ON ORDER OF MAGNITUDE REASONING AND QUALITATIVE PROBABILITY

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Abstract

In recent years there has been a spate of papers describing systems for probabilistic reasoning which do not use numerical probabilities. In some cases these systems are unable to make any useful inferences because they deal with changes in probability at too high a level of abstraction. This paper discusses one of the problems this level of abstraction can cause, and shows how the use of a technique for order of magnitude reasoning can solve it.

1 INTRODUCTION

In the past few years there has been a good deal of interest in qualitative approaches to reasoning under uncertainty—approaches which do not make use of precise numerical values. These approaches range from systems of argumentation [1, 5, 9] to systems for nonmonotonic reasoning [10, 14] and abstractions of precise quantitative systems [7, 22]. Qualitative abstractions of probabilistic networks, in particular, have proved popular, finding use in areas in which the full numerical formalism is neither necessary nor appropriate. Applications have been reported in explanation [11], diagnosis [6, 12], engineering design [13], and planning [22].

In qualitative probabilistic networks (QPNs) [22], the focus is rather different from that of ordinary probabilistic systems. Whereas in probabilistic networks the main goal is to establish what the probabilities of hypotheses are when particular observations are made, in qualitative systems the main aim is to establish how values change. Since the approach is qualitative, the size of the changes are not the focus. It only matters whether a given change is positive, written as [+], negative [−], or zero [0]. In some cases it is not possible to resolve the change with any precision so that its value remains unknown, and it is written as [?]. Clearly this information is rather weak, but as the applications show it is sufficient for some tasks. Furthermore, reasoning with qualitative probabilities is much more efficient than reasoning with precise probabilities, since computation is quadratic in the size of the network [7], rather than NP-hard [2].

The popularity of qualitative probabilistic networks prompted work on abstractions of other uncertainty handling formalisms [17, 18]. This latter uses techniques from qualitative reasoning to generalize the approach provided by qualitative probabilistic networks to what are termed qualitative certainty networks (QCNs). Using this approach it is possible to propagate qualitative probability, possibility [8, 23] and Dempster-Shafer belief [21] in a uniform way.

The degree of abstraction in both QPNs and QCNs leads to situations in which certain changes may only be determined as [?] despite the presence of information that allows more precise inferences to be made. Whilst this is not always problematic, there are situations in which it causes difficulties, and in such situations techniques from order of magnitude reasoning may be helpful [15]. In this paper we extend the order of magnitude approach, providing a means of resolving problems of over-abstraction that goes beyond anything suggested so far.

2 QUALITATIVE PROBABILITY

QCNs are built around the notion of influences between variables, where the influence may be given a probabilistic semantics, as in QPNs, or a semantics in terms of possibility or Dempster-Shafer theory. Formally, a QCN is a pair $G = (V, Q)$, where V is a set of variables or nodes in the graph, represented by a capital letter, and Q is a set of sets of qualitative relations among the values of the variables which reflect the influences between the variables. The qualitative relations are expressed in terms of the derivatives that relate the different values of the variables together. In the case of a probabilistic QCN (QP/CN) we have:

Definition 1 (qualitative derivative) *The qualitative derivative* $\left[\frac{\partial \Pr(c_1)}{\partial \Pr(a_1)}\right]$ *relating the probability of* C *taking value* c_1 *to the probability of* A *taking value* a_1 *has the value* $[+]$*, if, for all* a_2 *and* X*:*

$$\Pr(c_1 \,|\, a_1, X) \geq \Pr(c_1 \,|\, a_2, X)$$

⊗	[+]	[0]	[−]	[?]
[+]	[+]	[0]	[−]	[?]
[0]	[0]	[0]	[0]	[0]
[−]	[−]	[0]	[+]	[?]
[?]	[?]	[0]	[?]	[?]

Table 1: Sign multiplication.

