cisc3665
game design
fall 2011
lecture # V.1
game balance and game theory

topics:

- game balance
- game theory

references:

- notes from:
  - *On Game Design*, by Andrew Rollings and Ernest Adams. New Riders Publishing / Pearson Education, 2003, chapter 8.
  - *An Introduction to MultiAgent Systems*, by Michael Wooldridge, Wiley, 2009 (2nd edition), chapter 6.
- additional notes from:
  - Prof Simon Parsons, CUNY Brooklyn College
  - Prof Michael Wooldridge, University of Liverpool, UK

game balance

- a *balanced* game is a game where the skill level of a player is the main factor that determines the player's success at the game
- *static* vs *dynamic* balance
- static balance:
  - balance does not change during the course of the game
  - properties:
    - *dominant strategy:*
      the strategy with the highest likelihood of winning
    - *symmetry:*
      there are no player handicaps in the game
    - *transitivity:*
      if $A \succ B$ and $B \succ C$, then $A \succ C$
      Note on notation: $\succ$ means "beats"
      A means "player A", B means "player B", and so on
    - *intransitivity:*
      games where $A \succ B$ and $B \succ C$ and $C \succ A$...

    - *emergence:*
      simple rules produce in complex results
      (example: Prisoner's Dilemma, discussed below)
    - *feedback loops:*
      - *positive feedback:*
        the more points you get, the easier it is to get more points; e.g., the rich get richer while the poor get poorer
      - *negative feedback:*
        the more points you get, the harder it is to get more points
- dynamic balance:
  - balance changes during the course of the game
  - i.e., game balance may "tip" in favor of one player or another; and there is a need to be able to bring the game back to *equilibrium*, where players have equal, unbiased chances of winning (this is expressed as a "need", in order to keep players interested in the game)

example game: rock-paper-scissors

- this is a 2-player game
- there are three possible *moves* in the game: *Rock*, *Paper* and *Scissors*
- each player makes their move simultaneously
- and then the moves are compared to determine the result, according to the following rules:
  - *Rock* $\succ$ *Scissors*
  - *Scissors* $\succ$ *Paper*
  - *Paper* $\succ$ *Rock*
- this is an *intransitive* game, since:
  Rock $\succ$ *Scissors* $\succ$ *Paper* $\succ$ *Rock*...

game theory

- game theory is a field studied in economics, mathematics and computer science
- concepts:
  - strategy
  - payoff matrix
  - Nash equilibrium
- classic sample games:
  - Prisoner's Dilemma
  - Stag Hunt
  - Chicken

strategy

- a *strategy* is the set of internal rules that an agent (or human player) uses to decide what moves to make in a game
- if you are creating an agent player for a game, then that agent's behavior in the game is governed by its strategy
- for example, if you were creating a Tic-Tac-Toe agent, you might say that the agent's opening move strategy is to go in the middle if they go first
- some strategies are good, and lead to the player winning games
- other strategies are less good...
- typically, we measure the effectiveness of a strategy by how well a player does when playing a game using that strategy

payoff matrix

- we can measure the effectiveness of a strategy using a *payoff matrix*
- this is a table that shows the points that a player would earn in a game if s/he makes a particular move when his/her opponent makes their move
- by definition, a payoff matrix is a 2-dimensional table and is used to express the payoffs for 2-player games
- an example is shown below, where each player can either *cooperate* or *defect*:

		column player	
		defect	cooperate
row player	defect	$1,1$	$1,4$
	cooperate	$4,1$	$4,4$

- each cell represents the reward (or number of points) given to each player, according to the moves they make; the first reward in the pair goes to the *row player* and the second reward goes to the *column player*
- for example, if the row player's move is "cooperate" and the column player's move is "defect", then the row player's reward is 4 points and the column player's reward is 1 point

dominant strategy

- a strategy, s_1, *dominates* another strategy, s_2, if every outcome possible by a player playing s_1 is better than every outcome possible by the player playing s_2.
- thus we can analyze the game on the previous slide:
  - if the row player defects and the column player defects, then the result will be a tie
  - if the row player defects and the column player cooperates, then the row player will lose
  - if the row player cooperates and the column player defects, then the row player will win
  - if the row player cooperates and the column player cooperates, then they will tie
- thus the dominant strategy for the row player is to cooperate—because s/he will always either win or tie (whereas if s/he defects, s/he will either tie or lose)
- note that the dominant strategy is the same for the column player, because the payoffs in this game are symmetrical
- we say that a "rational agent" will always play the dominant strategy, if one exists
- unfortunately, there is not always a unique dominant strategy
- actually, maybe this is better—because otherwise game play would be boring!

