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QUALITATIVE DEMPSTER-SHAFER THEORY

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ABSTRACT

This paper introduces the idea of using the Dempster-Shafer theory of evidence with qualitative values. Dempster-Shafer theory is a formalism for reasoning under uncertainty which may be viewed as a generalisation of probability theory with special advantages in its treatment of ambiguous data and the ignorance arising from it. Here we are interested in applying the theory when the numbers that it usually operates over are not universally available. To cope with this lack of numbers, we use qualitative, linguistic and relative values.

1. INTRODUCTION

Dempster-Shafer theory is a numerical method for evidential reasoning. The theory originated with a paper by the statistician Arthur Dempster [4] who wanted to free probability theory from the need to attach a measure of uncertainty to every hypothesis under consideration. His work remained hidden in the statistics literature until Glenn Shafer, one of Dempster's students, brought the material to a wider audience in his doctoral dissertation [11]. The method has become popular, and the basic model has been extended in a number of directions in recent years [12], [13], [14].

In this paper we propose another adaptation of the model. Our area of interest is reasoning under uncertainty when all the numerical information required by methods such as Dempster-Shafer theory are not available, handling such a lack of information [8] [9] by using techniques from qualitative reasoning [1]. Extending the approach first suggested in [10], we consider replacing the numerical operands of more usual applications of Dempster-Shafer theory with qualitative values. These give us degraded, but still useful, results which are illustrated by a number of examples.

In Section 2, the basics of Dempster-Shafer theory are explained for the benefit of those who are not familiar with the approach. Section 3 introduces the qualitative version of the theory, and Section 4 demonstrates the kind of results that may be obtained by reference to a simple example. Section 5 then discusses some heuristic extensions that may be useful when it is employed in more complex domains, Section 6 applies the theory to linguistic values, and Section 7 assesses the use of relative values. Section 8 concludes.

2. DEMPSTER-SHAFER THEORY

The basic idea of the theory is that numerical measures of uncertainty, termed basic probability masses, may be assigned to sets of hypotheses as well as individual hypotheses. Consider the following example, adapted from the work of Philippe Smets [12]. Mr Jones has been murdered. We know that the murderer was one of three

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notorious assassins, Peter, Paul and Mary, so we have a set of hypotheses Θ = {Peter, Paul, Mary}. The only evidence that we have initially is that of Mrs Jones who saw the killer leaving the scene of the murder and is 80% sure that it was a man. Thus all we know is that $p(Man) = 0.8$. If we were using probability theory we would have to:

(a) allocate $p(\neg Man) = p(Mary) = 1 - 0.8 = 0.2$
(b) allocate $p(Man) = 0.8 = p(Peter) = 0.4 + p(Paul) = 0.4$ by some principle such as the principle of maximum entropy.

With evidence theory, however, we are not limited to allocating probability to the members of the set {{Peter}, {Paul} , {Mary}}. We have instead a mass assignment function $m(\cdot)$ where $m: 2^{\Theta} \rightarrow [0, 1]$ assigns probabilities to any set which is a member of the power set of Θ, that is the set 2^{Θ} = {{Peter, Paul, Mary}, {Peter, Paul}, {Peter, Mary}, {Paul, Mary}, {Peter}, {Paul}, {Mary}, $\emptyset$}. The only restrictions on $m(\cdot)$ are:

$$\sum_{x \in 2^{\Theta}} m(x) = 1$$
$$m(\emptyset) = 0$$

so that all the assigned probabilities sum to unity, and there is no belief in the empty set. Thus in the case of Mr Jones murder we can assign values to equate with what we know and nothing more. We know that $p(Man) = 0.8$ so that $m(\{Peter, Paul\}) = 0.8$, and the remaining probability we know nothing about so that $m(\{Peter, Paul, Mary\}) = 0.2$.

