delta \vdash_{QU} S(St, G, \mathsf{neg}(Sg))}$$

$$\neg\text{-I}\frac{\Delta \vdash_{QU} S(St, G, Sg)}{\Delta \vdash_{QU} S(\neg St, G, \mathsf{neg}(Sg))}$$

$$\rightarrow\text{-E}\frac{\Delta \vdash_{QU} S(St, G, Sg) \quad \Delta \vdash_{QU} S(St \rightarrow St', G', Sg')}{\Delta \vdash_{QU} S(St', G \cup G', \mathsf{imp}_{\mathsf{elim}}(Sg, Sg'))}$$

$$\stackrel{v}{\rightarrow}\text{-E}\frac{\Delta \vdash_{QU} S(St, G, Sg) \quad \Delta \vdash_{QU} S(St \stackrel{v}{\rightarrow} V, G', Sg')}{\Delta \vdash_{QU} S(V, G \cup G', \mathsf{val}_{\mathsf{prop}}(Sg, Sg'))}$$

Y-rules

$$\text{Ax2}\frac{}{\Delta \vdash_{QU} Y((St'', St, St'), \{i\}, Sg)}\ (i : St \uplus St' \leadsto St'' : Sg) \in \Delta$$

$$\text{Ax3}\frac{}{\Delta \vdash_{QU} Y((V, St, St'), \{i\}, Sg)}\ (i : St \uplus St' \stackrel{v}{\leadsto} V : Sg) \in \Delta$$

$$\text{Y-I1}\frac{\Delta \vdash_{QU} S(St \rightarrow St', G, Sg) \quad \Delta \vdash_{QU} Y((St, St'', St'''), G', Sg')}{\Delta \vdash_{QU} Y((St', St'', St'''), G \cup G', \mathsf{syn}_{\mathsf{prop}}(Sg, Sg'))}$$

$$\text{Y-I2}\frac{\Delta \vdash_{QU} S(St \rightarrow St', G, Sg) \quad \Delta \vdash_{QU} Y((St'', St', St'''), G', Sg')}{\Delta \vdash_{QU} Y((St'', St, St'''), G \cup G', \mathsf{syn}_{\mathsf{prop}}(Sg, Sg'))}$$

$$\text{Y-I3}\frac{\Delta \vdash_{QU} S(St \rightarrow St', G, Sg) \quad \Delta \vdash_{QU} Y((St'', St''', St'), G', Sg')}{\Delta \vdash_{QU} Y((St'', St, St'''), G \cup G', \mathsf{syn}_{\mathsf{prop}}(Sg, Sg'))}$$

Fig. 1. The consequence relation $\vdash_{QU}$

Definition 2. *A* synergy argument *for a well-formed formula p from a database Δ is a triple $Y((p, q, r), G, s)$ such that $\Delta \vdash_{QU} Y((p, q, r), G, s)$*

Such an argument indicates that q and r have a synergistic effect on p. The sign gives the synergy of q on the relation between r and p, or, equivalently, the synergy of r on the relation between q and p.

To see how the idea of an argument fits in with the proof rules in Figure 1, consider the rules 'Ax1', and '$\rightarrow$-E'. The first says that from a triple $(i : l : s)$ it is possible to build an argument for l which has sign s and a set of grounds $\{i\}$ (the grounds thus identify which elements from the database are used in the derivation). The rule is thus a kind of bootstrap mechanism to allow the elements of the database to be turned into arguments to which other rules can

s	$\uparrow$	$\leftrightarrow$	$\downarrow$	$\updownarrow$
$\mathsf{neg}(s)$	$\downarrow$	$\leftrightarrow$	$\uparrow$	$\updownarrow$

$\mathsf{imp}_{\mathsf{elim}}$	$+$	0	$-$	$?$
$\uparrow$	$\uparrow$	$\leftrightarrow$	$\downarrow$	$\updownarrow$
$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$
$\downarrow$	$\downarrow$	$\leftrightarrow$	$\uparrow$	$\updownarrow$
$\updownarrow$	$\updownarrow$	$\leftrightarrow$	$\updownarrow$	$\updownarrow$