Nash equilibrium

- named for John Forbes Nash, who was a famous professor at Princeton
- in general, we say that two strategies s_1 and s_2 are in Nash Equilibrium (NE) if:
  1. under the assumption that the row player uses s_1, then the column player can do no better than use s_2; and
  2. under the assumption that the column player uses s_2, then the row player can do no better than to use s_1.
- key principle: *neither agent has any incentive to deviate from a NE*
- for example, consider the following payoff matrix:

		column player	
		defect	cooperate
row player	defect	$3, 5$	$2, 1$
	cooperate	$2, 0$	$1, 0$

- here, the Nash Equilibrium is *defect,defect*
- in a game like this, you can find the NE by cycling through the outcomes, asking if either player can improve its payoff by switching its strategy

- for example, from the point of view of the row player:
  - if the row player defects and the column player defects, then the row player $\leftarrow 3$
  - if the row player defects and the column player cooperates, then the row player $\leftarrow 2$
  - if the row player cooperates and the column player defects, then the row player $\leftarrow 2$
  - if both players cooperate, then the row player $\leftarrow 1$
  - so no matter what the column player does, the row player does better by defecting (payoff of 3 or 2, versus 2 or 1)

  and from the point of view of the column player:
  - if the row player defects and the column player defects, then the column player $\leftarrow 5$
  - if the row player defects and the column player cooperates, then the column player $\leftarrow 1$
  - if the row player cooperates and the column player defects, then the column player $\leftarrow 0$
  - if both players cooperate, then the column player $\leftarrow 0$
  - so no matter what the row player does, the column player does better by defecting (payoff of 5 or 0, versus 1 or 0)

multiple NE

- some games have more than one "pure strategy" NE (pure strategy means that a player does not change its strategy during the game, e.g., it always cooperates or always defects)
- for example, the game below has two NEs: *cooperate,cooperate* and *defect,defect*

		column player	
		defect	cooperate
row player	defect	$3, 5$	$2, 1$
	cooperate	$2, 0$	$3, 3$

- in both cases, a single player cannot improve its reward on its own

no NE

- some games do not have any pure strategy NEs

		column player	
		defect	cooperate
row player	defect	$1, 2$	$2, 1$
	cooperate	$2, 0$	$1, 1$

- for every outcome, one of the players will improve its reward by switching its strategy

mixed strategy

- a *mixed strategy* implies that a player changes its strategy during the game
- e.g., sometimes it cooperates and sometimes it defects
- remember *Rock-Paper-Scissors*
- when you play that game, do you always play *Rock*? or *Paper*? or *Scissors*? OR do you change your move randomly? OR do you change your move according to what you guess your opponent will do?

classic game theory games

- let's look at three class games from the game theory literature
- these are all 2-player, 2-move games
- as with the games we looked at above, the two moves are labeled in the literature as "cooperate" and "defect"
- now you will see why, with the first game
- the games we will look at are:
  1. Prisoner's Dilemma
  2. Stag Hunt
  3. The Game of Chicken

Prisoner's Dilemma

- here is the back story for the game of *Prisoner's Dilemma*:

  Two men are collectively charged with a crime and held in separate cells, with no way of meeting or communicating. They are told that:
  - if one confesses and the other does not, the confessor will be freed and the other will be jailed for three years;
  - if both confess, then each will be jailed for 2 years.