Now, consider that a second piece of evidence comes to light. It is reported with confidence 0.6 that Peter was leaving on a jet plane when the murder occurred, so that we have $m'(\{Paul, Mary\}) = 0.6$, $m'(\{Peter, Paul, Mary\}) = 0.4$. We would like to combine these two pieces of evidence, and this may be done by combining the mass assignments using Dempster's rule to create a new mass assignment m'' defined by:

$$m''(A) = \sum_{\substack{i,j \\ A_i \cap B_j = A}} m(A_i)\, m'(B_j)$$

when the result is unnormalised as advocated by Smets [12]. There is some controversy about the use of normalisation, and since the qualitative version of the theory is simpler without it we will follow Smets in not normalising. Put simply, the result of combining two assignments is that for any intersecting sets A and B, where A has mass M from assignment m and B has mass M' from assignment m', the belief accruing to their intersection is the product of M and M'. So for our example:

		m	
		{Peter, Paul} 0.8	{Peter, Paul, Mary} 0.2
m'	{Paul, Mary} 0.6	{Paul} 0.48	{Paul, Mary} 0.12
	{Peter, Paul, Mary} 0.4	{Peter, Paul} 0.32	{Peter, Paul, Mary} 0.08

Having established the final mass assignments of the set of hypotheses we can assess the belief and plausibility of any set of hypotheses as follows:

$$\mathrm{Bel}(A) = \sum_{B \subset A} m(B)$$

$$\mathrm{Pl}(A) = \sum_{B \cap A \neq \emptyset} m(B)$$

These measures are clearly related to one another:

$$\mathrm{Bel}(A) = 1 - \mathrm{Pl}(\neg A)$$
$$\mathrm{Pl}(A) = 1 - \mathrm{Bel}(\neg A)$$

The belief in any set is the lower bound of the probability of that set which is the sum of all the probabilities of all the subsets of that set. The plausibility is the upper bound on the probability, namely the sum of all the values not accruing to any sets that are exclusive of the one in question. Thus:

$$\begin{aligned}
\mathrm{Bel}(\{Paul\}) &= 0.48, \\
\mathrm{Bel}(\{Peter, Paul\}) &= \mathrm{Bel}(\{Peter\}) + \mathrm{Bel}(\{Paul\}) \\
&\quad + \mathrm{Bel}(\{Peter, Paul\}) \\
&= 0 + 0.48 + 0.32 \\
&= 0.8, \\
\mathrm{Bel}(\{Peter, Paul, Mary\}) &= 1.
\end{aligned}$$

While:

$$\begin{aligned}
\mathrm{Pl}(\{Peter\}) &= \mathrm{Pl}(\{Peter, Paul\}) \\
&\quad + \mathrm{Pl}(\{Peter, Paul, Mary\}) \\
&= 0.4, \\
\mathrm{Pl}(\{Mary\}) &= 0.2.
\end{aligned}$$

3. QUALITATIVE EVIDENCE THEORY

As it stands, evidence theory is fine as long as all the necessary numerical information is available. Provided that we can put a basic probability number on any piece of evidence that comes to light then everything is alright, and the theory gives us nice, intuitive, results. However, a problem arises when we do not have easily quantifiable evidence. For instance we may be taking readings from faulty sensors, or we may be dealing with data which relates to occurrences that happen so rarely that no accurate numbers are available. In such cases all we can say about a particular piece of evidence is that it indicates that certain hypotheses are true to a certain degree. To what degree "a certain degree" is we have no idea. What we would like is to use the intuitive evidence theory style of reasoning to combine such pieces of evidence to give us some idea of what the evidence implies.

In a previous paper [10] it was argued that one way of coping with unknown probability values was to represent them as qualitative numbers since assuming that unknown values are equal to qualitative values such as $+$ (which represents "positive") is an assumption that takes nothing for granted. If I know that a probability value exists, then assuming that it is in the range $[0, 1]$ is a fact that follows trivially from knowledge that the value is a probability [7]. What is proposed in "qualitative evidence theory" is that all basic probability values are assumed to be 0 or $+$, that is some unknown value between 0 and 1. It is, of course possible to use intermediate values to model whatever numerical information we possess [9], but here we are only interested in the case where we have no numerical information whatsoever. We assign probabilities and combine using Dempster's rule as before, carrying out arithmetic using restricted versions of the combinator tables for qualitative addition ⊕ and qualitative multiplication ⊗ [1]:

⊕	+	0
+	+	+
0	+	0

⊗	+	0
+	+	0
0	0	0

Thus the example of Mr Jones' murder from the first piece of evidence we have p(Man) = + so that m({Peter, Paul}) = + and m({Peter, Paul, Mary}) = +, and from the second piece of evidence we have m'({Paul, Mary}) = +, m'({Peter, Paul, Mary}) = +. Combining these:

	{Peter, Paul} +	{Peter, Paul, Mary} +
{Paul, Mary} +	{Paul} +	{Paul, Mary} +
{Peter, Paul, Mary} +	{Peter, Paul} +	{Peter, Paul, Mary} +

$$
\begin{aligned}
\text{Bel}(\{\text{Paul}\}) &= + \\
\text{Bel}(\{\text{Peter, Paul}\}) &= \text{Bel}(\{\text{Peter}\}) + \text{Bel}(\{\text{Paul}\}) \\
&\quad + \text{Bel}(\{\text{Peter, Paul}\}) \\
&= + \\
\text{Bel}(\{\text{Peter, Paul, Mary}\}) &= +
\end{aligned}
$$

and:

$$
\begin{aligned}
\text{Pl}(\{\text{Peter}\}) &= \text{Pl}(\{\text{Peter, Paul}\}) \\
&\quad + \text{Pl}(\{\text{Peter, Paul, Mary}\}) \\
&= + \\
\text{Pl}(\{\text{Mary}\}) &= +
\end{aligned}
$$

which doesn't seem to have a great deal going for it at all, since all the sets of hypotheses have the same degree of support from the evidence.

However, this is not as useless as it seems at first sight. What this stripping away of the numbers makes extremely clear is that the beautiful and intuitive mechanism of evidence theory works just as well without numbers as it does with them, and it continues to lay bare the implication of the evidence. What we can see from this, just as well as we can see from the numerical example, is that there is only one singleton hypothesis that is indicated by the evidence, {Paul}, and that if we want to consider hypotheses of the form "A or B", then there is evidence against {Paul, Mary} and {Paul, Peter}. The method will even detect evidence for solutions other than those in the frame of discernment {Peter, Paul, Mary} by the accruing of a + to the empty set ∅ [12]when the focal elements of the mass functions (that is the sets of hypotheses that the mass functions support) do not intersect.

Of course it is possible to invent pathological cases where the intuitive result is the wrong one. Consider what would happen if m'({Paul, Mary}) were 0.1. The final result of the weighing of the evidence would be Bel({Paul}) = 0.08, Bel({Peter, Paul}) = 0.8 which suggests that there is little evidence against Paul alone, while the qualitative solution would be the same as before. However, this does not mean that there is no virtue in using the qualitative approach to establish which way the evidence points in particular situations where no numerical probability masses may be established.

4. A SIMPLE EXAMPLE

By way of illustrating the usefulness of qualitative evidence theory we will consider an example from decision making in gastroenterology which was also studied in [10]. We consider a clinic specialising in gastroenterological complaints. These complaints have a number of possible origins which may be classified as gastric cancer, peptic ulcers (both gastric and duodenal ulcers), gallstones, and functional disorders. The latter are conditions with no identifiable organic cause, and are often stress related. Over many years, a number of symptoms and signs which provide useful information for discriminating between complaints have been recorded from many patients. These are signs of jaundice, pain after meals, weight loss and the age of the patient.

The clinic's research into gastric disorders has progressed since it was reported in [10]. The clinic has now established that particular symptoms point to particular sets of diseases. Thus jaundice indicates gallstones or functional disorder {gs, fd}, pain after meals indicates gastric cancer, peptic ulcer or functional disorder {gc, pu, fd}, weight loss indicates gastric cancer {gc} and if the patient is elderly then he is likely to be suffering from gastric cancer, peptic ulcer or gallstones {gc, pu, gs}. We are interested in the case of Jack, an elderly patient who shows no signs of jaundice but has recently lost weight and often has pain after eating. Considering the evidence of Jack's age and the fact that he suffers pain after eating we have:

	{gc, pu, fd} +	{gc, pu, fd, gs} +
{gc, pu, gs} +	{gc, pu} +	{gc, pu, gs} +
{gc, pu, fd, gs} +	{gc, pu, fd} +	{gc, pu, fd, gs} +

Which suggests that the most specific diagnosis is that Jack is suffering from either gastric cancer or peptic ulcer. Now we consider the evidence that Jack has recently lost weight. This gives:

	{gc} +	{gc, pu, fd, gs} +
{gc, pu} +	{gc} +	{gc, pu} +
{gc, pu, gs} +	{gc} +	{gc, pu, gs} +
{gc, pu, fd} +	{gc} +	{gc, pu, fd} +
{gc, pu, fd, gs} +	{gc} +	{gc, pu, fd, gs} +

which strongly indicates that Jack has gastric cancer since not only is gastric cancer the only singleton hypothesis indicated by the evidence, but it is also a member of every single other set of hypotheses that the evidence points to.