$\mathsf{val}_{\mathsf{prop}}$	$+$	0	$-$	$?$
$\uparrow$	$\uparrow$	$\leftrightarrow$	$\downarrow$	$\updownarrow$
$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$
$\downarrow$	$\downarrow$	$\leftrightarrow$	$\uparrow$	$\updownarrow$
$\updownarrow$	$\updownarrow$	$\leftrightarrow$	$\updownarrow$	$\updownarrow$

Fig. 2. The functions neg, $\mathsf{imp}_{\mathsf{elim}}$ and $\mathsf{val}_{\mathsf{prop}}$.

be applied. The second rule '$\rightarrow$-E' can be thought of as analogous to modus ponens. From an argument for a and an argument for $a \rightarrow c$ it is possible to build an argument for c once the necessary book-keeping with grounds and signs has been carried out. The proof procedure used here has an important difference from other similar logical proof systems which stems from the fact that $\mathcal{QUR}$ is dealing with probability values (albeit changes in probability) rather than just truth and falsity as is the case in classical logic. In logic, once there is a valid proof for a formula, the formula is known to be true. Here we may have several arguments which suggest different things about the probability of a formula and it is necessary to establish all the arguments and then combine them.

3.2 Combination functions

In order to apply the proof rules to build arguments, it is necessary to supply the functions used in Figure 1 to combine signs. Broadly speaking, all these functions are exactly those introduced by Wellman [25] for the analogous operations in QPNs[3]. The rules for handling negation are applicable only to *swff*s and permit negation to be either introduced or eliminated by altering the sign, for example allowing $(i : \neg a : \uparrow)$ to be rewritten as $(i : a : \downarrow)$. This leads to the definition of neg:

Definition 3. *The function* $\mathsf{neg} : Sg \in \{\uparrow, \leftrightarrow, \downarrow, \updownarrow\} \mapsto Sg' \in \{\uparrow, \leftrightarrow, \downarrow, \updownarrow\}$ *is specified in Figure 2.*

To deal with implication we need the function $\mathsf{imp}_{\mathsf{elim}}$ to establish the sign of formulae generated by the rule of inference $\rightarrow$-E. This means that $\mathsf{imp}_{\mathsf{elim}}$ is used to combine the change in probability of a formula a, say, with the constraint that the probability of a imposes upon the probability of another formula c.

Definition 4. *The function* $\mathsf{imp}_{\mathsf{elim}} : Sg \in \{\uparrow, \leftrightarrow, \downarrow, \updownarrow\} \times Sg' \in \{+, 0, -, ?\} \mapsto Sg'' \in \{\uparrow, \leftrightarrow, \downarrow, \updownarrow\}$ *is specified in Figure 2.*

We also need the function $\mathsf{val}_{\mathsf{prop}}$ which makes it possible to determine the changes in utility.

Definition 5. *The function* $\mathsf{val}_{\mathsf{prop}} : Sg \in \{\uparrow, \leftrightarrow, \downarrow, \updownarrow\} \times Sg' \in \{+, 0, -, ?\} \mapsto Sg'' \in \{\uparrow, \leftrightarrow, \downarrow, \updownarrow\}$ *is specified in Figure 2.*

[3] The reason our notation differs is to allow our system to be extended to handle categorical information exactly as in [17].