⊕	[+]	[0]	[−]	[?]
[+]	[+]	[+]	[?]	[?]
[0]	[+]	[0]	[−]	[?]
[−]	[?]	[−]	[−]	[?]
[?]	[?]	[?]	[?]	[?]

Table 2: Sign addition.

Derivatives with values [−] and [0] are defined by replacing $\geq$ with $\leq$ and $=$. If a derivative cannot be determined to be [+], [−], or [0], then it takes the value [?]. If A has possible values $\{a_1, a_2, a_3\}$ and C has possible values $\{c_1, c_2\}$, and if we write the probability of A taking value a_1 as $[\Delta \Pr(a_1)]$ (the square brackets denoting that it is the qualitative value of the quantity that we are interested in), then we have:

$$[\Delta \Pr(c_1)] = \left[\frac{\partial \Pr(c_1)}{\partial \Pr(a_1)}\right] \otimes [\Delta \Pr(a_1)] \qquad (1)$$

where $\otimes$ is qualitative multiplication, as defined in Table 1 and the overall effect of multiple changes on a single node is calculated using $\oplus$, as defined in Table 2. QCNs with possibilistic or Dempster-Shafer belief semantics handle changes in value in a similar way, but define qualitative derivatives differently [17].

To allow belief propagation it is necessary to propagate qualitative changes in value in both directions. This is made possible by the following theorem [17]:

Theorem 2 (symmetry of influences) $\left[\frac{\partial \Pr(c_1)}{\partial \Pr(a_1)}\right] = \left[\frac{\partial \Pr(a_1)}{\partial \Pr(c_1)}\right]$ *if* $\left[\frac{\partial \Pr(c_1)}{\partial \Pr(a_1)}\right] = [+]$ *or* $[-]$ *or if* $\left[\frac{\partial \Pr(c_1)}{\partial \Pr(a_1)}\right] = [0]$ *and* $\left[\frac{\partial \Pr(c_i)}{\partial \Pr(a_1)}\right] = [0]$ *for all* c_i, $i \neq 1$.

The impact of evidence on a given node can be calculated by taking the sign of the change in value at the evidence node and multiplying it by the sign of every link in the sequence that connects it to the node of interest. To see how this works, consider the example in Figure 1 (from [12]) in which the value labeling each arc is the value of the qualitative derivative linking the probabilities of the events represented by the nodes at the end of the arc. If we observe that the radio is dead, so that the probability of the radio being ok decreases, $[\Delta \Pr(\textit{radio ok})] = [-]$, and we want to know the impact of this on the probability of the battery being good we calculate the effect as $[-] \otimes [+] \otimes [+]$. With the definition of sign multiplication in Table 1 this gives a change in $\Pr(\textit{battery good})$ of [−]. If we also observed that the lights were not ok, and wanted to assess the impact of both pieces of evidence on the probability that the battery was good, we would establish the two individual effects and sum them using $\oplus$ (Table 2).

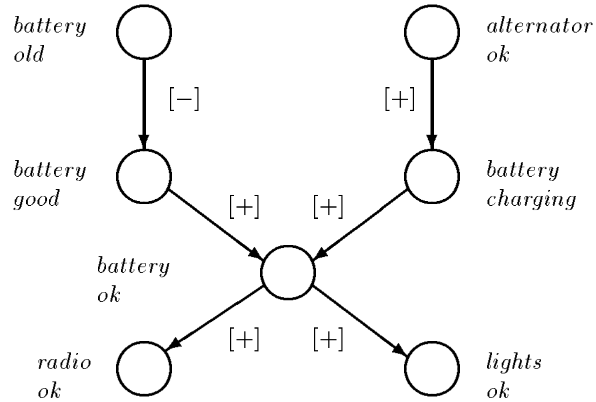

Figure 1: Part of a car diagnosis network

Described in these terms, QP/CNs are essentially equivalent to QPNs, the only difference being that the relation between two variables is described by a single qualitative value in a QPN and by a set of qualitative values in a QP/CN. However, QP/CNs can also go somewhat further. In particular, we can describe the propagation of values in terms of "separable" derivatives [17] where the effect of a change in the probability of one value of A on the probability of a value of C is calculated without considering its effects on the other values of A. We have:

Definition 3 (separable qualitative derivative) *The separable qualitative derivative* $\left[\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_1)}\right]$ *relating the probability of* C *taking value* c_1 *to the probability of* A *taking value* a_1 *has the value* $[\Pr(c_1 \mid a_1)]$.