5. EXTENDING THE BASIC APPROACH

The basic approach discussed in Sections 3 and 4 is fine when dealing with simple sets of evidence. When there is only one singleton hypothesis then it is clear which hypothesis is most favoured by the evidence. Similarly, when there is one set of two hypotheses which is smaller than any other supported set, then it is clear which hypotheses the evidence points to. However, this will not always be the case. Take the case of Irwin, another patient who comes to the clinic of Section 4 as an example of the problems that might arise. When Irwin's symptoms are related to the diseases that the clinic specialises in, it is clear that there is evidence for two possible sets of diseases, gallstones and functional disorder {gs, fd} and gastric cancer or peptic ulcer {gc, pu}. The result of combining this evidence is:

	{gs, fd} +	{gc, pu, fd, gs} +
{gc, pu} +	∅ +	{gc, pu} +
{gc, pu, fd, gs} +	{gs, fd} +	{gc, pu, fd, gs} +

From which it is clear that there is good evidence for Irwin suffering from none of the usual diseases. This seems a natural reasonable conclusion, but consider how this changes with a third piece of evidence which indicates that Irwin is suffering from gastric cancer:

	{gc} +	{gc, pu, fd, gs} +
{gs, fd} +	∅ +	{gs, fd} +
{gc, pu} +	{gc} +	{gc, pu} +
∅ +	∅ +	∅ +
{gc, pu, fd, gs} +	{gc} +	{gc, pu, fd, gs} +

As a result of this third piece of evidence, there are two singleton hypotheses identified; Irwin is suffering from gastric cancer, or Irwin is suffering from none of the usual diseases. How can we decide which to choose? Well, there is good evidence [2], [3] that when it is not possible to put accurate numerical weights on terms, it is reasonable to use an improper linear model to sum them up. Using such a model in this case is trivial, and tells us that since there are more indications that Irwin is suffering from a disease outside the frame of discernment (3) than there are that he is suffering from gastric cancer (2), then our belief in him suffering from gastric cancer is less than our belief in him suffering from a disease that is not under consideration.

6. USING LINGUISTIC QUANTIFIERS

It is possible to further refine the approach if we have some idea of the magnitude of the mass assignments to the focal elements. For instance, consider that we have information that allows us to quantify the mass assignments in terms of a set of linguistic labels, which correspond to a subintervals of the unit interval [0, 1], in a similar manner to that considered by Dubois *et al.* [1992]. We take the following symmetric set of subintervals: $\mathcal{P} = \{0, (\varepsilon, \alpha], [\alpha, 1-\alpha], [1-\alpha, 1-\varepsilon), 1\}$ whose names are, respectively; None, Little, (About) Half, Much and All. These subintervals are ordered:

$$\text{None} \leq \text{Little} \leq \text{Half} \leq \text{Much} \leq \text{All}$$

As in Dubois *et al.* [1992] we take ε to be some positive infinitesimal quantity while α is some number in $(0, 0.5)$. In practice we take $\alpha = 0.3$, so that it is appropriate to note (as Dubois *et al.* do) that “about half” is short for “neither little, nor much but somewhere in between”.

Given that the mass assignments are quantified with this set of labels, we can use interval arithmetic and Dempster’s rule to compute the result of combining evidence in terms of the linguistic labels and their combinations, such as $[\text{Few}, \text{Most}] = \{X \in \mathcal{P} \mid \text{Few} \leq X < \text{Most}\}$. For instance, to hark back to Jack’s trip to the clinic, consider what would have happened if we had the information that if a patient has pain after meals we should have much belief in his having gastric cancer, peptic ulcer or functional disorder $\{gc, pu, fd\}$, whilst if a patient is elderly then we should have a middling belief that he is suffering from gastric cancer, peptic ulcer or gallstones $\{gc, pu, gs\}$. For Jack, we have:

	{gc, pu, fd} Much	{gc, pu, fd, gs} Little
{gc, pu, gs} Half	{gc, pu} [Little, Half}	{gc, pu, gs} Little
{gc, pu, fd, gs} Half	{gc, pu, fd} [Little, Half]	{gc, pu, fd, gs} Little

which is rather more precise than before. We could, of course, add in the third mass assignment to make use of the fact that if the patient has recently suffered a weight loss then we should have much belief in his having gastric cancer. The reader is encouraged to check that this yields $\text{Bel}(\{gc\}) = [\text{Little}, \text{Much}]$ while all other sets of hypotheses have belief Little. This method would work equally well for larger sets of subintervals, or for any set of combinations of subintervals.

Another way of reasoning with linguistic quantifiers is to “pre-compile” the results of all possible assignments of linguistically quantified mass assignments. To demonstrate the idea, we deal with the case of mass assignments where $m(\cdot)$ assigns belief to a single subset of Θ, and we have just two such assignments. The concept can be extended, but at too great a length for inclusion in this paper. For two mass assignments m_1 and m_2 each with a single focal element F_1 and F_2, such that $m_1(F_1) = M_1$, $m_2(F_2) = M_2$, there are four sets to which the combined mass is assigned; $F_1 \cap F_2$, F_1, F_2 and Θ. These have belief masses $M_1.M_2$, $M_1.(1 - M_2)$, $(1 - M_1).M_2$ and $(1 - M_1).(1 - M_2)$ respectively, assigned to them. The set of hypotheses (which can under the open world assumption include the hypothesis $\emptyset$ indicating something outside Θ) which has the largest belief is the one preferred by the evidence. With a set of linguistic labels, we can compute the most believable set of hypotheses for every possible mass assignment, and the belief in that hypothesis. We have:

Mass of focal elements	Preferred Hypothesis	Mass of focal elements	Preferred Hypothesis
$m_1(F_1)$ = all $m_2(F_2)$ = all	$Bel(F_1 \cap F_2)$ = all	$m_1(F_1)$ = all $m_2(F_2)$ = much	$Bel(F_1 \cap F_2)$ = much
$m_1(F_1)$ = all $m_2(F_2)$ = half	$Bel(F_1 \cap F_2)$ = half $Bel(F_1)$ = half	$m_1(F_1)$ = all $m_2(F_2)$ = few	$Bel(F_1)$ = much
$m_1(F_1)$ = all $m_2(F_2)$ = none	$Bel(F_1)$ = all	$m_1(F_1)$ = much $m_2(F_2)$ = much	$Bel(F_1 \cap F_2)$ = [half, all]
$m_1(F_1)$ = much $m_2(F_2)$ = half	$Bel(F_1 \cap F_2)$ = [little, half] $Bel(F_1)$ = [little, half]	$m_1(F_1)$ = much $m_2(F_2)$ = little	$Bel(F_1)$ = [half, much]
$m_1(F_1)$ = much $m_2(F_2)$ = none	$Bel(F_1)$ = much	$m_1(F_1)$ = half $m_2(F_2)$ = half	$Bel(F_1 \cap F_2)$ = [little, half] $Bel(F_1)$ = [little, half] $Bel(F_2)$ = [little, half] $Bel(\Theta)$ = [little, half]
$m_1(F_1)$ = half $m_2(F_2)$ = little	$Bel(F_1)$ = [little, half] $Bel(\Theta)$ = [little, half]		
$m_1(F_1)$ = half $m_2(F_2)$ = none	$Bel(F_1)$ = half $Bel(\Theta)$ = half	$m_1(F_1)$ = little $m_2(F_2)$ = little	$Bel(\Theta)$ = [half, most]
$m_1(F_1)$ = few $m_2(F_2)$ = none	$Bel(\Theta)$ = most	$m_1(F_1)$ = none $m_2(F_2)$ = none	$Bel(\Theta)$ = all

Thus when Old Bull Hubbard attends the clinic with symptoms that indicate that $m_1(\{fd, gs, pu\})$ = All and $m_1(\{fd, gc\})$ = Few we can say that the most likely diagnosis is {fd, gs, pu} and that we have much belief in this.