$\mathsf{syn}_{\mathsf{prop}}$	$+$	0	$-$	$?$
$+$	$+$	0	$-$	$?$
0	0	0	0	$?$
$-$	$-$	0	$+$	$?$
$?$	$?$	$?$	$?$	$?$

$\mathsf{flat}_{\mathsf{S}}$	$\uparrow$	$\leftrightarrow$	$\downarrow$	$\updownarrow$
$\uparrow$	$\uparrow$	$\uparrow$	$\updownarrow$	$\updownarrow$
$\leftrightarrow$	$\uparrow$	$\leftrightarrow$	$\downarrow$	$\updownarrow$
$\downarrow$	$\updownarrow$	$\downarrow$	$\downarrow$	$\updownarrow$
$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$

$\mathsf{flat}_{\mathsf{Y}}$	$+$	0	$-$	$?$
$+$	$+$	$+$	$?$	$?$
0	$+$	0	$-$	$?$
$-$	$?$	$-$	$-$	$?$
$?$	$?$	$?$	$?$	$?$

Fig. 3. Synergy propagation $\mathsf{syn}_{\mathsf{prop}}$ and flattening functions $\mathsf{flat}_{\mathsf{S}}$ and $\mathsf{flat}_{\mathsf{Y}}$.

This function is virtually identical to $\mathsf{imp}_{\mathsf{elim}}$, differing only in that it combines a change in probability with a utility to give a change in expected utility, whereas $\mathsf{imp}_{\mathsf{elim}}$ derives a change in probability from a change in probability and a relationship between probabilities. We also need the function $\mathsf{syn}_{\mathsf{prop}}$ in order to be able to reason with synergies.

Definition 6. *The function* $\mathsf{syn}_{\mathsf{prop}} : Sg \in \{+, 0, -, ?\} \times Sg' \in \{+, 0, -, ?\} \mapsto Sg'' \in \{+, 0, -, ?\}$ *is specified in Figure 3.*

These functions are sufficient to apply $\vdash_{QU}$ to build both influence and synergy arguments.

3.3 Flattening

In general it is possible to build several arguments for a single proposition. To get firm conclusions we need to *flatten* all the arguments for a proposition to get a single sign which tells us the combined change in the probability of that proposition. We can describe this in terms of a function $\mathsf{Flat}_{\mathsf{S}}(\cdot)$ which maps from a set of influence arguments $\mathbf{A_S}$ for a proposition St built from a particular database Δ to the pair of that proposition and some overall measure of validity:

$$\mathsf{Flat}_{\mathsf{S}} : \mathbf{A_S} \mapsto \; S\langle St, v\rangle$$

where $\mathbf{A_S}$ is the set of all influence arguments which are concerned with St, that is:

$$\mathbf{A_S} = \{S(St, G_i, Sg_i) \mid \Delta \vdash_{QU} S(St, G_i, Sg_i)\}$$

and v is the result of a suitable combination of the Sg that takes into account the structure of the arguments. Since in the precise case we are considering here, the structure is unimportant (though in very similar cases it must be taken into consideration [17]) we can ignore the grounds and define v as:

$$v = \mathsf{flat}_{\mathsf{S}}(\{Sg_i \mid (St, G_i, Sg_i) \in \mathbf{A_S}\})$$

where $\mathsf{flat}_{\mathsf{S}}$ is as defined in Figure 3. We can formalise a similar notion for synergy arguments in terms of a function $\mathsf{Flat}_{\mathsf{Y}}(\cdot)$ which maps from a set of synergy arguments $\mathbf{A_Y}$ for a proposition St to the pair of that synergistic relationship and some overall measure of validity:

$$\mathsf{Flat}_{\mathsf{Y}} : \mathbf{A_Y} \mapsto \; Y\langle (St, St', St''), v\rangle$$

where $\mathbf{A_Y}$ is the set of all synergy arguments which give the synergistic effect of St' and St'' on St:

$$\mathbf{A_Y} = \{Y((St, St', St''), G_i, Sg_i) \mid \Delta \vdash_{QU} Y((St, St', St''), G_i, Sg_i) \\ \text{or } \Delta \vdash_{QU} Y((St, St'', St'), G_i, Sg_i)\}$$

and v is defined by:

$$v = \mathsf{flat}_\mathsf{Y}(\{Sg_i \mid ((St, St', St''), G_i, Sg_i) \in \mathbf{A_Y}\})$$

where flat_Y is given in Figure 3.