Qualitatively this value will always be [+], but it is the numerical value that will be important in the application of order of magnitude techniques. When using separable derivatives $[\Delta \Pr(c_1)]$ is calculated as:

$$\begin{aligned}[\Delta \Pr(c_1)] \quad = \quad & \left[\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_1)}\right] \otimes [\Delta \Pr(a_1)] \\ & \oplus \left[\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_2)}\right] \otimes [\Delta \Pr(a_2)] \\ & \oplus \left[\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_3)}\right] \otimes [\Delta \Pr(a_3)] \qquad (2)\end{aligned}$$

3 OVER-ABSTRACTION

Now, the problem of over-abstraction with which we are concerned stems from the definition of qualitative derivatives. The problem is that for a broad class of networks

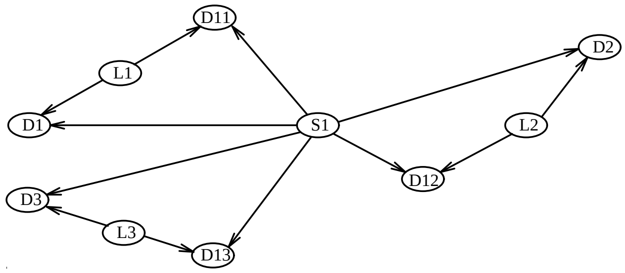

Figure 2: The causal network representation of the electricity distribution problem.

there are values of C for which it is not possible to predict the effect of a change in the probability of a given value of A using Definition 1 because the values of the conditional probabilities are such that the derivative which links the two has value [?]. Some of these networks will be genuinely ambiguous in the sense that it would take a detailed calculation to determine what the influence of the given value of A is. However, others will be such that the effect of a change in the value of A will be immediately obvious, and it is these which we consider to be instances of over-abstraction and which we can resolve using order of magnitude techniques.

The problem which we address is thus different from, and complementary to, that discussed in [15] where order of magnitude techniques were used to resolve indeterminacy in the change at a given node due to conflicting influences from two of more nodes to which it is connected.

As an example, consider the network of Figure 2 which drawn from the domain of fault diagnosis in electricity networks. The domain knowledge is greatly simplified. Nodes $L1$, $L2$ and $L3$ represent the fault states of three transmission lines, and have possible values fault and ok. Node $S1$ represents the fault states of a large conductor (a "busbar") which connects transmission lines together and also has possible values fault and ok. Nodes $D1$, $D2$, $D3$, $D11$, $D12$ and $D13$ represent the state of the circuit breakers that detect short-circuits on the lines and in the busbar. They have possible values reflecting the three states that they may be in—instantaneous alarm (inst), delayed alarm (del) and no alarm (ok). The exact meaning of the alarms does not concern us here (but see [19, 20] for more detail).

Now, considering Figure 2 as a QP/CN we are interested in how the probabilities of the line and busbar faults change when the probabilities of circuit breaker alarms change due to the observation of alarm states. To determine the way in which they change we need information about the conditional probabilities. For a fraction of the network these are given below, not as numerical values but instead, as is often appropriate for problems which are handled using qualitative methods, in terms of the relative magnitude of the values:

$$\Pr(L1{=}\mathsf{fault}\,|\,D11{=}\mathsf{ok}) \ll \Pr(L1{=}\mathsf{fault}\,|\,D11{=}\mathsf{inst})$$
$$\Pr(L1{=}\mathsf{fault}\,|\,D11{=}\mathsf{del}) \approx \Pr(L1{=}\mathsf{fault}\,|\,D11{=}\mathsf{inst})$$

where $\ll$ indicates a difference of at least an order of magnitude. Information about the prior values of the alarm conditions is also available:

$$\Pr(D11{=}\mathsf{del}) \approx \Pr(D11{=}\mathsf{inst})$$
$$\Pr(D11{=}\mathsf{del}) \ll \Pr(D11{=}\mathsf{ok})$$

