cisc3665
game design
fall 2011
lecture # III.1
game physics and simple motion

topics:

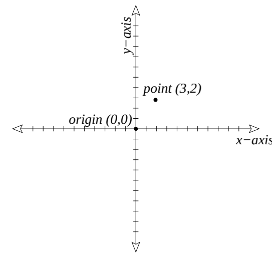
- game physics
- simple motion

references:

- notes on math from: *Programming Game AI by Example*, by Mat Buckland. Worldware Publishing (2005), chapter 1.

Cartesian coordinate system

- origin
  - where x-axis crosses the y-axis, in a 2-dimensional world; or where x-axis, the y-axis and the z-axis all intersect, in a 3-dimensional world
  - known as the point $(0,0)$ (or "coordinate pair"), in a 2-dimensional world; or as point $(0,0,0)$, in a 3-dimensional world



example 2-dimensional Cartesian coordinate system

functions and equations

- a *function* expresses the relationship between 2 (or more) *variables*. typically a function is written in the form of an *equation*—two things on either side of an equals sign (=)
- a typical equation has one variable on the left side of the equals sign, and an expression containing another variable on the other side of the equals sign; for example:

$$y = mx + b$$

- in this case, y is called the *dependent* variable, because its value depends on the value of x (the independent variable). this is a *single-variable* function, because there is only one independent variable.
- the values m and b are called *constants*. they may also be called *coefficients*. their values do not change.
- thus, given any value of x, and constant values for m and b, you can use algebra to determine the value of y. for example, if:

$$y = 2x + 1$$

and $x = 3$, then

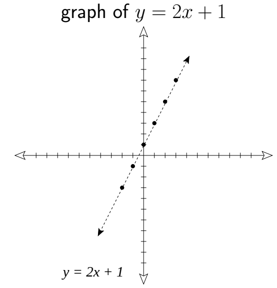
$$y = 2(3) + 1 = 7$$

- you can create a graph of the function by computing some of the values for y, given selected values of x. for example:

table of (x, y) values

x	$2x+1$	y
0	$2(0)+1$	1
1	$2(1)+1$	3
2	$2(2)+1$	5
3	$2(3)+1$	7
-1	$2(-1)+1$	-1
-2	$2(-2)+1$	-3

graph of $y = 2x + 1$



- you might see the same thing written as $f(x) = 2x + 1$, in which case the notation $f(x)$ takes the place of y
- *NOTE: for simplicity, we fill focus only on 2-dimensional environments in this class.*

exponents and powers

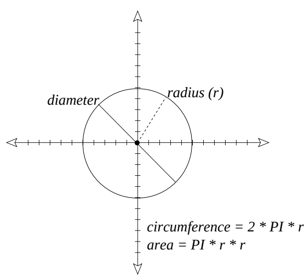
- the equation:
$$y = a^x$$
means "a raised to the power x", where a is called the "base", and x is called the "power", or the "exponent". the equation means $a \times a \times \ldots \times a$, x times; i.e.,
$$y = a^5 = a \times a \times a \times a \times a$$
- the equation:
$$y = \sqrt[x]{a}$$
means "the x root of a". this is sometimes also called a *radical*. it can also be written as:
$$y = a^{1/x}$$
and it means: $a = y \times y \times \ldots \times y$, x times.
- for example:
$$2^3 = 8$$
and
$$\sqrt[3]{8} = 8^{1/3} = 2$$

solving equations

- you'll probably need to solve some equations when creating games. so, try to create equations where you only have to solve for (find the value of) one variable
- *a handy rule: you can add, subtract, divide or multiply terms to either side, as long as you do the same thing on both sides at the same time.*
- for example, you want to solve for y in the following equation:
$$9y = 3x + 27$$
divide all terms on both sides by 9, to get y by itself on the left side of the equation:
$$\begin{aligned} (y/9) &= (3x/9) + (27/9) \\ y &= (1/3)x + 3 \end{aligned}$$
- *some more handy rules:*
$$\begin{aligned} x/y &= (1/y) \times x \\ a/x + b/y &= (ay + bx)/xy \\ (x+y)^2 &= x^2 + 2xy + y^2 \\ (x/y)^2 &= x^2/y^2 \\ \sqrt{x/y} &= \sqrt{(x)}/\sqrt{(y)} \end{aligned}$$