7. RELATIVE ORDERS OF MAGNITUDE

This exhaustive analysis of the different outcomes of the application of Dempster's rule suggests that it might be possible to extend the approach. Rather than being able to predict which will be the most likely set of hypotheses when the weight of the evidence is placed into some interval, surely it is possible to carry out a more precise analysis to determine the conditions on the masses assigned to two sets of hypotheses so that, for instance, the intersection of their focal elements is the most credible set of hypotheses? In other words, what are the conditions upon M_1 and M_2 such that the most credible set of hypotheses is $F_1 \cap F_2$? Or, for that matter, F_1, F_2 or Θ? Once again, we are constrained by the space available to restrict our attention to the combination of two mass functions each with a single focal element, but the analysis may be simply extended to more complex cases. For the restricted case we have the following.

It is clear that $F_1 \cap F_2$ is one of the most credible hypotheses if belief in it is greater than or equal to the belief in F_1, F_2 or Θ. This will be the case if $M_1 . M_2 \geq M_1 . (1 -$

M_2), $(1 - M_1). M_2$ and $(1 - M_1). (1 - M_2)$. Thus we can say that $F_1 \cap F_2$ is one of the most credible hypotheses if:

$$M_1 \geq 0.5 \text{ and } M_2 \geq 0.5$$

while F_1 is one of the most credible hypotheses if:

$$M_1 \geq 0.5 \text{ and } M_2 \leq 0.5$$

From these and similar calculations, we can determine a set of rules that specify the result of combining a pair of mass functions based only on the relative sizes of the masses distributed.

IF $M_1 \geq 0.5$ and $M_2 \geq 0.5$
THEN $F_1 \cap F_2$ is one of the preferred hypotheses.

IF $M_1 \geq 0.5$ and $M_2 \leq 0.5$
THEN F_1 is one of the preferred hypotheses.

IF $M_2 \geq 0.5$ and $M_1 \leq 0.5$
THEN F_2 is one of the preferred hypotheses.

IF $M_1 \leq 0.5$ and $M_2 \leq 0.5$
THEN Θ is one of the preferred hypotheses.

Clearly, these rules are not mutually exclusive, and we can have sets of preferred hypotheses. Note also that the rules are very similar to the *specified* form of rules for combining evidence discussed by Cohen *et al.* [1989], while the use of the relative size of the masses is reminiscent of absolute order of magnitude reasoning as discussed by Dubois and Prade [1989].

To illustrate the kind of reasoning that it is possible to perform with these rules, consider what happens when Cody visits the clinic. Cody's symptoms fall into two groups, one of which suggests that he is suffering from $\{fd, gs\}$ and the other of which suggests that he has $\{gs, pu\}$. While it is not possible to put precise numbers on the degree to which the symptoms suggest the sets of diseases, the physician who examines Cody is confident that the belief mass that she assigns to the set $\{fd, gs\}$ is at least 0.5, and she is even more sure that the second set of symptoms indicate $\{gs, pu\}$. Thus the first rule may be applied to obtain the fact that the most credible diagnosis of Cody's problem is that he is suffering from $\{fd, gs\} \cap \{gs, pu\} = \{gs\}$. In other words the disease that it is most believable that Cody is suffering from is gallstones.

8. SUMMARY

This paper has introduced a number of different ways in which the Dempster-Shafer theory of evidence may be applied when precise numerical weights are not given for the various pieces of evidence. Firstly, the idea of a completely qualitative theory of evidence was introduced. In this approach, all numbers are abstracted away to be replaced by the qualitative values + and 0. This strips the theory down to its bare bones, which may still prove useful in identifying which hypotheses are indicated by the evidence in situations where numerical weights may not easily be identified. By assuming that the smallest set of hypotheses is the most likely set, a sort of single fault hypothesis, it is possible to combine evidence to identify the most likely hypothesis. Adopting the heuristic approach of the improper linear model allows us to choose between several smallest sets. The paper also discussed two ways of using the Dempster-Shafer theory with limited numerical information. Firstly, the idea of using "linguistic quantifiers" in the sense of [Dubois *et al.* 1992] was introduced, and results given for the combination of a pair of consonant belief functions. Secondly, a similar analysis was performed for belief functions where the size of the mass assignments are known relative to one another, so that it is possible to tell, for instance, that hypothesis X is more likely that hypothesis Y since belief mass M is greater than belief mass N.

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