In this situation applying Definition 1 gives:

$$\left[\frac{\partial \Pr(L1{=}\mathsf{fault})}{\partial \Pr(D11{=}\mathsf{inst})}\right] = [?] \qquad \left[\frac{\partial \Pr(L1{=}\mathsf{fault})}{\partial \Pr(D11{=}\mathsf{del})}\right] = [?]$$

$$\left[\frac{\partial \Pr(L1{=}\mathsf{fault})}{\partial \Pr(D11{=}\mathsf{ok})}\right] = [-]$$

which, when we apply (1) by writing $L1{=}\mathsf{fault}$ for c_1 and $D11{=}\mathsf{inst}$, $D11{=}\mathsf{del}$ and $D11{=}\mathsf{ok}$ for a_1, a_2 and a_3 gives:

Report	inst	delayed	ok
[Δ Pr(L1=fault)]	[?]	[?]	[−]

Here the [?] indicates that it is not possible to predict precisely how the probability of a line fault will change when the probability of a line fault changes.

We consider this failure to produce an unambiguous result to be over-abstraction since an unambiguous result can easily be obtained using the information that is to hand. The way in which this may be done is discussed below. It should be noted that exactly the same problem will occur if the situation is modelled using QPNs, so it is clear that this is a problem of purely qualitative methods in general rather than of QP/CNs in particular.

4 ORDER OF MAGNITUDE REASONING

The ambiguous inferences made by the purely qualitative approach can be resolved by a slightly less abstract form of reasoning which considers the relative magnitudes of the quantities rather than their signs. If we take Definition 3 along with the information about the magnitudes of the conditional values we find that:

$$\frac{\partial_s \Pr(L1 = \mathsf{fault})}{\partial_s \Pr(D11 = \mathsf{ok})} \ll \frac{\partial_s \Pr(L1 = \mathsf{fault})}{\partial_s \Pr(D11 = \mathsf{inst})}$$
$$\frac{\partial_s \Pr(L1 = \mathsf{fault})}{\partial_s \Pr(D11 = \mathsf{del})} \approx \frac{\partial_s \Pr(L1 = \mathsf{fault})}{\partial_s \Pr(D11 = \mathsf{inst})}$$

Along with information about the change in value of $\Pr(D11 = \mathsf{inst})$, $\Pr(D11 = \mathsf{del})$, and $\Pr(D11 = \mathsf{ok})$ this is sufficient to tell us what the result of a delayed alarm will be.

The argument is as follows. Initially $\Pr(D11{=}\mathsf{ok}) \approx 1$ and $\Pr(D11 = \mathsf{inst}) \approx \Pr(D11 = \mathsf{del}) \approx 0$ (this follows from the relative magnitudes of the priors). Since the observation of a delayed alarm means that the probability distribution over the states of $D11$ will alter so that $\Pr(D11 = \mathsf{del}) = 1$ while $\Pr(D11 = \mathsf{inst}) = \Pr(D11 = \mathsf{ok}) = 0$, clearly $|\Delta \Pr(D11{=}\mathsf{inst})| \ll |\Delta \Pr(D11{=}\mathsf{ok})| \approx$