circles

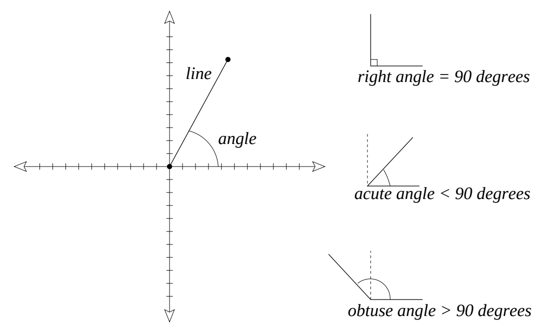
- a *circle* is defined by its *center* and its *radius*



- the *circumference* (or "perimeter", i.e., distance around the edge) of a circle is $2\pi r$
- the $diameter$ of a circle is equal to $2r$
- a circle inscribes an arc of 2π *radians* or $360°$ (degrees)
thus, $1° = 2\pi/360 = \pi/180$ radians

angles

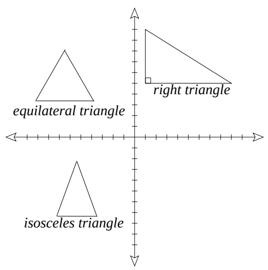
- an *angle* is defined by two *lines* that share one *endpoint*



- angles are measured in *degrees* or *radians*

triangles

- a triangle is defined by three points

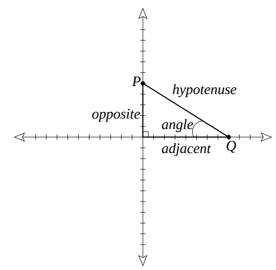


  - the *perimeter* of a triangle is the sum of the length of its 3 sides
  - the *area* of a triangle is $1/2 \times base \times height$
  - the sum of the *angles* of any triangle $= 180°$

- we note three special kinds of triangles:
  - *equilateral* (all sides are the same length, all angles are $60°$)
  - *iscosceles* (two sides are the same length)
  - *right triangle* (one of the angles is $90°$)

trigonometry

- *trigonometry* is largely based on the special properties of a right triangle and functions that are defined using those properties



given h = hypotenuse (the side opposite the right angle). select angle α and sides a (the side adjacent to α) and b (the side opposite to α). then we define:

$$\begin{aligned} sin(\alpha) &= opposite/hypotenuse &= b/h \\ cos(\alpha) &= adjacent/hypotenuse &= a/h \\ tan(\alpha) &= opposite/adjacent &= b/a \end{aligned}$$

and their inverses, to find the angle α:

$$\begin{aligned} \alpha &= sin^{-1}(b/h) &= asin(b/h) \\ \alpha &= cos^{-1}(a/h) &= acos(a/h) \\ \alpha &= tan^{-1}(b/a) &= atan(b/a) \end{aligned}$$

- the Pythagorean Theorem says that $h^2 = a^2 + b^2$
  which gives us a good way to find the straight-line distance between two points (e.g., P and Q in the example)

useful equations

- rectangle:
  $perimeter = 2 \times length + 2 \times width$
  $area = length \times width$
- circle:
  $circumference = perimeter = 2 \times \pi \times radius$
  $area = 2 \times \pi \times radius^2$
- slope of a line:
  $slope = rise/run = \Delta y/\Delta x = (y_2 - y_1/x_2 - x_1)$

to do

- read ch 1 of Buckland book (handout)
- work on assignment for unit II (labII.1), which was due on Oct 9—AND HAS NOW BEEN EXTENDED TO OCT 14