  Both prisoners know that if neither confesses, then they will each be jailed for 1 year.
- here is the payoff matrix for Prisoner's Dilemma:

		column player	
		defect	cooperate
row player	defect	$2, 2$	$4, 1$
	cooperate	$1, 4$	$3, 3$

- is there a Nash Equilibrium?
- for example, from the point of view of the row player:
  - if the row player defects and the column player defects, then the row player $\leftarrow 2$
  - if the row player defects and the column player cooperates, then the row player $\leftarrow 4$
  - if the row player cooperates and the column player defects, then the row player $\leftarrow 1$
  - if both players cooperate, then the row player $\leftarrow 3$
  - so no matter what the column player does, the row player does better by defecting (payoff of at least 2, versus of at least 1)

  and from the point of view of the column player:
  - if the row player defects and the column player defects, then the column player $\leftarrow 2$
  - if the row player defects and the column player cooperates, then the column player $\leftarrow 1$
  - if the row player cooperates and the column player defects, then the column player $\leftarrow 4$
  - if both players cooperate, then the column player $\leftarrow 3$
  - so no matter what the row player does, the column player does better by defecting (payoff of at least 2, versus of at least 1)

- so what should you do?
  - the *individual rational* action is *defect*: this guarantees a payoff of at least 2, whereas cooperating guarantees a payoff of at least 1
  - so defection is the best response to all possible strategies: both agents defect, and get payoff $= 2$
  - but *intuition* says this is *not* the best outcome: Surely they should both cooperate and each get payoff of 3!
  - this is why the Prisoner's Dilemma game is interesting — the analysis seems to give us a paradoxical answer.
- solution summary:
  - there is no dominant strategy (in the game theory sense)
  - *defect,defect* is the only Nash Equilibrium
  - *cooperate,cooperate* maximises *social welfare*, i.e., the combined, or "group" reward

Stag Hunt

- here is the back story for the *Stag Hunt* game:

  A group of hunters goes stag hunting. If they all chase the stag, they will catch it and all have a lot of food. If some of them instead decide to catch rabbits, the stag will escape. In this case the rabbit hunters will have some small amount of food and the (remaining) stag hunters will go hungry. What should each hunter do?
- here is the payoff matrix for Stag Hunt:

		column player	
		defect	cooperate
row player	defect	$2, 2$	$3, 1$
	cooperate	$1, 3$	$4, 4$

- the difference between Stag Hunt and Prisoner's Dilemma is that now it is better for both players to cooperate than if either player defects
- there are two Nash Equilibrium solutions in this game: *cooperate,cooperate* and *defect,defect*
- social welfare is maximized by *cooperate,cooperate*

the Game of Chicken

- this game is exemplified in movies like "Rebel without a Cause" or "American Graffiti"
- here is how the game is played:

  Two players drive their cars towards each other. Each player can continue to drive straight head (toward the other car) or can swerve to miss the other car. If either player swerves, then nobody dies. If neither player swerves, then both players die.
- the idea is that the player(s) who swerve(s) loses "face" and is considered not as brave as a player who does not swerve...
- here is the payoff matrix for the Game of Chicken:

		column player	
		defect	cooperate
row player	defect	$1, 1$	$4, 2$
	cooperate	$2, 4$	$3, 3$

- the difference between Chicken and Prisoner's Dilemma is that mutual defection is the worst outcome; whereas the "sucker's payoff" is the worst outcome in Prisoner's Dilemma

- there is no dominant strategy (in the game theory sense)
- there are two NE: *cooperate,defect* and *defect,cooperate*:
  - If I think you will drive straight, I should swerve.
  - If I think you will swerve, I should drive straight.
- all outcomes except *defect,defect* maximize social welfare

other symmetric 2x2 games

- given the 4 possible outcomes of (symmetric) 2-move (e.g., cooperate/defect) games, there are 24 possible orderings on outcomes
- below are some examples with dominant solutions:
  - cooperation dominates:
    * $CC \succ_i CD \succ_i DC \succ_i DD$
    * $CC \succ_i CD \succ_i DD \succ_i DC$
  - defecting dominates:
    * $DC \succ_i DD \succ_i CC \succ_i CD$
    * $DC \succ_i DD \succ_i CD \succ_i DD$
  - Prisoner's Dilemma:
    * $DC \succ_i CC \succ_i DD \succ_i CD$
  - Game of Chicken:
    * $DC \succ_i CC \succ_i CD \succ_i DD$
  - Stag Hunt:
    * $CC \succ_i DC \succ_i DD \succ_i CD$

to do

- read chapter 8 from *On Game Design*