(A1)	$A \approx A$
(A2)	$A \approx B \rightarrow B \approx A$
(A3)	$A \approx B, B \approx C \rightarrow A \approx C$
(A4)	$A \sim B \rightarrow B \sim A$
(A5)	$A \sim B, B \sim C \rightarrow A \sim C$
(A6)	$A \approx B \rightarrow A \sim B$
(A7)	$A \approx B \rightarrow C.A \approx C.B$
(A8)	$A \sim B \rightarrow C.A \sim C.B$
(A9)	$A \sim 1 \rightarrow [A] = [+]$
(A10)	$A \ll B \leftrightarrow B \approx (B + A)$
(A11)	$A \ll B, B \sim C \rightarrow A \ll C$
(A12)	$A \approx B, [C] = [A] \rightarrow (A + C) \approx (B + C)$
(A13)	$A \sim B, [C] = [A] \rightarrow (A + C) \sim (B + C)$
(A14)	$A \sim (A + A)$
(A15)	$A \not\simeq B \leftrightarrow (A - B) \sim A$ or $(B - A) \sim B$
(P1)	$A \sim B \rightarrow [A] = [B]$
(P3)	$A \ll B \rightarrow C.A \ll C.B$
(P4)	$A \ll B, A \sim C \rightarrow C \ll B$
(P13)	$A \ll B \rightarrow \|A\| < \|B\|$
(P26)	$A \sim B \rightarrow B \sim A$
(P31)	$A \ll B \rightarrow A \not\simeq B$
(P35)	$A \not\simeq B \rightarrow C.A \not\simeq C.B$
(P36)	$A \not\simeq B, C \ll A \rightarrow C \ll (A - B)$
(P38)	$A \not\simeq B, C \approx A, D \approx B \rightarrow C \not\simeq D$

Table 3: Some of the axioms and properties of ROM[K].

$|\Delta \Pr(D11 = \mathsf{del})|$. Thus when we compare the magnitudes of the terms in the quantitative version of (2):

$$\Delta \Pr(c_1) = \frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_1)} \otimes \Delta \Pr(a_1) \oplus \frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_2)} \otimes \Delta \Pr(a_2) \oplus \frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_3)} \otimes \Delta \Pr(a_3)$$

to establish the change in line fault probability for a delayed alarm, the second term dominates and we have $\Delta \Pr(L1 = \mathsf{fault}) = [+]$. This result suggests that providing a means of formalising the kind of reasoning performed above would be useful.

Now, handling this kind of reasoning is precisely what order of magnitude systems such as ROM[K] [4] were designed to do. ROM[K] is based on the idea that the order of magnitude of two quantities, Q_1 and Q_2, is usually expressed in terms of their relative sizes, and there are four possible ways of expressing this relation: Q_1 is *negligible wrt* Q_2, $Q_1 \ll Q_2$, Q_1 is *distant from* Q_2, $Q_1 \not\simeq Q_2$, Q_1 is *comparable to* Q_2, $Q_1 \sim Q_2$, and Q_1 is *close to* Q_2, $Q_1 \approx Q_2$. Once the relation between pairs of quantities is specified, it is possible to deduce new relations by applying the axioms and properties of ROM[K], some of which are reproduced in Table 3.

Using these rules we can formalise the process of deducing the fact that $\Delta \Pr(L1 = \mathsf{fault}) = [+]$. In the notation of ROM[K] we know that:

$$\frac{\partial_s \Pr(L1 = \mathsf{fault})}{\partial_s \Pr(D11 = \mathsf{ok})} \ll \frac{\partial_s \Pr(L1 = \mathsf{fault})}{\partial_s \Pr(D11 = \mathsf{inst})} \quad (3)$$

$$\frac{\partial_s \Pr(L1 = \mathsf{fault})}{\partial_s \Pr(D11 = \mathsf{del})} \approx \frac{\partial_s \Pr(L1 = \mathsf{fault})}{\partial_s \Pr(D11 = \mathsf{inst})} \quad (4)$$

$$\Delta \Pr(D11 = \mathsf{inst}) \ll \Delta \Pr(D11 = \mathsf{del}) \quad (5)$$

$$\Delta \Pr(D11 = \mathsf{ok}) \approx \Delta \Pr(D11 = \mathsf{del}) \quad (6)$$

