cis32 homework # 3

sklar/spring-2006

- The homework is due in class on **Wednesday April 26**.
- This homework is worth **9 points** in total. The distribution of points is indicated below.

1. *(*1 point)
   Using the following propositions:

p	It is raining
q	I have an umbrella
r	I get wet

   formulate the following expressions in words:

   (a) $p \wedge (q \vee r)$

   (b) $\neg p \vee r$

2. *(*1 point)
   Write the truth table for:
   $((p \wedge (q \Rightarrow r)) \vee s) \wedge t$

3. *(*2 points)
   For each of the following propositional logic formulae, determine whether it is: a *tautology* (always true), *consistent* (true for some domains), *inconsistent* (never true).

   (a) $p \vee q \vee \neg r$

   (b) $(p \wedge q) \Rightarrow r$

   (c) $p \Rightarrow (q \Rightarrow p)$

   (d) $(p \wedge (q \vee p) \Leftrightarrow \neg p$

4. *(*1 point)
   Convert the following sentences to predicate logic form:

   (a) Every cloud has a silver lining.

   (b) Nobody knows the trouble I seen.

5. *(*4 points)
Using the proof rules in the lecture notes and those given below, try to prove the following:

(a) $(p, p \Rightarrow (q \wedge r)) \vdash (p \vee r)$

(b) $(p \wedge (p \Rightarrow (q \wedge r))) \vdash (p \vee r)$

Some proof rules that aren't in the lecture notes are:

$$\frac{\vdash \phi \Leftrightarrow \psi}{\vdash \phi \Rightarrow \psi; \vdash \psi \Rightarrow \phi} \Leftrightarrow\text{-E}$$

$$\frac{\vdash \phi \Rightarrow \psi; \vdash \psi \Rightarrow \phi}{\vdash \phi \Leftrightarrow \psi} \Leftrightarrow\text{-I}$$

$$\frac{\vdash \neg\neg\phi}{\vdash \phi} \neg\text{-E}$$

$$\frac{\phi \vdash \bot}{\vdash \neg\phi} \neg\text{-I}$$

For the last of these rules, remember that $\bot$ stands for any formula which is inconsistent, i.e., false (for example $\phi \wedge \neg\phi$).