4 Soundness and Completeness

We can show that $\mathcal{QUR}$ is sound with respect to decision theory, and determine bounds on what it can deduce. First consider soundness[4]:

Theorem 1. *The construction and flattening of influence and synergy arguments in $\mathcal{QUR}$ using $\vdash_{QU}$ is sound with respect to decision theory.*

To prove completeness, one first needs to establish a proof procedure. The procedure for computing the effect on some formula p is:

1. Add a triple $(i : q : s)$ for every formula q whose change in probability is known.
2. Build $\mathbf{A_S}$, the set of all influence arguments for p.
3. Flatten this set to $S\langle p, v_S\rangle$.
4. Build $\mathbf{A_Y}$, the set of all synergy arguments for p.
5. Flatten this set to $Y\langle p, v_Y\rangle$.

This naturally backward chaining procedure can obviously be extended to compute the effect on a whole set of propositions. Now, we also need to define the sense in which we consider the system to be complete.

Definition 7. *A well-formed formula p is said to be a* cause *of a well-formed formula q if and only if it is possible to identify an ordered set of* iwffs *$\{p \rightarrow a_1, a_1 \rightarrow a_2, \ldots, a_n \rightarrow q\}$. If q is the value proposition V, the final member of the set is $a_n \xrightarrow{v} V$.*

In other words p is a cause of q if it is possible to build up a trail of (causally directed) implications which link p to q. We have a similar notion for synergies:

Definition 8. *A well-formed formula p is said to be a* synergistic cause *of a well-formed formula q if there is a* ywff *$a \uplus b \rightsquigarrow c$ such that p is a cause of either a or b and c is a cause of q. If q is the value proposition V, then the* ywff *in question is of the form $a \uplus b \overset{v}{\rightsquigarrow} V$.*

[4] All the proofs in this section are straightforward but lengthy, and so have been omitted to save space. They may be found in [19] and are simple extensions of those in [17].

Definition 9. *A well-formed formula q is said to be an* effect *(respectively a* synergistic effect*) of a well-formed formula p if and only if p is a cause (respectively a synergistic cause) of q.*

Definition 10. *The construction and flattening of arguments is said to be* causally complete *in some system of qualitative utility with respect to some formula p if it is possible to use that system to compute the changes in probability of all the effects of p.*

Given these definitions we can prove that $\mathcal{QUR}$ is complete in the following sense:

Theorem 2. *The construction and flattening of influence arguments in $\mathcal{QUR}$ using $\vdash_{QU}$ is causally complete with respect to any simple well-formed formula.*

We also need to deal with synergy arguments. For them we need the following notion of completeness:

Definition 11. *The construction and flattening of arguments is said to be* synergistically causally complete *in some system of qualitative utility with respect to some formula p if it is possible to use that system to compute the synergies involving p and all its synergistic effects.*

Given this we can show that:

Theorem 3. *The construction and flattening of synergy arguments in $\mathcal{QUR}$ using $\vdash_{QU}$ is synergistically causally complete with respect to any simple well-formed formula.*

Note that completeness is defined only in terms of *swff*s. This restriction is considered in detail in [19].

5 Example

This section presents a short example of the kind of reasoning possible in $\mathcal{QUR}$. Since the example is one used in [25], it also helps to informally demonstrate the fact that $\mathcal{QUR}$ captures the kind of reasoning possible in QPNs.

The example concerns the decisions made about digitalis therapy, and comes initially from [22]. An increased dosage of digitalis (dig) has a negative effect on conduction (con) ($r1$) and a positive effect on automaticity (aut) ($r2$). A negative effect on conduction is the aim of the therapy since the conduction has a positive effect on heart rate (hr) ($r3$) and a reduction in heart rate is what is required ($r4$). Automaticity has a positive effect on ventricular fibrillation (vf) ($r5$), a life threatening state ($r6$). High calcium levels (Ca) also have a positive effect on automaticity ($r7$). Increasing the digitalis dose makes automaticity more sensitive to calcium level ($r8$), and an increased heart rate means that ventricular fibrillation has a more severe effect on the patient's well-being. This information can be expressed as:

$$
\begin{array}{llll}
(r1 : dig \rightarrow con : -) & (r4 : hr \overset{v}{\rightarrow} V : -) & (r7 : Ca \rightarrow aut : +) & \Delta_1 \\
(r2 : dig \rightarrow aut : +) & (r5 : aut \rightarrow vf : +) & (r8 : dig \uplus Ca \rightsquigarrow aut : +) & \\
(r3 : con \rightarrow hr : +) & (r6 : vf \overset{v}{\rightarrow} V : -) & (r9 : hr \uplus vf \overset{v}{\rightsquigarrow} V : +) &
\end{array}
$$

Adding $(f1 : dig, \uparrow)$, indicating increased digitalis dosage, to this database, we can build the influence arguments:

$$
\begin{array}{l}
S(V, \{r1, r3, r4\}, \uparrow) \\
S(V, \{r2, r5, r6\}, \downarrow)
\end{array}
$$

These indicate, respectively, that there are reasons to both think that overall utility will increase and that it will decrease. These flatten to give $S\langle V, \updownarrow\rangle$ indicating, exactly as with the equivalent QPN, that there is no conclusive argument. We can also build two synergy arguments connecting dig and Ca with V:

$$
\begin{array}{l}
Y((V, Ca, dig), \{r8, r5, r6\}, -) \\
Y((V, dig, Ca), \{r9, r5, r7, r3, r1\}, -)
\end{array}
$$

These flatten to give $Y\langle(V, dig, Ca), -\rangle$, indicating that digitalis dosage and calcium level have a negative synergistic effect on overall utility. Thus increasing digitalis dosage reduces the effect that an increase in calcium level has on utility.

6 Discussion

The system introduced in this paper has its roots in Wellman's QPNs [25], the first attempt to build a qualitative decision theory, and draws its notion of "qualitative" from QPNs. This is a notion close to that in qualitative physics [14] where the basic abstraction is that which distinguishes between positive, negative and zero quantities and the derivatives of those quantities. The main focus in both QPNs and $\mathcal{QUR}$ is on the way in which values change with evidence. These two factors, the extreme abstraction and the concentration on change, distinguishes both $\mathcal{QUR}$ and QPNs from other qualitative systems.

As mentioned in the introduction, there have been a number of attempts to devise qualitative decision theories where "qualitative" is taken to means some form of relative order of magnitude based upon infinitesimal quantities. The first such effort was that of Pearl [21] which abstracted utility values in this way (earlier work, such as that of Darwiche [3] and Goldszmidt [12] had dealt with probabilities of this form). In doing this, Pearl thus provided an order of magnitude version of classical decision theory. This was then extended by Tan [23, 24] to deal with conditional preferences, so that it is possible to base decisions on statements like "if β is preferred to α". Around the same time Wilson [26] provided an alternative way to formulate Pearl's original qualitative version of classical decision theory, and more recently Lehmann [15] has made a similar proposal. The strand of this work which is most similar to ours is that of Bonet and Geffner [2], who also keep track of the reasons behind the decision, in terms of the information used to reach it.

The use of a different notion of "qualitative" is that investigated by Dubois, Prade and colleagues [7–9]. Their system has a possibilistic rather than a probabilistic semantics and is qualitative in the sense that only the ordinal rank of quantities is important. It should be noted, however, that the values they use are not infinitesimal (though one could build an infinitesimal version of their theory), and so can be considered more expressive than those of Pearl *et al.* It should also be noted that while, as described here, our system has a probabilistic semantics, we can give it alternative semantics, as discussed in [19].

7 Summary

This paper has extended our previous work on proof theoretic approaches to qualitative probabilistic reasoning [17] in two important ways. First this paper has extended it to deal with statements of utility, making it possible to reason about changes in expected utility as well as about changes in probabilities. This is an important step in developing a qualitative decision theory. Second, this paper has dealt with the concept of additive synergy, which is important in determining dominating decision options [25].

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