Now, if we only take into consideration the magnitude of the quantities, and if for convenience we write $\partial_s \Pr(L1 = \mathsf{fault})/\partial_s \Pr(D11 = \mathsf{inst})$ as A, $\Delta \Pr(D11 = \mathsf{inst})$ as B, $\partial_s \Pr(L1 = \mathsf{fault})/\partial_s \Pr(D11 = \mathsf{del})$ as C, $\Delta \Pr(D11 = \mathsf{del})$ as D, $\partial_s \Pr(L1 = \mathsf{fault})/\partial_s \Pr(D11 = \mathsf{ok})$ as E, and $\Delta \Pr(D11 = \mathsf{ok})$ as F, then the quantity we want to establish the sign of is $C.D - (A.B + E.F)$. The derivation is as follows:

(D1)	$B.C \approx B.A$	(A7)(4)
(D2)	$B.C \ll C.D$	(P3)(6)
(D3)	$B.A \approx B.C$	(A2)(D1)
(D4)	$B.A \sim B.C$	(A6)(D3)
(D5)	$B.A \ll C.D$	(P4)(D2) (D4)
(D6)	$E.D \approx E.F$	(A7)(5)
(D7)	$E.F \approx E.D$	(A2)(D6)
(D8)	$C.D \ll E.D$	(P3)(3)
(D9)	$C.D \not\simeq E.D$	(P31)(D8)
(D10)	$C.D \approx C.D$	(A1)
(D11)	$C.D \not\simeq E.F$	(P38)(D6)(D8)(D10)
(D12)	$A.B \ll (C.D - E.F)$	(P36)(D5)(D11)
(D13)	$\|A.B\| < \|(C.D - E.F)\|$	(P13)(D12)

from which it is clear that $A.B - (C.D - E.F)$ is negative, so that $[C.D - (A.B + E.F)] = [+]$.

One may generalize this result so that it is possible to calculate the sign of any qualitative change in a probability $\Pr(c_1)$ for which order of magnitude information about the conditionals $\Pr(c_1 \mid a_1)$, $\Pr(c_1 \mid a_2)$, and $\Pr(c_1 \mid a_3)$ which relate it to the node A which influences it, and about changes in the values of the $\Pr(a_j)$. Given initial information:

$$\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_1)} \, \mathrm{rel}_1 \, \frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_2)} \, \mathrm{rel}_2 \, \frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_3)}$$

$$\Delta \Pr(a_1) \, \mathrm{rel}_3 \, \Delta \Pr(a_2) \, \mathrm{rel}_4 \, \Delta \Pr(a_3)$$

where $\mathrm{rel}_i \in \{\ll, \not\simeq, \sim, \approx\}$ we can use the following procedure.

Step 1 Establish the relations between the products of separable derivative and change:

$$\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_1)}.\Delta \Pr(a_1) \quad \mathrm{rel}_5 \quad \frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_2)}.\Delta \Pr(a_2)$$

$$\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_2)}.\Delta \Pr(a_2) \quad \mathrm{rel}_6 \quad \frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_3)}.\Delta \Pr(a_3)$$

using the following result:

$\mathrm{rel_a}$ \ $\mathrm{rel_b}$	$\approx$	$\sim$	$\not\simeq$	$\ll$
$\approx$	$\approx$	$\sim$	$\not\simeq$	$\ll$
$\sim$	$\sim$	$\sim$	U	$\ll$
$\not\simeq$	$\not\simeq$	U	U	$\ll^{\ddagger}$
$\ll$	$\ll$	$\ll$	$\ll^{*}$	$\ll$

Table 4: How to establish $\mathrm{rel_c}$ (Theorem 4)—U indicates that the relation may not be established.

$\mathrm{rel_d}$ \ $\mathrm{rel_e}$	$\approx$	$\sim$	$\not\simeq$	$\ll$
$\approx$	$\not\simeq$	$\not\simeq$	$\sim, \not\simeq^{\ddagger}$	$\ll$
$\sim$	$\not\simeq^{*}$	$\not\simeq^{*}$	$\sim, \not\simeq^{\ddagger}$	$\ll$
$\not\simeq$	$\not\simeq^{*}$	$\not\simeq^{*}$	$\not\simeq^{\dagger}$	$\not\simeq$
$\ll$	U	U	$\ll^{\dagger}$	$\not\simeq$

Table 5: How to establish $\mathrm{rel_f}$ (Theorem 5)—U indicates that the relation may not be established.

Theorem 4 (relative magnitude) *If we are given that* $\partial_s \Pr(x)/\partial_s \Pr(y)\ \mathrm{rel_a}\ \partial_s \Pr(w)/\partial_s \Pr(z)$ *and* $\Delta \Pr(y)\ \mathrm{rel_b}\ \Delta \Pr(z)$*, where* $\mathrm{rel_a}, \mathrm{rel_b} \in \{\approx, \sim, \not\simeq, \ll\}$*, then the relation* $\mathrm{rel_c}$ *that holds between* $\partial_s \Pr(x)/\partial_s \Pr(y).\Delta \Pr(y)$ *and* $\partial_s \Pr(w)/\partial_s \Pr(z).\Delta \Pr(z)$ *is given by Table 4 and the obvious symmetrical results where* * *indicates that the relation holds provided that* $\Pr(y) < \Pr(z)$ *and* ‡ *indicates that the relation holds provided that* $\partial_s \Pr(x)/\partial_s \Pr(y) < \partial_s \Pr(w)/\partial_s \Pr(z)$*.*

Proof: See [15].

Step 2 From the result of the first step, establish the relationship between one product and the difference of the others since this is the general pattern of all solutions:

$$\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_1)}.\Delta \Pr(a_1) \quad \mathrm{rel_7} \quad \left(\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_2)}.\Delta \Pr(a_2) - \frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_3)}.\Delta \Pr(a_3)\right)$$

using Theorem 5.

Theorem 5 (difference) *Given:*

$$\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_1)}.\Delta \Pr(a_1) \quad \mathrm{rel_d} \quad \frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_2)}.\Delta \Pr(a_2)$$
$$\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_2)}.\Delta \Pr(a_2) \quad \mathrm{rel_e} \quad \frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_3)}.\Delta \Pr(a_3)$$

then the relation $\mathrm{rel_f}$ *such that:*

$$\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_1)}.\Delta \Pr(a_1) \quad \mathrm{rel_f} \quad \left(\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_2)}.\Delta \Pr(a_2) - \frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_3)}.\Delta \Pr(a_3)\right)$$

is given by Table 5. Where two relations are given for $\mathrm{rel_f}$ *it indicates that either of them may hold, while* * *indicates that the relation holds if* $|\partial_s \Pr(c_1)/\partial_s \Pr(a_1).\Delta \Pr(a_1)| > |\partial_s \Pr(c_1)/\partial_s \Pr(a_2).\ \Delta \Pr(a_2)|$*,* † *indicates that the relation holds if* $|\partial_s \Pr(c_1)/\partial_s \Pr(a_2).\Delta \Pr(a_2)| > |\partial_s \Pr(c_1)/\partial_s \Pr(a_3).\Delta \Pr(a_3)|$*, and* ‡ *indicates that the relation holds if* $|\partial_s \Pr(c_1)/\partial_s \Pr(a_1).\ \Delta \Pr(a_1)| > |\partial_s \Pr(c_1)/\partial_s \Pr(a_3).\ \Delta \Pr(a_3)|$*.*

Proof: (sketch) The proof proceeds by using results such as A15, P4, P36 and P38 from Table 3 to establish relationships between one product and the difference of the others. The full proof may be found in [16].

Step 3 From the result of the previous step, establish the sign of $\partial_s \Pr(c_1)/\partial_s \Pr(a_1).\Delta \Pr(a_1) - (\partial_s \Pr(c_1)/\partial_s \Pr(a_2).\Delta \Pr(a_2) + \partial_s \Pr(c_1)/\partial_s \Pr(a_3).\Delta \Pr(a_3))$ using Theorem 6.

Theorem 6 (signs) *Given:*

$$\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_1)}.\Delta \Pr(a_1) \quad \mathrm{rel_g} \quad \left(\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_2)}.\Delta \Pr(a_2) - \frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_3)}.\Delta \Pr(a_3)\right)$$

the sign of

$$\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_1)}.\Delta \Pr(a_1) - \left(\frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_2)}.\Delta \Pr(a_2) - \frac{\partial_s \Pr(c_1)}{\partial_s \Pr(a_3)}.\Delta \Pr(a_3)\right)$$

is $[-]$ *if* $\mathrm{rel_g}$ *is* $\ll$ *or if* $\mathrm{rel_g}$ *is* $\not\simeq$ *and* $|\partial_s \Pr(c_1)/\partial_s \Pr(a_2).\Delta \Pr(a_2)| \leq |\partial_s \Pr(c_1)/\partial_s \Pr(a_3).\Delta \Pr(a_3))|$ *and* $[+]$ *if* $\mathrm{rel_g}$ *is* $\not\simeq$ *and* $|\partial_s \Pr(c_1)/\partial_s \Pr(a_2).\Delta \Pr(a_2)| \geq |\partial_s \Pr(c_1)/\partial_s \Pr(a_3).\Delta \Pr(a_3))|$*. Otherwise the sign is* $[?]$*.*

Proof: If $\mathrm{rel_g}$ is $\ll$ then the application of P13 gives the result immediately, while if $\mathrm{rel_g}$ is $\not\simeq$ then we apply A15 to get $(A.B - C.D + E.F) \sim A.B$ or $(C.D - E.F - A.B) \sim (CD - EF)$. In both cases the result follows from P1. If $\mathrm{rel_g}$ is $\sim$ or $\approx$ then the magnitudes of the two quantities are too close to give a result using ROM[K] [4].

5 SUMMARY

The main results of this paper are to show that order of magnitude reasoning can be used to resolve indeterminate qualitative probabilities, and to give formal results that allow indeterminate values of three valued variables to be resolved (for the extension to variables with more possible values see [16]). This work is far from being the final word on the subject, but does go further in resolving indeterminate qualitative probabilities reasoning than any similar work.

There are three points which should be made about the method presented in this paper. Firstly, the generality of the QCN framework means that the results can be applied to resolve indeterminate values when qualitative versions of possibility and Dempster-Shafer theories are used. Secondly, it should be noted that the method is heuristic. As with other order of magnitude techniques, there is a trade-off between drawing safe conclusions which are correct but unhelpful and drawing more aggressive conclusions which are more useful but which can be wrong. In the case of the technique employed here the trade-off emerges from the mapping from numerical values to ROM[K] relations. The more aggressive the mapping—the more small relative differences are mapped in to $\ll$ and $\not\simeq$ relations—the more the ambiguity that can be resolved, but also the larger the chance of an error. Conversely, the more that the mappings are made safe—the more that large relative differences are mapped into $\approx$ and $\sim$ relations—the less the ambiguity can be resolved, but the safer the conclusions are guaranteed to be. Here the information is provided in order of magnitude terms, but it will often be provided numerically. Providing maximally safe mappings is the goal of future research, and seem likely to make use of Dague's system ROM[$\Re$] [3] which permits numerical order of magnitude reasoning. The third point is related to this. When the approach concludes that the change is [?] it does not represent a failure, but the conclusion that it is not safe to make any more precise inference about the change.

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